Simulation Study Of Super-Resolution In Hydrophone Measurements Of Pulsed Ultrasonic Fields

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Résumé

Afin de réaliser des mesures fiables du champ ultrasonore impulsionnel, l'utilisation d'un hydrophone piézoélectrique comme récepteur est recommandé. Cependant, en raison de la taille finie de l'ouverture du récepteur, la pression acoustique mesurée est affectée par le moyennage spatial sur sa surface active. Le but de ce travail est de déconvoluer les effets d'ouverture de l'hydrophone récepteur afin de reconstruire le champ ultrasonore impulsionnel avec une bonne résolution spatiale. Pour cela, nous considérons le champ de pression impulsionnel rayonné dans l'eau par des transducteurs à large bande de 19 mm de diamètre, avec des fréquences centrales f_c = 2,25 MHz et f_c = 15 MHz. Les récepteurs sont des hydrophones à membrane en PVDF de 25 µm d'épaisseur avec des ouvertures rectangulaires et circulaires. Les résultats de cette étude montrent la forte dépendance de la qualité de la reconstruction du rapport signal sur bruit (SNR). Généralement, la qualité de la reconstruction diminue avec la réduction du SNR. Une bonne qualité de reconstruction a été obtenue avec un coefficient de corrélation supérieur à 0,9936 lorsque les signaux "acquis" ne sont pas trop bruités (SNR = 60 dB). Dans ce cas, l'amélioration de la résolution spatiale par un facteur de 5 et 9, respectivement, pourrait être atteinte. La qualité de la reconstruction dépend également des dimensions de l'hydrophone, de la distance axiale par rapport à la source et de la fréquence centrale de l'impulsion ultrasonore ainsi que de sa bande passante spectrale.

Mots clefs : Super-résolution, filtre inverse spatial, filtre de Wiener spatial, hydrophone en PVDF, champ ultrasonore impulsionnel, champ ultrasonore reconstruit

Abstract

In order to carry out reliable measurements of pulsed ultrasonic fields, the use of a piezoelectric hydrophone as receiver is recommended. However, due the finite size of the receiver aperture the measured acoustic pressure is affected by spatial averaging on the surface active face. The aim of this work is to deconvolve the spatial effects of the receiver hydrophone in order to reconstruct the pulsed ultrasonic field with a better spatial resolution. Hereby, the linear pulsed pressure field radiated in water by wideband planar transducers of 19 mm diameter, with central frequencies f_c=2.25 MHz and f_c=15 MHz are considered. The receivers are PVDF membrane hydrophones of 25 µm - thickness with rectangular and circular apertures. The results of this study show the strong dependency of the reconstruction quality upon the signal-to-noise ratio (SNR). Generally, the quality of the reconstruction decreases with decreasing SNR. Good reconstruction quality has been obtained with correlation coefficient larger than 0.9936 when the “acquired” signals are not too much noisy (SNR=60dB). In this case, improvement of the spatial resolution by a factor of 5 and 9 respectively could be reached. The reconstruction quality depends also upon the hydrophone dimensions, the axial distance to the source, the central frequency and the spectral frequency bandwidth of the pressure pulse.

Keywords: Super-resolution, spatial inverse filter, spatial Wiener filter, PVDF hydrophone, pulsed ultrasonic field, ultrasonic field reconstruction

1 Introduction

In order to carry out reliable measurements of pulsed ultrasonic fields, different techniques are used. However, the most used standardized technique consists in the utilization of piezoelectric PVDF membrane hydrophones as receivers. That is principally because of their advantageous acoustic properties [1-4] and commercial availability. However, when high frequencies are used, spatial averaging due to their finite–size aperture and the variations of their frequency response have to be considered [5-8]. These spatio-temporal transmission properties may strongly affect the electric signal delivered by these devices.

In this work, and in order to reconstruct the impulse ultrasonic field with a high spatial resolution we propose to deconvolve the spatial effects of the hydrophone. The application of deconvolution methods has already been applied in different fields to signals of various forms [9-10]. Methods for correcting the spatial averaging effect have also been proposed [11-16].

These are however mostly developed by using idealized models. It should be mentioned that a fundamental difference of the work here proposed compared to the works cited above is that these are based on a correction of the

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value of the pressure taking into account the general form of the field. The method suggested in this paper allows to find the original value of the pressure, which should be received by a point transducer, by spatial deconvolution of the signal received by a large-size hydrophone and thus permits a super-resolution measurement of this field. Also, the possibility of deconvolving the spatial effects has been shown for harmonic ultrasonic fields only [17-18]. The present contribution is concerned with the study of pulsed ultrasonic fields. In this work, we are interested only in the spatial effects of the hydrophone. It should be noted that the deconvolution of the temporal effect is well established in the literature [19-20].

2 Response of the hydrophone receiving chain to the ultrasonic pressure field

In order to develop appropriate procedures through which the spatial effects of the receiving hydrophone can be inverted, the study of the direct problem is necessary. First, the output signal of the receiving hydrophone, $v(x,y,z_0,t)$, when it is placed in the transverse plane, $z=z_0$, has to be determined; z_0 is the distance from the source.

The receiving system (hydrophone with its receiving chain) being supposed linear and space- and time-invariant, can be characterized by its spatio-temporal impulse response, $h(x,y,t)$. The electric output voltage of the hydrophone chain, $v(x,y,z_0,t)$, can then be obtained by convolving this response with the radiated pressure field. Furthermore, if the signal is assumed to be corrupted by an additive noise, $n(x,y,z_0,t)$, this voltage will be given by:

$$v(x,y,z_0,t) = p(x,y,z_0,t) \otimes_{xyt} h(x,y,t) + n(x,y,z_0,t) \quad (1)$$

Where $p(x,y,z_0,t)$ is the acoustic pressure radiated by the transducer at the field point $M(x,y,z_0,t)$.

