

A NOVEL APPROACH TO COUNTING WAVES IN A ROOM

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1 Introduction

Humankind is used to describing the world in three discrete spatial dimensions, with a fourth one for time. Nature however does not observe discrete dimensions. Whether it be a leaf, a tree or a coastline, nature works between the integer dimensions, typically between the dimensions of two and three or sometimes between one and two. These are called fractal dimensions. And they are the hallmark of what is called chaotic behaviour. One of the methods that has been developed to determine the fractal dimension of a signal is to plot it against a time delayed version of itself [1]. This begs the question: is the acoustic field in a room chaotic. The author has tried, unsuccessfully so far, to determine if sound in a room is chaotic. D'Antonio [2] is of the opinion that it is not. It would seem, however, to make sense that it is chaotic; most natural systems are, although to author's knowledge no one has yet proved that with measurements. Whether it is or isn't is not the topic of this paper. But one can use the methods of detecting chaotic behaviour to other aspects of an acoustic field; namely counting waves. Indeed, it was while trying to identify chaotic behaviour in impulse response functions that the method was developed.

2 Method

2.1 Measurements

Measurements were performed in 25 venues over the space of 25 years. Only 12 will be reported here ranging in type from concert halls, recital halls, proscenium arch theatres and rehearsal halls. All 12 were stage acoustics measurements, as opposed to audience chamber measurements. Further details on the measurement procedure may be found in [3] and [4]. The list of venues is shown in Table 1.

2.2 Analysis

Following the methods of chaos theory analysis, one needs to plot a given measurement, in this case $p(t)$, against a time delayed version of itself. If one takes a typical impulse response function, and wants to look at a time delayed version of it, the simplest way is to take the time derivative, in this case dp/dt .

The result is a series of circles, each one representing a wave. Using a simple Matlab routine, the circles – and hence the waves – can be counted.

Table 1: List of venues

Venue	City ¹	Vol. (m ³)
Centre in the Square	Kitchener	16,632
Glenn Gould Studio	Toronto	4,587
Hamilton Place	Hamilton	29,907
NAC Opera House	Ottawa	37,452
NAC Rehearsal Hall	Ottawa	1,398
Sony Centre Rehearsal Hall	Toronto	1,918
The Orpheum	Vancouver	19,200
Playhouse Theatre	Vancouver	6,533
Queen Elizabeth Theatre	Vancouver	32,452
Royal Theatre	Victoria	15,240
Theatre Aquarius	Hamilton	12,526
Theatre Aquarius Reh. Hall	Hamilton	1,822

At this point the difference between reflections and waves should be clarified. The number of reflections is a theoretical construct. One of the things discovered in this study is that the number of reflections in a room cannot be measured. This is due to constructive and destructive interference of one reflection with another. The author chooses the word 'wave' because that is what results when two or more reflections interfere with each other.

In 1950, Bolt et al. [4] developed a formula to predict the number of reflections in a room:

$$N = \frac{4 c^3 t^3}{3V} \quad (1)$$

Measurements reported below show a vast difference between predictions from Equation (1) and what can be measured.

In 1995, the author suggested Revised Theory [6] might not be the most accurate predictor of sound levels on a stage. Revised Theory assumes an unoccluded direct sound and a diffuse reverberant field. Both assumptions are often violated on a stage. And with a wave counter, one can approximate the temporal division between the discrete and reverberant fields. Two sets of comparisons were made between Revised Theory predictions and linear regressions of the measurements. In the first set, unadulterated versions of the impulse response functions were compared with calculations based on an unadulterated versions of the Revised Theory calculations. In the second set, the direct sound was omitted from both the measurements and the calculations. This was done by nulling the first 5 ms on both the signal and in the calculation. Results are shown below.

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3 Results and conclusions

A typical plot of $p(t)$ vs. dp/dt is shown in Figure 1. Note that the time axis is vertical, $t = 0$ is at the bottom of the graph. It's often said that a 2-dimensional plot of an impulse response looks like a Christmas turned on its side. Well if one plots $p(t)$ against its time derivative dp/dt one gets a 3-dimensional plot and it really does look like a Christmas tree!

