GEOACOUSTIC INVERSION OF REFLECTION DATA IN A WATER TANK

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1 Introduction

Geoacoustic inversion surveys conducted in ocean environments often require expensive resources and challenging logistics and sometimes involve computationally intensive calculations. For example, moored or towed horizontal arrays or vertical arrays of hydrophones are often employed and complicated setups can introduce operational difficulties and increase costs in shallow-water inversion methods such as matched-field inversion, reverberation inversion, transmission-loss inversion, etc. However, inversion of reflectivity data can be carried out by employing a single receiver, and holds considerable promise owing to more convenient instrument deployments and recoveries and less time-consumption for forward-model computations as full-wave calculations may not be required.

This paper describes a nonlinear Bayesian inversion approach applied to broadband reflection data [1] at multiple frequencies for geoacoustic properties of river sand at the base of an anechoic water tank at Harbin Engineering University, China.

2 Bayesian Inversion Theory and Algorithms

2.1 Bayesian formulation

The nonlinear approach to invert reflectivity data for geoacoustic properties is based on Bayes rule, which can be expressed as [1]

$$P(\mathbf{m}|\mathbf{d},F)=\frac{P(\mathbf{d}|\mathbf{m},F)P(\mathbf{m}\mid F)}{P(\mathbf{d}\mid F)}=\frac{P(\mathbf{d}|\mathbf{m})P(\mathbf{m})}{P(\mathbf{d})}\ . \quad (1)$$

The goal of Bayesian inversion is to interpret the multi-dimensional posterior probability density (PPD), $P(\mathbf{m}|\mathbf{d},F)$, for optimal estimates of the model parameters $\mathbf{m}$ and their uncertainties based on observed data $\mathbf{d}$ and prior information $P(\mathbf{m})$; here F represents the choice of model parameterization (e.g., the number of sediment layers). These properties include the maximum *a posteriori* (MAP) model and marginal probability distributions etc, which are defined, respectively, as

$$\hat{\mathbf{m}}=\mathrm{Arg}_{\max}\left\{P(\mathbf{m}|\mathbf{d})\right\} \quad (2)$$

$$P(m_i\mid\mathbf{d})=\int\delta\left(m_i-m_i'\right)P(\mathbf{m}|\mathbf{d})d\mathbf{m}'\ . \quad (3)$$

Joint (two dimensional) marginal densitites can be computed analogously to Eq. (3). Parameter uncertainties are also quantified in terms of credibility intervals. For example, the highest probability density credibility interval represents the interval of minimum width containing of the area of the marginal distribution (50% and 95% credibility intervals are used in this paper).

2.2 Numerical Optimization

In this paper, delayed rejection adaptive Metropolis (DRAM), a Markov Chain Monte Carlo (MCMC) sampling method, is applied to nonlinear Bayesian inversion of reflectivity data. It can provide an effective numerical approach to determine MAP estimates and to evaluate integrals Eq. (2)-Eq. (3) to quantify parameter estimates, uncertainties and inter-relationships [2].

DRAM is a combination of two ideas for improving the efficiency of Metropolis-Hastings type MCMC algorithms, i.e. adaptive Metropolis (AM) and delayed rejection (DR). In adaptive Metropolis, the covariance matrix of a Gaussian proposal distribution is adapted on the history of the Markov chain. In delayed rejection, upon rejecting a proposed candidate model, a second (smaller) move is proposed instead of simply rejecting the initial move. Moreover, the acceptance probability of the second candidate move is formulated so as to preserve reversibility of the Markov chain. Once convergent MCMC chains are established, the MAP parameter estimates can be determined by Eq. (2), while parameter uncertainties and inter-relationships will be further analyzed according to Eq. (3).

2.3 Model Choice

Choosing an appropriate model parameterization F is an important aspect for Bayesian reflectivity inversion. For parameterized models, measuring the probability of observed data arising under the assumed model is an effective way to estimate the most appropriate model parameterization. The Bayesian information criterion (BIC) is applied here to approximate Bayesian evidence in terms of minimizing

$$\mathrm{BIC}=-2\log P(\mathbf{d}|\mathbf{m}^*,F)+M\log N\ . \quad (4)$$

3 Inversion Results and Analysis

3.1 Model Selection Results

MAP models were estimated by DRAM for model parameterization including 1, 2, and 3 layers (including a halfspace). The results of the model choice are shown in Figure 1. Figure 1(a) shows the misfit function (negative log likelihood) decreases rapidly with layers changing from 1 to

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2, while decreases much less in going from 2 to 3 layers. Considering the penalty term for the number of parameters for every possible model shown in Figure 1 (b), Figure 1 (c) indicates BIC has its minimum at 2 layers, showing this is the maximum number of layers that can be reliably estimated from the observed data.