$\otimes_{xyt}$ designates the spatio-temporal convolution operator. Since the hydrophone surface vibrates synchronously with the incident wave and as we are only interested in the spatial properties of the hydrophone, equation (1) becomes:

$$< p_n(x,y,z_0,t) > = p(x,y,z_0,t) \otimes_{xy} h(x,y) + n_p(x,y,z_0,t) \quad (2)$$

Where $\otimes_{xy}$ designates the spatial convolution operator. $h(x,y)$ represents the two-dimensional spatial impulse response of the hydrophone (its aperture function). In this case, $< p_n(x,y,z_0,t) >$ represents the noisy spatially averaged pressure and $n_p(x,y,z_0,t)$ the noise corrupting this pressure. It should be noted that $< p_n(x,y,z_0,t) >$ can be obtained by temporal deconvolution of the received voltage $v(x,y,z_0,t)$ by the temporal impulse response $h(t)$ of the hydrophone. This can be derived from its complex receiving transfer function $H(f)$.

2.1 Simulation of the radiated pressure field

For the simulation of the ultrasonic field, the latter is supposed to be radiated in water by a planar circular piston of diameter 19 mm set in a rigid baffle. The pulsed ultrasonic pressure generated at the surface of the transmitter is assumed to be, in a first step of the study, as a gaussian modulated sinusoid of central frequency f_c =2.25 MHz with a wide fractional frequency bandwidth pulse (B=60% at -6dB); f_c corresponds to a wavelength λ_c =0.67 mm of the ultrasonic wave in water. In a second step, a transmitted pressure pulse of $f_c = 15$ MHz central frequency ($\lambda_c = 0.1$mm) with the same fractional frequency bandwidth will be considered. At the axial distance z_0, the temporal variations of ultrasonic pressure, for different radial positions r, shows the well-known contributions of plane wave and edge waves in the "direct radiation" region $(0 \le r < a)$ and edge wave exclusively in the "shadow region ($r \ge a$).

2.2 Spatial Impulse Response of the Hydrophone

Our receiver is constituted of a PVDF membrane hydrophone of finite-size aperture. This latter is characterized by the spatial transfer function $\mathcal{H}(f_x, f_y)$ in the corresponding frequency domain. f_x and f_y are the spatial frequencies in the x- and y- directions respectively. Firstly, a hydrophone of rectangular aperture has been chosen with the dimensions $l_x = 1.4$mm, $l_y = 0.6$mm, that means $l_x = 2.1\lambda_c$, $l_y = 0.89\lambda_c$, at $f_c = 2.25$ MHz, in order to demonstrate the geometry-dependent averaging effect for both dimensions (the two co-ordinates). The sensitivity is supposed to be constant over the hydrophone aperture (ideal aperture). It should be noted that equation (2) supposes that the ultrasonic field is being scanned by means of the hydrophone in the two orthogonal directions x and y. A scanning step width $\Delta x = \Delta y = 0.2$ mm, that is $\Delta x = \Delta y = 0.3\lambda_c$, at $f_c = 2.25$ MHz, leads to the discretization of the hydrophone aperture in 3x7 square receiving cells. The corresponding spatial transfer function is represented in Figure 1.

The latter was obtained using the two-dimensional spatial Fourier transform of the impulse response of the rectangular aperture $h(x,y)$ defined above.

2.3 Noise of the Measuring System

Noise is an important item when considering any deconvolution problem. Indeed, measured signals are more or less corrupted by noise. Moreover, measurement uncertainties have to be taken into account [21-22]. Therefore, in order to approach real measurement conditions, the acoustic pressure spatially averaged by the

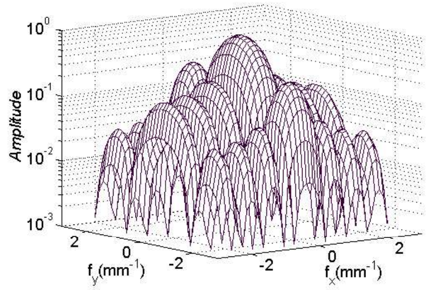

Figure 1: Amplitude of the spatial transfer function of a hydrophone of rectangular aperture (l_x=1.4mm=2.1λ_c, l_y =0.6mm=0.89λ_c; λ_c=0.67mm).

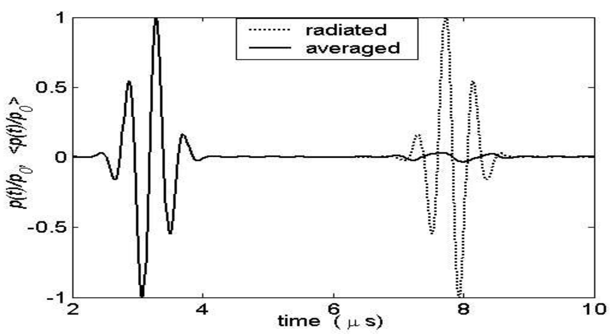

Figure 2: Radiated pressure (dotted curve) and spatially averaged pressure (thick curve) by a hydrophone of a rectangular aperture (l_x=1.4mm=2.1λ_c, l_y =0.6mm=0.89λ_c; λ_c=0.67mm) on transducer axis (x=0mm, y=0mm) at z_0=3mm.

hydrophone is supposed to be corrupted by a stationary and non-correlated noise. In addition, the noise is supposed to be white with a Gaussian distribution. This noise has an average value n_m = 2.32×10^{-6}P$_0$ (in Pa) and a standard deviation $\sigma = 7.1\times10^{-3}\,\mathrm{P}_0$ (in Pa). These values correspond to a signal-to-noise ratio (SNR) of 40 dB with a reference level of 1P$_0$ (in Pa). Po is a reference pressure amplitude taken as the maximal amplitude of the radiated plane wave pressure pulse.

