Operational Transfer Path Analysis: Practical Considerations For Selecting Sensor Positions

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1 Introduction

Operational transfer path analysis (OTPA) is an alternative to classical transfer path analysis (TPA) as a method used to predict the noise or vibration source/path contributions to the response of a system. While the classical TPA method uses a known input to compute frequency response functions and contributions at the receiver; the OTPA method uses operational measurable quantities to compute both the transmissibilities and the response at the receiver.

Although OTPA is currently used predominantly for vehicle noise, vibration and harshness (NVH) assessment, the method is useful for any noise and vibration assessment where a ranking of the source/path contributions is desired, e.g. industrial installations, building services installations, complex machinery and appliances, trains, aircraft, ships, submarines, construction equipment.

The goal of this paper is to introduce the underlying theory behind the OTPA method, as well as to highlight some practical considerations for selecting sensor positions during the OTPA setup and post-processing. The practical considerations are highlighted through the description of a case study and by recreating the results of the case study in a simple OTPA numerical simulation.

2 Background

Comparable to classical TPA, OTPA is based on a linear relationship between the source(s) and receiver(s), which can be described as:

$$\mathbf{Y}(j\omega) = \mathbf{X}(j\omega) \cdot \mathbf{H}(j\omega) \tag{1}$$

Where $\mathbf{Y}(j\omega)$ is a matrix of the output responses at the receiver measurement positions, $\mathbf{X}(j\omega)$ is a matrix of the measured quantities at input reference measurement positions (MP) and $\mathbf{H}(j\omega)$ is a matrix of the transfer functions. Important for the computation of OTPA is the setup of the matrices, where $\mathbf{X}$ and $\mathbf{Y}$ are organized such that the columns are the measurement positions (MPs) and the rows consist of blocks of measurement data.

Prior to computing the transfer function, $\mathbf{H}$, the cross-talk (i.e. the contributions to the measurement at a reference MP from noise/vibration acting at other reference MP's) must also be minimized. This is done by a singular value decomposition (SVD) of $\mathbf{X}$, which is also an efficient method to compute the least-squares estimate of the inverse of a matrix.

A principal component analysis (PCA) is then conducted where the lowest ranked principal components (PCs), which constitute measurement noise, are disregarded from the analysis. The result is a "noise removed and cross-talk cancelled" estimate of the transfer function matrix [1, 2, 5].

The statement "noise removed and cross-talk cancelled" should be taken with a degree of skepticism – many factors come into play which may impede the effectiveness of the cross-talk cancellation method [3 - 5], such as:

- Neglected sources/paths in the measurement setup
- Cross-coupling between input measurements
- Incorrect estimation of the transfer paths

2 Case study: OTPA of a road tractor

An OTPA study was conducted on an idling road tractor: Microphones were placed to cover the airborne sources and accelerometers were mounted to cover the structure-borne sources and paths. The microphone positions are shown in Figure 1, while the response measurement position was a microphone at the driver position.

The contribution analysis produced some strange results in the low frequency, specifically at 25 Hz (dominating 3rd order): The results indicated that the airborne sources were the significant contributors to the overall sound level at the response position in the tractor cab, while the structure-borne paths were insignificant contributors, at approximately 20 dB lower than the airborne contribution.

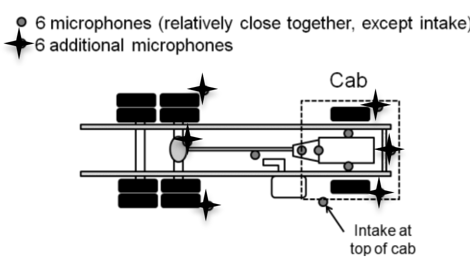

Figure 1: OTPA microphone positions

Given that the analysis was conducted at the engine idle operating condition (i.e. stationary), it was determined that several airborne reference MPs were not actually measuring any significant airborne sources/paths (refer to Figure 1, the MPs denoted as *6 additional microphones*). The OTPA post-processing was therefore repeated with the *6 additional microphone* MPs excluded from the analysis. The contribution analysis results indicated that the airborne and structure-borne contributions were equal. The total computed sound pressure level remained constant for both analysis cases.

