

# SHAPE OPTIMIZATION OF EXTENDED TUBE MUFFLER USING THRESHOLD ACCEPTANCE, SIMULATED ANNEALING AND FEM METHODS

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## Résumé

Le bruit d'échappement des moteurs industriels doit répondre aux attentes des clients, aux objectifs législatifs et à la réduction des coûts. Dans cet article, l'optimisation du bruit acoustique d'un silencieux à chambre d'expansion unique doté d'un tube allongé dans un espace limité est effectuée par la maximisation de la perte de transmission sonore (STL) et cela en utilisant la méthode de la matrice de transfert (TMM) et la méthode des éléments finis (FEM). L'optimisation de la géométrie est réalisée à l'aide de nouveaux algorithmes appelés Threshold Acceptance (TA) et Simulated Annealing (SA). Les résultats montrent que les meilleures performances acoustiques sont obtenues avec l'algorithme TA et que la perte de transmission acoustique maximal est précisément localisée sur la tonalité souhaitée. Par conséquent, l'approche optimale sur la conception d'un silencieux à chambre d'expansion unique avec tube allongé présentée dans cette étude fournit une méthode rapide et précise pour l'optimisation de la forme géométrique du silencieux acoustiques dans un espace limité.

**Mots clés :** Silencieux réactif, Méthode des éléments finis, Acceptation du seuil, Recuit simulé, Méthode de la matrice de transfert et Puissance acoustique.

## Abstract

Exhaust noise must meet customer expectations, legislation targets, and cost reduction that call for design optimization of the exhaust systems. In this paper, a numerical assessment of single expansion-chamber muffler with extended tube used under limited space is performed by the maximization of the sound transmission loss (STL) using the Transfer Matrix Method (TMM) and Finite Element Method (FEM). This shape optimization analysis is performed using novel schemes called Threshold Acceptance (TA) and Simulated Annealing (SA) algorithms. Results show that the best acoustical performance is obtained with TA optimizer and the maximal STL is precisely located at the desired targeted tone. Consequently, the optimal approach on the design of single expansion-chamber muffler with extended tube presented in this study provides quick and novel schemes for the shape optimization of muffler under limited space.

**Keywords:** Reactive muffler, Finite element method, Threshold Acceptance, Simulated Annealing, Transfer Matrix Method, and Sound Acoustic Power

## 1 Introduction

The most common element used to reduce generator exhaust noise are reactive mufflers. Reactive mufflers are available in a wide range regarding cost and performance. The noise is reduced by forcing the exhaust air to pass through a series of tubes and chambers. Each element of the muffler has sound reduction properties that vary greatly with acoustic frequency.

Research works on the sound attenuation of mufflers started by Davis and al. in 1954 [1]. To predict the acoustic performance of mufflers Bilawchuk and Fyfe [2] presented a comparison of various numerical methods to analyze the sound behavior of mufflers.

The most common type of linear acoustic model applies classical electrical filter theory widely known as the transfer matrix method [3] also referred as the method of 4-poles parameters Munjal 1987 [4-6]. Later Craggs in 1989 [7-9]

developed a technique that combines the Transfer Matrix method and the Finite Elements Method to study the acoustic attenuation of a duct.

The space volume of mufflers is often limited for maintenance and operation reasons. Therefore, there has been an increasing interest in designing mufflers used under space constraints by optimizing the STL using shape optimization methods. In 1986, Bernhard [10] proposed the shape optimization of simple expansion muffler using a non-constrained space.

In 2002, Yeh et al [11] presented a shape optimization method of a simple-chamber muffler, designed to work in a limited space, by using Transfer Matrix Method (TMM) conjugated with a three-dimensional graphic analysis. To obtain a good acoustic performance for the shape optimisation of mufflers, novel schemes have appeared such as Genetic Algorithm (GA) and Simulating Annealing (SA) [12-14]. Min-Chie and Yeh [15] used Boundary Element Method, Mathematic Gradient Method and Genetic Algorithm to optimize a constrained muffler composed of a single chamber connected to inlet/outlet sides. The results of this optimization showed that those methods are very

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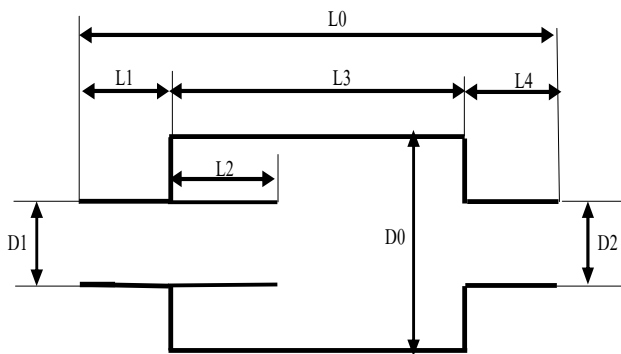
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important to predict and attenuate the noise of mufflers. This paper discusses the noise attenuation.