3 Spatial transmission effects of the hydrophone system (Averaging)

By convolving the radiated pressure $p(x,y,z_0,t)$ with the aperture function of the hydrophone $h(x,y)$ the spatially averaged pressure over the receiver face can be obtained. That is:

$$< p(x,y,z_0,t) >= p(x,y,z_0,t) \otimes_{xy} h(x,y) \qquad (3)$$

Taking account of equations (2) and (3), the output voltage of the receiver hydrophone becomes:

$$< p_n(x,y,z_0,t) >= < p(x,y,z_0,t) > + \, n_p(x,y,z_0,t) \quad (4)$$

The effects of hydrophone spatial properties on the "measured" pressure field are illustrated in Figure 2. This figure shows the variations of the radiated and the averaged pressure when the hydrophone is placed on the transducer axis (x =0 mm, y = 0 mm) at the distance $z_0 = 3\,\mathrm{mm}$. It can be noticed that, compared with the radiated pressure field, the plane wave is not affected by spatial averaging. On the contrary, the amplitude of the edge wave is considerably reduced.

4 Deconvolution of the spatial effect of the hydrophone

In order to retrieve the radiated pressure field $\hat{p}(x,y,z_0,t)$ outgoing from the ''measurement data", that is, from the acoustic pressure spatially averaged by the hydrophone after temporal deconvolution $< p_n(x,y,z_0,t) >$, a spatial deconvolution method is proposed [17-18]. This method permits the spatial effects to be inverted.

The acquisition system being supposed linear, the estimated value $\hat{p}(x,y,z_0,t)$ of the ultrasonic pressure can be obtained by using a spatial reconstruction filter with the spatial impulse response $h_{\mathcal{F}}(x,y)$, that is:

$$\hat{p}(x,y,z_0,t) = h_{\mathcal{F}}(x,y) \otimes_{xy} < p_n(x,y,z_0,t) > \qquad (5)$$

As a criterion for the evaluation of the quality of the deconvolution procedure, the normalized correlation coefficient, $r_{\hat{p}p}$, between the reconstructed pressure, $\hat{p}(x,y,z_0,t)$, and the radiated pressure, $p(x,y,z_0,t)$, is used. On transducer axis, this coefficient is given by [23]:

$$r_{\hat{p}p} = A/B \qquad (6)$$

With:

$$A = \sum_{k=1}^{N} p^*(i_0, j_0, k)\,\hat{p}(i_0, j_0, k) \qquad (7)$$

$$B = \sqrt{\sum_{k=1}^{N} \left| p(i_0, j_0, k) \right|^2 \sum_{k=1}^{N} \left| \hat{p}(i_0, j_0, k) \right|^2} \qquad (8)$$

Where i_0, j_0 and k are the indices related to the position $x = 0$, $y = 0$ and, the time t respectively. N is the number of temporal samples.

4.1 Spatial two-dimensional inverse filter

The intuitive method for the reconstruction of the ultrasonic field is the inverse filtering, which simply compensates for the spatial aperture effects. The spatial transfer function of this filter is simply the inverse of that of the hydrophone, that is:

$$\mathcal{H}_F(f_x, f_y) = 1/\mathcal{H}(f_x, f_y) \qquad (9)$$

The pressure field can not be reconstructed from the spatially averaged data by using a spatial inverse filter in the case of an idealized aperture. Indeed, the calculations showed that the obtained acoustic pressure is completely submerged by the system noise. According to equation (9), the zeros of the spatial transfer function of the hydrophone correspond to infinite values of the transfer function of the inverse filter. In this case, inverting the hydrophone aperture by using this filter may be not possible. This characterizes the ill-posedness of this inverse problem.

Even if the zeros are numerically avoided, the problem remains ill-conditioned [23] because of the drastic amplification of the noise level in the deconvolved signals. In addition to this inverse filter, a low pass filter with a cut-off frequency limited to useful frequencies of the signal has been applied. The deconvolution result, when using this additive filter, is shown in Figure 3, which still exhibits a poor quality of the reconstructed signal.

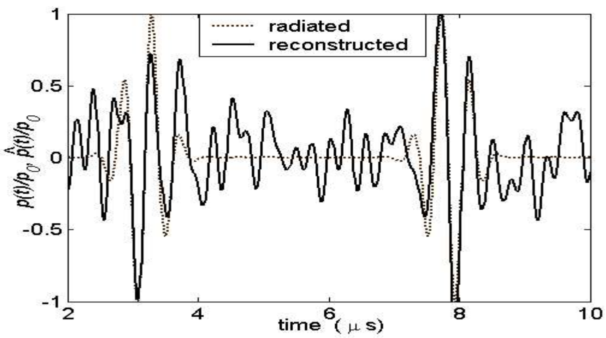

Figure 3: Acoustic pressure reconstructed by using a spatial inverse filter associated to a low pass filter on transducer axis at z_0 =3mm. Hydrophone of rectangular aperture: (l_x=1.4mm=2.1λ_c, l_y =0.6mm=0.89λ_c ; λ_c=0.67mm, SNR = 40dB).