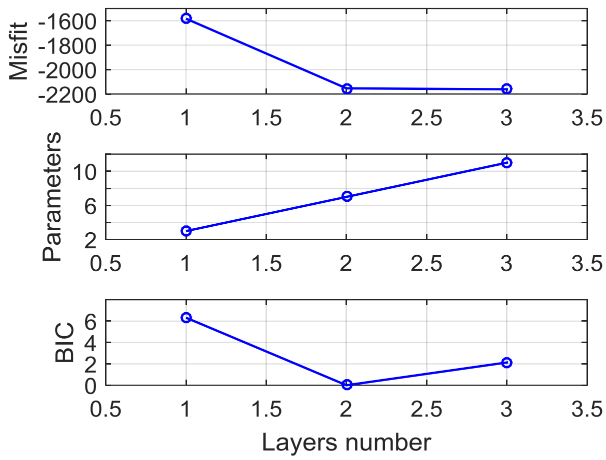

Figure 1: Results of the model choice in terms of: (a) misfits, (b) number of parameters, (c) BIC as a function of layers numbers with its value shifted to unity at its minimum and shown in log scale for display purposes.

3.2 Inversion Results

Figure 2 shows good agreement between the observed data and data predicted for the inversion results at 31.5 kHz, with 50% and 95% credibility intervals quantified. Figure 3 shows joint marginal probability distributions of parameters of interest for the two-layer model to illustrate parameter uncertainties and inter-parameter relationships.

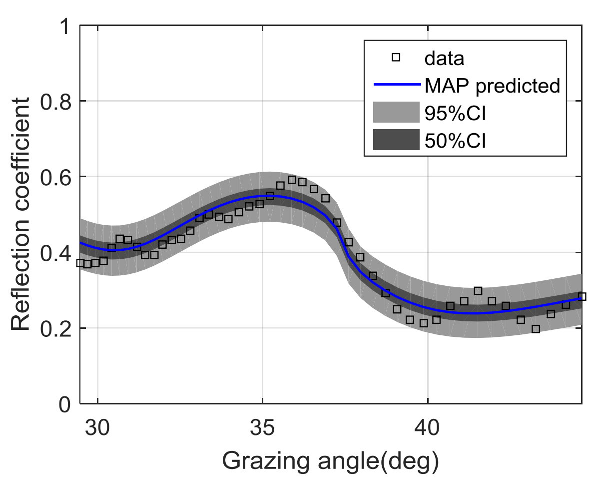

Figure 2: Measured reflection coefficient data (squares) and predicted data for the MAP model (line) and 50% and 95% credibility intervals from the inversion at 31.5 kHz.

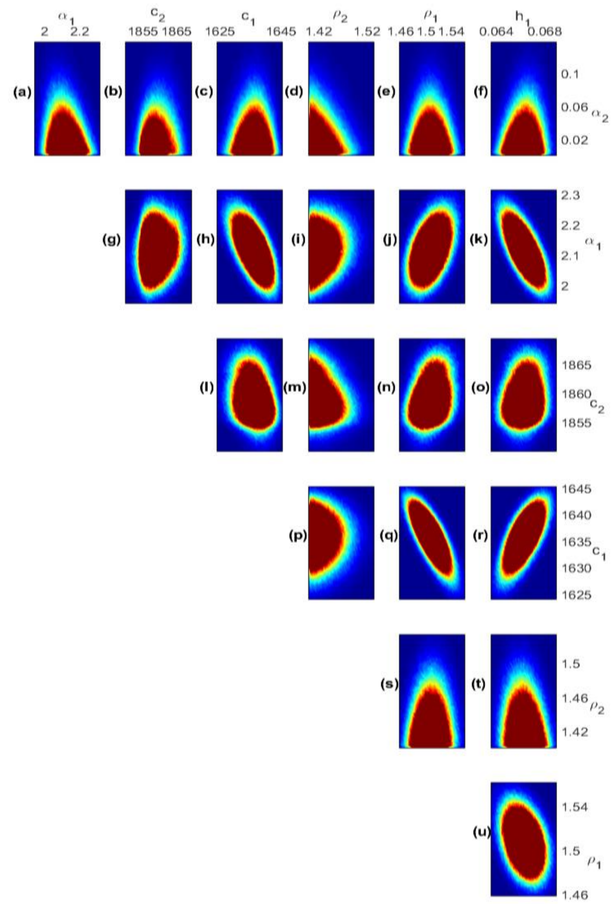

Figure 3: Joint marginal probability distributions of parameters of interest

4 Summary and Discussion

This paper presented a nonlinear Bayesian inversion for reflection coefficient data to estimate geoacoustic properties and uncertainties of sand layers at the base of a water tank. The MAP inversion results for the first layer suggested that this layer is clayey sand which is consistent with a visual observation.

References

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