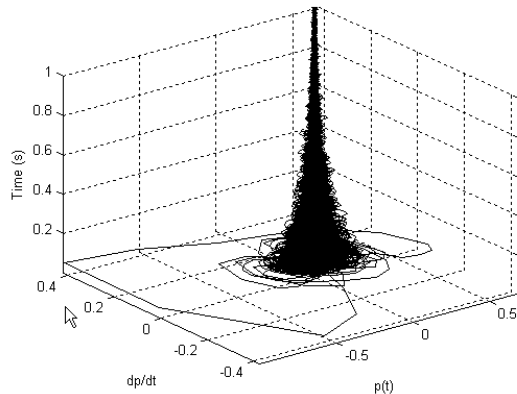


Figure 1: A 3 dimensional view of an impulse response function. The signal $p(t)$ is plotted against time and its time derivative dp/dt . Note that the vertical axis is time. Each circle of this “Christmas Tree” impulse response represents a reflection.

As previously mentioned, there is a wide discrepancy between the number of reflections predicted and the number of waves that can actually be measured. A comparison is shown in Figure 2. There are three things to notice here. On the right side of the graph, the wave count is much, much lower than the predicted reflection count. This is almost certainly due to interference effects. Note also how the wave count curve (solid line) flattens out slightly as it enters the diffuse field. On the left side of the graph, the wave count is higher than the predicted reflection count. There are two possible explanations for this. All the measurements shown here were done in the close confines of a stage, which might explain a higher reflection density close to the source. And it could be the source itself, a 12 sided dodecahedron with 75 mm speakers. Hardly a point source when measured within a 7 – 8 m range.

As mentioned, two sets of comparisons were made between measurements and calculations. (The results quoted here are from Hamilton Place but the other venues showed similar results). A typical example of the unadulterated version is shown in Figure 3. The rms error between measurements and predictions is 1.91 dB for Revised Theory and 2.96 dB for linear regression. This contradicts the original hypothesis of the study. For the second set of comparisons, the ones with the first 5 ms deleted, the rms error between Revised Theory increases greatly to 6.11 dB. The rms error between the measurements and the linear regression decreases to 1.21 dB. Thus to answer the question if Revised Theory is an appropriate predictor of sound levels on a stage, the answer is yes. However if the

direct sound is blocked, as it often is on a stage, linear regressions correlate better with measured sound levels.

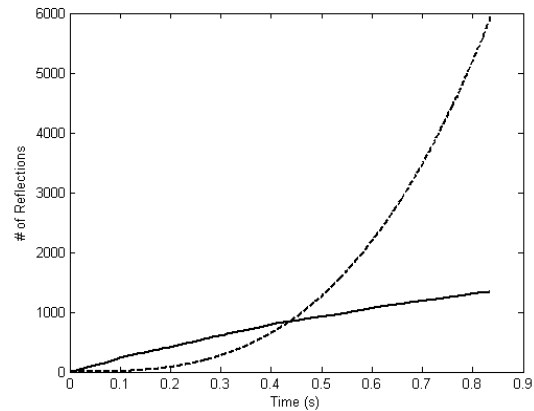


Figure 2: Comparison between measured reflection counts (solid line) and predictions from equation 1 (dashed line). The measured data came from the centre in the square, source at the soloist position, receiver at bass.

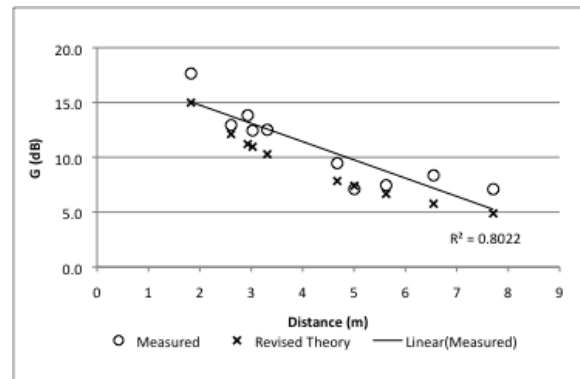


Figure 3: Typical comparison between measured data (circles) Revised Theory (x) and linear regression (line). The measurements were performed at Hamilton Place.

References

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