4.2 Spatial two-dimensional Wiener filter

Because of the ill-posedness of the problem, it is necessary to use a regularization procedure. From a purely mathematical point of view, the principle of the regularization consists in adding to the null or almost null values of the inversion operator a sufficient quantity so that calculation can be carried out with a satisfactory stability [21-22]. From a physical point of view, the aim of the regularization procedure is to lead to a convolved signal, which is a solution of the inverse problem satisfying some physical conditions [24]. The Wiener filter is based on the minimization of the mean square error between the pressure to be reconstructed, $p(x,y,z_0,t)$, and the estimated pressure value, $\hat{p}(x,y,z_0,t)$, (MMSE criterion), that is:

$$E\{[\hat{p}(x,y,z_0,t)-p(x,y,z_0,t)]^2\}\rightarrow \min \quad (10)$$

Where E designates the expected value. Its spatial transfer function is given as follows [26]:

$$\mathcal{H}_F(f_x,f_y)=\frac{\mathcal{H}^*(f_x,f_y)\,\Phi_{pp}(f_x,f_y,z_0)}{\left|\mathcal{H}(f_x,f_y)\right|^2\Phi_{pp}(f_x,f_y,z_0)+\Phi_{n_pn_p}(f_x,f_y,z_0)} \quad (11)$$

Where $\mathcal{H}^*(f_x,f_y)$ is the complex conjugated of the spatial transfer function of the hydrophone. $\Phi_{pp}(f_x,f_y,z_0)$ and $\Phi_{n_pn_p}(f_x,f_y,z_0)$ are the spatial power spectrum densities (PSDs) of the original acoustic pressure and of the pressure noise respectively. The use of this procedure permits a simultaneous deconvolution of the hydrophone aperture and the reduction of the noise level. For its implementation two cases have been considered. In the first one, a-priori knowledge of the PSDs of the acoustic pressure and of the pressure noise has been assumed. The filter implemented under these ideal conditions will be next designated as "ideal". In the second one, as no a-priori knowledge of these quantities was assumed, the PSDs have been estimated from the spatially averaged pressure after temporal deconvolution. The filter implemented under these conditions will be designated by "real". For both types of the Wiener filter, a supplementary low pass filter is implemented in order to ameliorate the quality of the signal reconstruction.

Wiener filter with a-priori known PSDs

In this case, a-priori knowledge of the PSDs is assumed. The comparison of the reconstructed pressure (Figure 4a) when using this "ideal" Wiener filter with the radiated pressure (Figure 4a) (dotted curve) on transducer axis ($x=0$, $y=0$) shows that, under these ideal conditions, good reconstruction results can be achieved ($\left|r_{\hat{p}p}\right|$=0.9931).

Wiener filter with estimated PSDs

Generally, for the investigation of ultrasonic fields, there is no sufficient information on the ultrasonic field to be investigated. In this case, there is no a-priori knowledge of the PSDs, Φ_{pp} and $\Phi_{n_pn_p}$. Therefore, these quantities are replaced in equation (11) by their estimated values, $\Phi_{\hat{p}\hat{p}}$ and $\Phi_{\hat{n}_p\hat{n}_p}$.

For an estimation of these quantities, the following procedure is adopted. First, an estimated mean level of $\Phi_{\hat{n}_p\hat{n}_p}$ for the pressure noise is determined from the spatial spectrum of the spatially averaged pressure. This level is estimated from the high spatial frequencies region of the spectrum, where the useful signal can be considered as negligible. i.e. the region in which no useful signal spectrum can be identified. An estimated PSD of the noiseless spatially averaged pressure, $\Phi_{<\hat{p}>_0<\hat{p}>_0}$ is then obtained by subtracting the estimated PSD of the pressure noise, $\Phi_{\hat{n}_p\hat{n}_p}$, from that of the noisy spatially averaged pressure, $\Phi_{<p_n><p_n>}$. That is:

$$\Phi_{<\hat{p}>_0<\hat{p}>_0}=\Phi_{<p_n><p_n>}-\Phi_{\hat{n}_p\hat{n}_p} \quad (12)$$

Finally, an estimated spatial PSD of the acoustic pressure to be reconstructed, $\Phi_{\hat{p}\hat{p}}$, is obtained from the quotient of the PSD of the estimated noiseless spatially averaged pressure $\Phi_{<\hat{p}>_0<\hat{p}>_0}$ and the square of the hydrophone transfer function $\mathcal{H}(f_x, f_y)$. That is:

$$\Phi_{\hat{p}\hat{p}} = \Phi_{<\hat{p}>_0<\hat{p}>_0} \Big/ \left|\mathcal{H}\right|^2 \qquad (13)$$

The temporal variations of the pressure so reconstructed on axis at z_0=3mm are shown in Figure 4b. Though the use of Wiener filter under these "real" conditions leads to results of less quality ($\left|r_{\hat{p}p}\right|$=0.9155) compared to those obtained in the "ideal" case, the reconstruction results are obviously more significant than those furnished by using a spatial inverse filter.

Effect of axial distance to the source

When studying the direct problem, it has been shown that the effect of spatial averaging decreases while moving the receiver away from the source [23]. That suggests studying the influence of the axial distance on the quality of reconstructed field. In this order, the same pressure field studied previously is considered.

This field is "acquired" by using the hydrophone of the same aperture (l_x=1.4mm, l_y=0.6 mm, with SNR = 40 dB and at particular axial distances z = 10 mm, z=15 mm and z=20 mm. The radiated acoustic pressure $p(x, y, z_0, t)$, and the spatially averaged one, $< p_n(x, y, z_0, t) >$, which have been obtained at these axial distances are illustrated in Figures 5a, 5c and 5e respectively.

