SURFACE DIPOLE STRENGTH GENERATED BY CYLINDRICAL STRUTS

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1 Introduction

The generation of noise by flow past rigid surfaces has been the subject of considerable speculation and controversy. This paper considers the spatial distribution of the acoustic dipole source strength over the surface of a rigid cylindrical strut located in a coannular cold subsonic air flow using the causality cross-correlation technique.

2 Method

2.1 Surface dipole strength by cross-correlation

The acoustic radiation $p^t(x_i,t^i)$ at receiver point x_i emanating from a region of unsteady flow containing an embedded surface is given by a simplification of Curle's generalized solution of the Lighthill equation [1]. The equation below assumes the Mach number is low, surfaces are stationary, shear stresses are negligible and $\lambda <<$ surface dimension:

$$p^{*}(x_{i},t) \stackrel{\simeq}{=} \frac{x_{i}}{4\pi x^{2}c_{\infty}} \int_{\varsigma} n_{i} \left[\frac{\partial}{\partial \hat{t}} p_{s}^{*}(y_{i},\hat{t})\right] \frac{ds(y_{i})}{\hat{t}=t-r/c_{\infty}}$$
(1)

Where n_i is the unit normal to surface S at surface point y_i , r is the distance of sound travel from any given point on surface S to the receiver x_i and c_{∞} is the celerity of sound.

The mean square pressure at x_i may be formed by multiplying equation 1 by the $p^t(x_i,t^t)$ and taking ensemble averages, or alternately, equivalent time averages assuming the flow parameters are ergodic random functions of time The result is shown in equation 2 below [2]:

$$\frac{\overline{p^{*}p^{*}}}{4\pi x^{2}c_{\infty}} \left(\begin{array}{c} x_{1} \\ s \end{array} \right) \xrightarrow{\simeq} \frac{-x_{1}}{4\pi x^{2}c_{\infty}} \left(\begin{array}{c} n_{1} \\ s \end{array} \right) \left(\begin{array}{c} \frac{\partial}{\partial \hat{\tau}} \end{array} \right) \overrightarrow{p_{1}^{*}(y_{1})p^{*}(x_{1})} \left(\begin{array}{c} \hat{\tau} \\ s \end{array} \right) \left(\begin{array}{c} ds(y_{1}) \\ \hat{\tau} = r/c_{\infty} \end{array} \right)$$
(2)

Introducing the source and receiver sound pressure levels (SPLs), the above may be written in differential form giving the distribution of surface dipole strength over surface S. The result is shown in equation 3 below:

$$\frac{d\overline{p^{*}p^{*}}}{dS} = (\mathbf{x}_{i}, \mathbf{y}_{i}) = \frac{-\mathbf{x}_{i} \mathbf{n}_{i} (\mathbf{y}_{i})}{4\pi \mathbf{x} \mathbf{c}_{\infty}} \left[\frac{\partial}{\partial \hat{\tau}} \overline{C} (\mathbf{x}_{i}, \mathbf{y}_{i}) (\hat{\tau}) \right] \qquad (3)$$

$$\hat{\tau} = \mathbf{r}/c_{\infty}$$

$$P_{Ree}^{2} \text{ antilog}_{io} \left\{ \frac{SPL(\mathbf{y}_{i}) + SPL(\mathbf{x}_{i})}{20} \right\}$$

Where $\overline{c}(x_i, y_i)$ is the dimensionless causality crosscorrelation coefficient for surface dipoles evaluated at the Kirchoff delay time $\hat{\tau}$ as shown in equation 4 below:

$$\overline{\mathbf{C}}(\mathbf{x}_{1},\mathbf{y}_{1}) \quad (\hat{\tau}) = \begin{bmatrix} \overline{\mathbf{p}_{1}^{*}(\mathbf{y}_{1})\mathbf{p}^{*}(\mathbf{x}_{1})} & (\hat{\tau}) \\ \overline{\mathbf{p}_{s}^{\mathsf{RMs}} \mathbf{p}^{\mathsf{RMs}}} \end{bmatrix} \qquad (4)$$

$$\widehat{\tau} = \mathbf{r}/\mathbf{c}_{\infty}$$

The source and far field RMS pressures may be

obtained from their respective autocorrelation functions evaluated at time zero.

2.2 Description of flow

Potential flow issuing from a coannular cold jet of air incident on a cylindrical strut separates from the strut at point s shown in Figure 1. The flow velocity at the jet nozzle exit plane was 72 m/s and the Reynolds number based on strut diameter was 6.4×10^4 for the analysis herein. Flow separation at s induces a periodic side force at the Strouhal vortex shedding frequency of 1.13 kHz [3].