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3 OTPA numerical simulation

To study the OTPA method with respect to the results discussed in the road tractor case study, a simplified, numerical simulation of an OTPA study was created. The purpose of the simulation was to investigate the OTPA method in a highly-controlled environment where all of the "measurement" parameters could be accurately defined.

A simple numerical simulation consisting of a source with one structure-borne path and one airborne path was created. The airborne source was modelled as a radiating spherical source of a size comparable to that of an engine, in a free field environment, and radiating a single tone at 25 Hz. Two reference microphones and one response microphone were positioned at a similar distance to the microphones for the road-tractor measurements, and one structure-borne reference MP was assumed.

The contribution analysis results indicated that an over-prediction of the airborne contribution (and under-prediction of the structure-borne contribution) occurs. The numerical simulation results represent the case where both the airborne reference MPs are at an equal distance from the source, thus they "measure" the same amplitude and phase. The calculated contribution was approximately +2.5 dB higher than the actual airborne contribution, and -3.5 dB lower for the structure-borne contribution. The total calculated response matched the actual response.

Upon review of the OTPA theory, it becomes apparent that because the reference measurements (two airborne and one structure-borne) are all fully correlated, the resulting SVD yields only one PC. When scaled, each reference MP is allotted the same contribution, and since the contribution for each path is summed, two-thirds of the contribution is presumed from the airborne path (two reference MPs, which are summed) and one-third from the structure-borne path (only one reference MP).

4 Discussion

The outcome of the OTPA simulation gives some insight into the likely cause of the erroneous contribution prediction that occurred in the road tractor OTPA case study: The additional microphone MPs were not actually measuring a significant additional source, and therefore were mainly measuring cross-talk (in this case, from the engine). Further, in the low frequency range it is likely that the structure-borne path is also highly correlated to the airborne path, thus the SVD does not effectively separate the contributions and the energy is simply spread out amongst the reference MPs, scaled by the relative amplitudes of the reference signals.

Recalling the potential sources of error for OTPA listed in section 2, the cause for the error in the road tractor case appears to be due to *cross-coupling between input measurement positions*. The results of the OTPA numerical simulation further support this conclusion.

The results also indicate that when two or more paths are highly correlated, and exhibit similar contributions at the response position, the number of reference MPs will influence the results: The paths will be weighted according to the number of reference MPs. In this case, PCA would show a strong contribution from very few PCs, which is an indication that the reference signals are highly correlated.

This highlights that consideration must be given to the physics when including MPs in the OTPA setup and post-processing. Particularly for airborne sources, the correct number of MPs and proper placement to ensure a good signal-to-noise ratio is critical. MPs that mainly measure cross-talk/noise (e.g. at a non-existent source), will lead to incorrect source contribution prediction results.

In general, it is proposed that the number of microphone positions used in the OTPA should be adapted according to frequency range: At low frequency (i.e. the size of the source is much smaller than one-sixth of the wavelength), the source radiates uniformly as a simple point source, thus fewer microphone positions are required; whereas at high frequency the source can be seen as a combination of multiple sources, and will therefore exhibit directivity in the radiation pattern, so several microphone positions are required to properly measure the source.

5 Summary and conclusion

It is important to keep in mind that the accuracy of the OTPA results depends on the correct placement and number of the sensors, and that it is advantageous to understand the system prior to setting up the measurement. A few practical considerations are summarized as follows:

- The correct number of sensors and proper placement to ensure a good signal-to-noise ratio is critical for accurate source contribution prediction results.
- The number of microphone positions per source used in the analysis should be adjusted during post-processing depending on frequency range.
- The correlation between reference measurement positions should be critically examined during the PCA.

By critically examining the OTPA data during post processing and keeping the suggestions listed above in mind, OTPA can be a convenient diagnostics tool leading to sufficiently accurate source/path contribution conclusions.

References

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