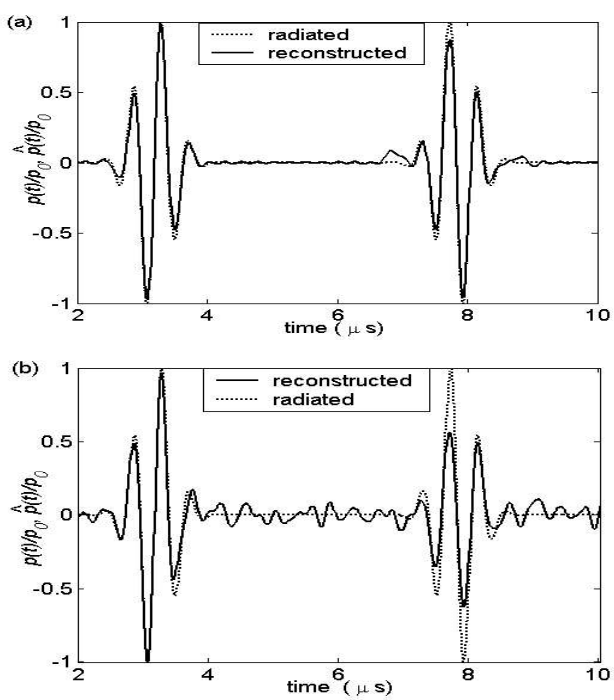

Figure 4: Acoustic pressure reconstructed by means of a spatial Wiener filter on transducer axis at z_0=3mm: (a) "ideal" Wiener filter, (b) "real" Wiener filter. Hydrophone of rectangular aperture: (l_x=1.4mm=2.1λ_c, l_y=0.6mm=0.89λ_c ; λ_c=0.67mm, SNR = 40dB).

Figures 5b, 5d and 5f show the reconstruction results at these distances respectively. These results have been obtained by using a spatial Wiener filter with estimated PSDs. It should be noted that in Figures 5c, 5d, 5e and 5f, the edge wave slightly interferes with the plane wave because of the distance considered and of the pulse width. By comparing these figures to those obtained at z = 3mm (Figure 4b), one notes that the quality of reconstruction depends upon the axial distance z.

This quality becomes better when moving the receiver away from the source. This improvement is confirmed quantitatively by the calculation of the normalized correlation coefficient between the reconstructed acoustic pressure and the radiated one. The values of this coefficient are $\left|r_{\hat{p}p}\right|$=0.9285, $\left|r_{\hat{p}p}\right|$=0.9430 and $\left|r_{\hat{p}p}\right|$=0.9789 at the distances z=10 mm, 15 mm and 20 mm respectively. This improvement of the reconstruction quality at farther axial distance from the source allows using greater aperture dimensions or higher transducer frequency at these distances before the limits of the procedure are reached.

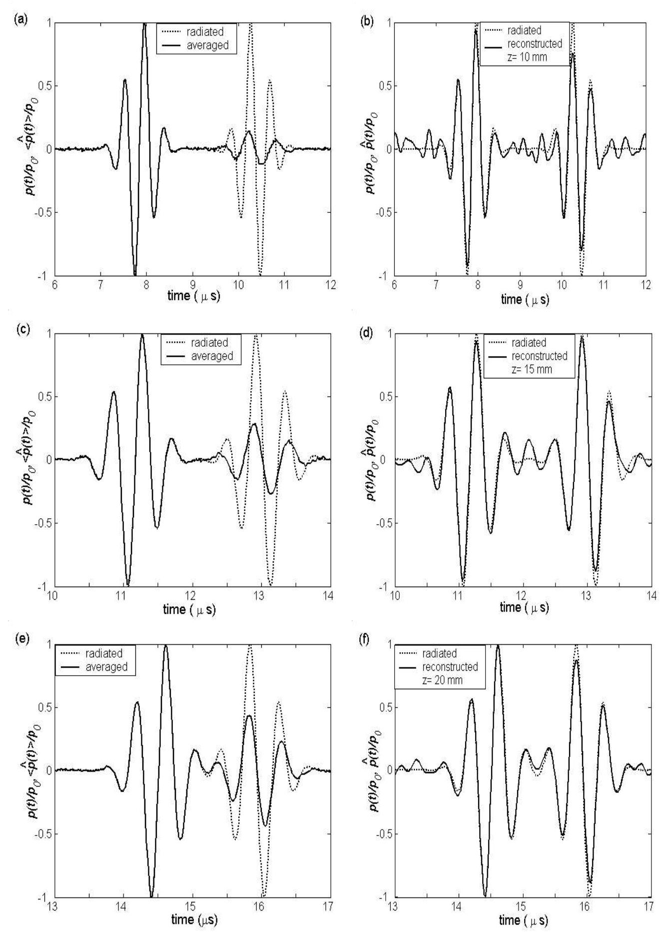

Figure 5: (a, c, e) Spatially averaged pressure and (b, d, f) Acoustic pressure reconstructed by using a "real" Wiener filter on transducer axis at different distances from the source. Hydrophone of rectangular aperture: (l_x=1.4mm=2.1λ_c, l_y =0.6mm=0.89λ_c ; λ_c=0.67mm, SNR = 40dB).

Effect of hydrophone dimensions

Another criterion for the evaluation of the effectiveness of the spatial deconvolution method is the size of the hydrophone aperture.

In this case, the less advantageous reconstruction procedure " Wiener filter with estimated PSDs " has been tested for a hydrophone of greater aperture ($l_x = 1.8\,\text{mm} = 2.7\,\lambda_c$, $l_y = 1\,\text{mm} = 1.5\lambda_c$; $\lambda_c = 0.67\text{mm}$), with an SNR= 40 dB at z=20mm. Figure 6a shows the radiated acoustic pressure on axis (dotted curve) as well as the spatially averaged one (solid curve).