Figure 1: Incident potential flow separates at s (Source unknown)

2.3 Procedure to evaluate surface dipole strength over the surface of the strut

A strut measuring 12.7 mm in diameter and 170 mm in length and was drilled out to accommodate a one quarter inch Brüel & Kjær (B&K) Condenser Microphone Type 4136 connected to a B&K Preamplifier Type 2618 forming a source region probe. The microphone cartridge was sealed in the small cavity by an O-ring, The O-ring was positioned between the microphone membrane and the relief vent as shown in Figure 2.



Figure 2: Cylindrical strut with ¹/₄ inch microphone probe

The air-filled cavity in front of the membrane was coupled to the surface of the cylindrical strut by a tiny capillary tube terminating at a surface pinhole. The tube measured 0.91 mm (0.036 in) in diameter at the surface of the strut and was 3.18 mm in length. The probe configuration essentially behaved as a Helmholtz resonator. These two Helmholtz parameters and volume were

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optimized to extend the high frequency response of the probe and minimize the phase shift at the highest frequency of interest at the receiver (measured to be 4000 Hz, $Re = 6.3 \times 10^4$). Cotton fibers were added to dampen the amplitude of the resonance peak. It was found the cylindrical strut fitted with a one eighth inch B&K Type 4138 microphone performed slightly better than a one quarter inch B&K Type 4135 microphone probe and introduced less phase shift (approximately one degree at 4000 Hz).

The pinhole on the strut surface was moved into different positions in the flow of the potential core of a cold air jet by varying ϕ and h (see Fig.3) to determine p^t(y_i,t).



Figure 3: Cylindrical strut at potential core

3 Results

Spectra of the acoustical pressure p^t were measured at surface point y_i given by $(R=d/2,\phi=0,h=0)$ and free field receiver point x_i given by $(x_1=0,x_2=-3m,x_3=0m)$. Peaks emerged from spectra at the center frequencies listed in Table 1.

 Table 1: 1/3 octave center frequencies of spectral peaks

Acoustical pressure	1/3 octave band center frequency (Hz)
Surface $p'(y_i)$	1250
Receiver $p^{\iota}(x_i)$	1250

Generating a circumferential profile of the surface dipole strength over a strut cross section involved evaluating equations (3) and (4). Cross correlations and auto correlations were generated from 24 measurements of $p^{t}(y_{i})$ and $p^{t}(x_{i})$ swept through successive ϕ with h=0.

 $\overline{c}(x_1, y_1)$ ($\hat{\tau}$) was evaluated using (4) at each position from which surface dipole strength was found by taking the slope of $\overline{c}(x_1, y_1)$ at the Kirchoff delay time ($\hat{\tau}$). Profiles resulting from (3) are depicted in Figure 4.

4 Discussion

Vortex shedding at the characteristic Strouhal frequency is evident from the Table 1. The spectrum of surface pressure in the potential core exhibited a peak within the one-third octave band centered at 1250 Hz. The peak was likely due to vortex shedding at the Strouhal frequency. Irregular turbulence superimposed on the vortices together with spanwise variation of the mean incident velocity acted to broaden the Strouhal peak.



Figure 4: Circumferential profile of surface dipole strength (left) with spanwise profile inset (right)

The overall SPL at x_i measured 70.1 \pm 0.5 dB, an increase of 20 dB over that before the strut was introduced into the flow. The observed increase is caused by the boundary obstructing the aerodynamic flow and inducing stress fluctuations in the neighboring fluid which formed dipole sources on the surface of the strut.

The circumferential profile of surface dipoles measured at a series of 24 points at midspan shown in Figure 4 bears the shape of $\cos^2\phi$ arising from two factors: surface geometry introduced a $\cos\phi$ factor; a fluctuating lift force introduced an additional $\cos\phi$ factor. Uncertainty is indicated by the error bars in Figure 4 and was largely due to uncertainty in the exact Kirchoff delay time.

The spanwise profile of surface dipoles (Figure 4 inset) was also measured, at a series of 10 points from midspan along the strut's lateral edge. The two profiles were used to approximate the complete surface dipole distribution over the strut. The surface integral of the dipole strength over the strut was found to be 67.0 to 71.7 dB, comparing favorably with the measurement of 70.1 ± 0.5 dB at the receiver.

5 Conclusions

The causality cross-correlation technique provided an efficient means for obtaining diagnostic information and insight on the mechanisms of aerodynamic noise generation. In this case, evidence of vortex shedding was found. The overall noise at the receiver x_i was predicted by integrating two surface dipole profiles for the strut.

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References

[1] M.J. Lighthill, 'On Sound Generated Aerodynamically, *Proc. Roy. Soc, A, 211, 564 (part 1) 1952.*

[2] T.E. Siddon, 'Surface Dipole Strength by Cross Correlation Method', *J. of Acoustical Soc. of America vol 53 No. 2 1973*.

[3] D.E. Marriner, Dipole Radiation Intensity Generated by Cylindical Struts, <u>BAC.ThesesCanada-</u> <u>ThesesCanada.LAC@canada.ca</u>, 1979