The reconstructed pressure by means of the "real" Wiener filter is illustrated in Figure 6b (solid curve). Although the edge wave has clearly greater amplitude than that of the spatially averaged one (Figure 6a), the reconstructed pressure field presents some differences with the original one ($\left|r_{\hat{p}p}\right|$ =0.8262). By comparing the reconstruction results obtained for this hydrophone aperture and those obtained in Figure 5f, it can be noticed that the quality of the reconstruction diminishes with increasing aperture dimensions.

Effect of SNR

Another critical parameter, which influences the quality of the reconstruction, is the signal-to-noise ratio (SNR). In order to investigate its effect, signals delivered by the hydrophone chain with other noise levels (SNR=60dB and SNR=20dB) are considered.

These signals have been "acquired" at the same axial distance from the source (z =20 mm) and for the same aperture dimension (l_x=1.8mm, l_y =1mm).

The curve in Figure 7a is obtained for an SNR= 60dB. It shows that the "real" Wiener filter furnishes excellent reconstruction results when the "acquired" signals are not too much noisy ($\left|r_{\hat{p}p}\right|$=0.993).

This result is equivalent to an improvement of the spatial resolution by a factor of 9 in the x- direction and of 5 in the y-direction. On the contrary, for a low SNR (20 dB), the reconstruction results become poor and $\left|r_{\hat{p}p}\right|$ =0.302 (Figure 7b). This result indicates that the limits of the reconstruction procedure are already attained for these aperture dimensions, at this axial distance and for this SNR.

Results for a circular aperture

Hydrophones with circular active area are mostly used. Therefore, the deconvolution procedure described in §4.2 has been also tested for hydrophones of circular aperture. The results presented here are obtained with a hydrophone aperture of diameter ∅=1 mm $\cong 1.4\,\lambda_c; \lambda_c \cong 0.67\text{mm}$.

By means of this hydrophone, the ultrasonic pressure field has been scanned in the transverse plane at z =20 mm from the source. The signal has been "acquired" with an SNR = 40dB.

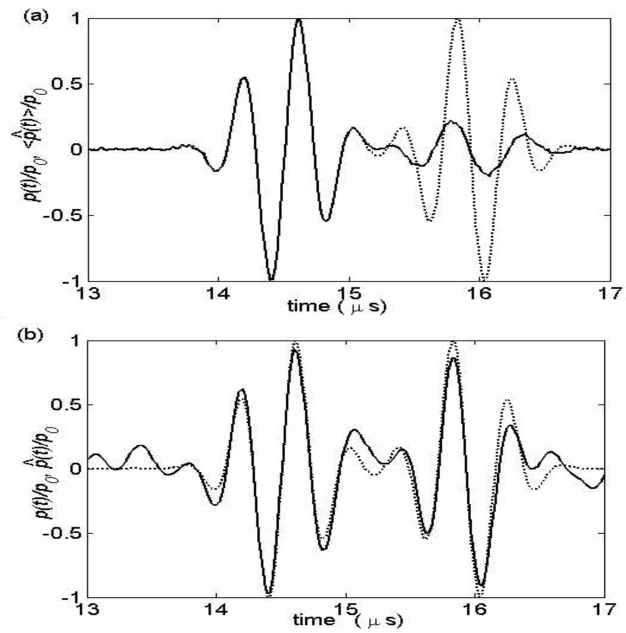

Figure 6: (a) Spatially averaged pressure and (b) Acoustic pressure reconstructed by using a "real" Wiener filter, on transducer axis at z_0 =20mm. Hydrophone of rectangular aperture: ($l_x = 1.8\,\text{mm} = 2.7\,\lambda_c$, $l_y = 1\,\text{mm} = 1.5\lambda_c$; $\lambda_c = 0.67\text{mm}$, SNR = 40 dB). Radiated pressure in dotted curve.

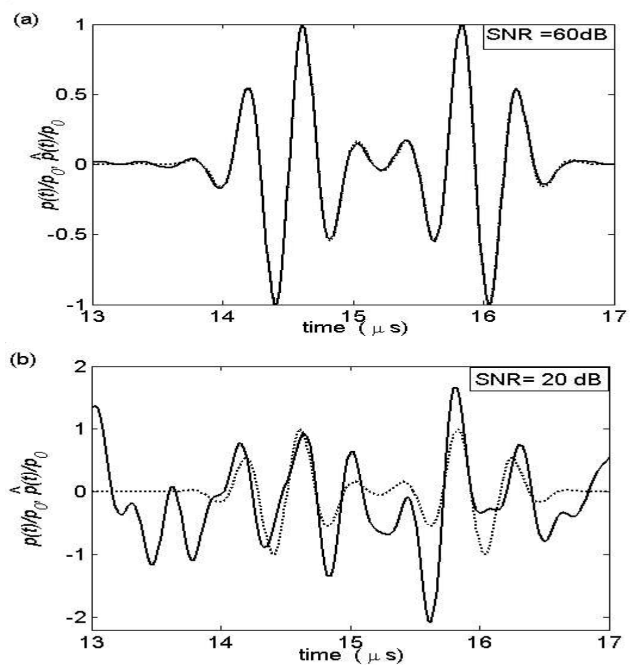

Figure 7: Acoustic pressure reconstructed by means of a "real" Wiener filter on transducer axis at z_0 =20mm for different SNR a) SNR=60dB, b) SNR= 20dB. (Hydrophone of rectangular aperture: 1.8mm x 1mm, SNR = 40dB). Radiated pressure in dotted curve.

Figure 8a illustrates the spatially averaged pressure (thick curve). The reconstruction result on axis obtained by using a spatial Wiener filter with estimated PSDs is shown in Figure 8b.

The comparison of this result with the radiated acoustic pressure (thin curve in Figure 8a and dotted curve in Figure 8b) shows that, for this aperture, the edge wave, which has been affected by spatial averaging has been correctly reconstructed. This is confirmed by the calculation of the normalized correlation coefficient $\left|r_{p\hat{p}}\right| = 0.9794$. The improvement of the resolution by using this deconvolution procedure has been thus achieved by a factor 5.

Effect of pulse central frequency and bandwidth

In order to study the effect of the signals frequency increase, a radiated pressure with a center frequency 15 MHz and 60% fractional bandwidth has been considered.

The receiver is a hydrophone of circular aperture of 1 mm diameter placed on axis at z=20mm and the signal is "acquired" with SNR=40dB. The quality of the spatial deconvolution obtaind by the application of the "ideal' Wiener filter (Figure 9a) is still good.

However, the use of a real Wiener filter (Figure 9b) shows that for an excitation with a higher frequency, the limits of the method are quickly reached.

Indeed, when the the center frequency and the bandwidth of the pressure pulse increases, the quality of the spatial deconvolution diminishes.

5 Conclusion

In this study, it has been shown that it is possible to deconvolve the effects of the spatial transmission properties of the receiver hydrophone of a pulsed ultrasonic field. The spatial deconvolution is an ill-posed problem. This has been overcome by using a regularization procedure such as by using a spatial Wiener filtering.

This permitted the pulsed pressure field to be reconstructed from "measurement data" with a better resolution. The spatial deconvolution carried out using the Wiener filter showed that the quality of the reconstruction of the pulsed ultrasonic field depends strongly upon the SNR, the spatial frequencies bandwidth of the field pressure investigated, which is related to the axial distance from the source, and the dimensions of the hydrophone aperture. In all cases, the greater is the SNR, the better are the results.

In addition, the farther is the scanned field region, the greater the aperture dimensions can be considered, before the limits of the reconstruction procedure are reached. The study has been achieved for the field of a circular planar transmitter and receivers of rectangular and circular apertures. It could, however, be generalized to other aperture geometries and any kind of ultrasonic field. Though the showed results concerned the on-axis region, the deconvolution allows the reconstruction of the original pressure at any region of the ultrasonic field, provided SNR is sufficient.

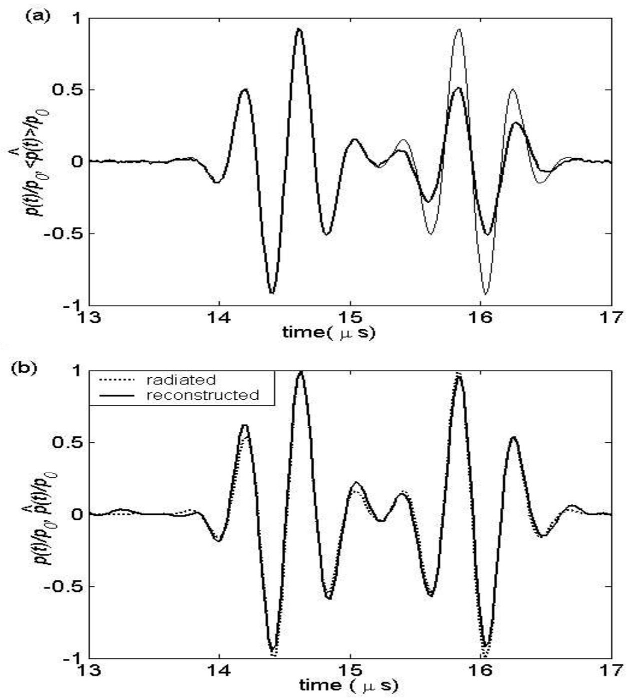

Figure 8: (a) Spatially averaged pressure, (b) Acoustic pressure reconstructed by using a "real" Wiener filter, on transducer axis at z_0 =20mm. (Hydrophone of circular aperture: ∅=1mm=1.4λ_c; λ_c=0.67mm, SNR = 40dB). Radiated pressure in dotted curve.

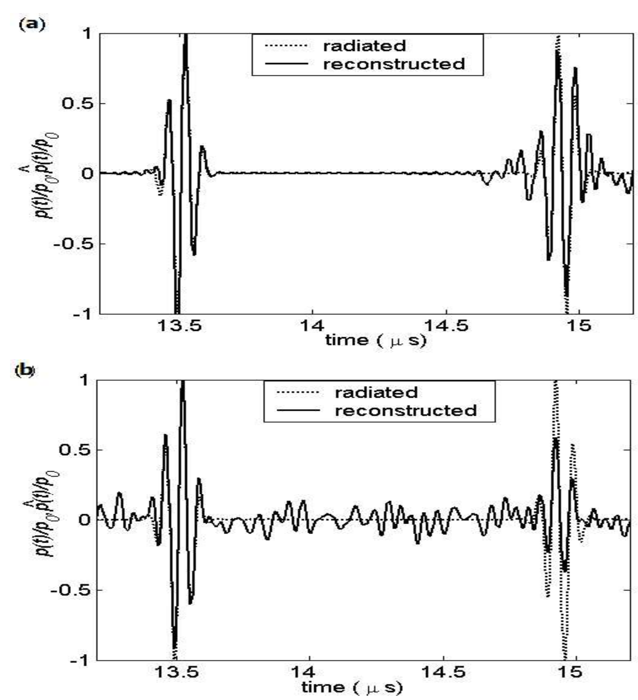

Figure 9: Acoustic pressure reconstructed by means of a spatial Wiener filter on transducer axis at z_0 =20 mm. (a) "ideal" Wiener filter and (b) "real" Wiener filter. (Hydrophone aperture: ∅=1mm=10λ_c; λ_c=0.1mm, SNR = 40dB). Radiated pressure (f_c=15MHz) in dotted curve.

References

[1] V. Wilkens, S. Sonntag and O. Georg. Robust spot-poled membrane hydrophones for measurement of large amplitude pressure waveforms generated by high intensity therapeutic ultrasonic transducers. *J. Acoustic. Soc. Am.*, 39 :1319, 2016.

[2] Nagle et al. Challenges and Regulatory Considerations in the Acoustic Measurement of High-Frequency (>20 MHz) Ultrasound. *J. Ultrasound Med.*, 32 :1897, 2013.

[3] P. Lum, M. Greenstein, C. Grossman and T.L Szabo. High frequency membrane hydrophones. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control UFFC.*, 34 :536, 1996.

[4] R.C. Preston, D.R. Bacon, A.J. Livett and K. Rajendran. PVDF membrane hydrophone performance properties and their relevance to the measurement of the acoustic output of medical ultrasonic equipment. *J. Phys. E: Sci. Instrum.,* 16 :786, 1983.

[5] C.J. Daly and N.A.H.K. Rao. Time-and frequency-domain description of spatially averaged one-way diffraction for an unfocused piston transducer. *Ultrasonics*, 37 :209, 1999.

[6] R. A. Smith. Are hydrophones of diameter 0.5 mm small enough to characterize diagnostic ultrasound equipment?. *Phys. Med. Biol*., 34 :1593, 1989.

[7] K. Beissner. Maximum hydrophone size in ultrasonic measurements. *Acustica,* 59 :61, 1985.

[8] A. Markiewicz and R.C. Chivers. Typical errors in using finite miniature ultrasonic probes for far field measurements. *Acoust. Lett*., 6 :142, 1983.

[9] H. Webster, M. R. Davenport and J.P. Ardouin. 3D Deconvolution of vibration corrupted hyperspectral images. *Spectral Sensing Research for Surface and Air Monitoring in Chemical. Biological and Radiological Defense and Security Applications*, 49 :521, 2009.

[10] R. Jirik and T. Taxt. Homomorphic deconvolution of utrasonics image. *Ultrasonic and Advanced Methods for Nondestructive Testing and Material Characterization*, 559 :590, 2007.

[11] B. Zeqiri, A.D. Bond. The influence of waveform distortion on hydrophone spatial averaging correction: theory and measurement. *J. Acoustic. Soc. Am*., 92 :1809, 1992.

[12] E. G. Radulescu, P.A. Lewin, A. Goldstein and A. Nowicki, Hydrophone spatial-averaging corrections from 1-40 MHz. IEEE *Trans. UFFC*., 48 :1575, 2001.

[13] E. G. Radulescu, P.A. Lewin, A. Nowicki. 1–60 MHz measurements in focused acoustic fields using spatial averaging corrections. *Ultrasonics*, 40 : 497, 2002.

[14] M. Yoshioka and T. Kikuchi. Evaluation of Hydrophone Spatial Averaging Effect in Near Field Measurement for Determining Mechanical Index. *The Japan Society of Applied Physics*, 51, 2012.

[15] C. Koch and W. Molkenstruck. Primary calibration of hydrophones with extended frequency range 1 to 70MHz using optical interferometry. *IEEE Trans. Ultrason. Ferroelectr. Freq. Contr. UFFC*., 46 : 885, 1999.

[16] G. Xing, P. Yang, P. Hu, K. H. Lam, L. He and Z. Zhang, "Field characterization of steady state focused transducers using hydrophones based on Fresnel approximation. *Measurement Science and Technology*, 28 :065005, 2017.

[17] T. Boutkedjirt and R. Reibold. Reconstruction of ultrasonic fields by deconvolving the hydrophone aperture effects. I. Theory and simulation. *Ultrasonics*, 39 :631, 2002.

[18] T. Boutkedjirt and R. Reibold. Reconstruction of ultrasonic fields by deconvolving the hydrophone aperture effects. II. Experiment. *Ultrasonics*, 39 :641, 2002.

[19] A. M. Hurrell and S. Rajagopal. The Practicalities of Obtaining and Using Hydrophone Calibration Data to Derive Pressure Waveforms. *IEEE Trans. Ultrason. Ferroelectr. Freq. Contro UFFC.*, 64 :126, 2017.

[20] S. Eichstädt, V. Wilkens, A. Dienstfrey, P. D. Hale, B. Hughes and C. Jarvis. On challenges in the uncertainty evaluation for time-dependent measurements. *Metrologia,* 53 :125, 2016.

[21] Mario Bertero and Patrizia Boccacci. Introduction to inverse problems in imaging. Institue of Physics Publishing Bristol and Philadelphia. 1998.

[22] David Colton and Rainer Kress. Inverse acoustic and electromagnetic scattering theory. Applied Mathematical Sciences, 1993.

[23] W. Djerir and T. Boutkedjirt. Reconstruction of pulsed ultrasonic fields by deconvolution of the spatio-temporal effects of the receiving hydrophone: Theory and simulation. *Proc.* 4^{th} *European Congress on Acoustics, Forum Acusticum*, 8430, 2005.

[24] W.K. Pratt. Generalized Wiener filtering computation techniques. *IEEE Transactions on Image Processing,* 21 :636, 1972.