## 2 Theoretical and numerical assessment

### 2.1 Silencer representation

Reactive silencers, commonly used in automotive applications, reflect the sound waves back towards the source and prevent sound from being transmitted along the pipe. The design of silencer is based on the principle of a Helmholtz resonator. It requires the use of acoustic transmission line theory. The Figure 1 shows an outline of the muffler used in this paper, it consists of a concentric expansion chamber with an extended inlet pipe and an end outlet pipe.



**Figure 1:** Sketches of the expansion-chamber muffler with extended inlet tube.

The most widely used performance characterizing mufflers is the transmission loss (STL), other indexes are however available such as insertion loss (IL) and the pressure loss (PL). STL only depends on the muffler and not on the source (inlet and outlet length) nor on impedance. It is considered as the best parameter to use when comparing different methods, and designs [7]. In this paper, two approaches were used to analyze the acoustic performance of single expansion-chamber muffler with extended tube under chosen limited space ( $L = 1.6$  m,  $D_0 = 0.5$  m) [11, 14]. These approaches are the Sound acoustic power and the Transfer matrices methods. The computational models are developed using Matlab Tool for the shape optimization by Threshold Acceptance and Simulated Annealing algorithms and the COMSOL Multiphysics Tools for the finite element analysis.

### 2.2 Shape optimization method

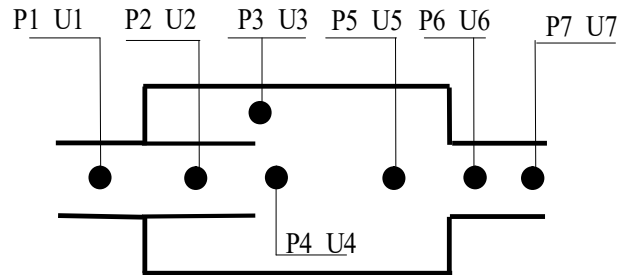
Transfer Matrix Method (TMM) is based on the plane waves models that can offer fast initial prototype solutions for the assessment of muffler's optimal shape design. In this paper, the four-pole system matrix evaluating the acoustic performance using sound transmission loss parameter (STL) is derived by using a decoupled numerical method [10]. Two optimization methods using Threshold Acceptance and Simulated Annealing algorithms are applied on the shape optimizations of the expansion-chamber muffler with inlet

### Notation

$dw$	acoustic energy
$D_i$	diameter of muffler's components (m)
$iter_{max}$	maximum iteration
$L_i$	length of muffler's components (m)
$M_i$	mean flow Mach number at $i$
$P$	total flow pressure (Pa)
$P_i$	pressure; acoustic pressure at $i$ (Pa)
$pb(T)$	transition probability
$Q$	volume flow rate of venting gas ( $m^3 s^{-1}$ )
$S_i$	section area at $i$ ( $m^2$ )
$T_0$	initial temperature ( $^{\circ}C$ )
$v_i$	Acoustic mass velocity at $i$ ( $kg s^{-1}$ )
$w_i$	Incoming power at the inlet
$w_o$	Outgoing power at the outlet

extended tube. Figure 2 represent the acoustical elements of the muffler, the acoustic pressure  $p$ , and the acoustic particle velocity  $u$ .

As shown in Figures 1 and 2, the single-expansion chamber muffler with extended inlet tube is composed of one-inlet tube elements, two straight ducts and one contraction duct element. Seven nodes represent the related acoustic pressure  $p$  and acoustic particle velocity  $u$  within the muffler. Because of the remarkably pure tone noise effect at 300 Hz [16], noise elimination at this frequency by shape optimization is applied.



**Figure 2:** Sketches of the one-dimensional plane wave propagation expansion-chamber muffler with extended inlet tube.

### 2.3 Theoretical formulation

The four-pole system matrix evaluating the acoustic performance known as TMM uses  $2 \times 2$  matrices to relate two variables at planes on either side of an acoustic component. The matrices for individual components can be readily combined to form a single, overall matrix that describes the behavior for a multi-component muffler system [6].

$$\begin{pmatrix} p_1 \\ u_1 \end{pmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{pmatrix} p_7 \\ u_7 \end{pmatrix}. \quad (1)$$

As per the plane wave theory, the individual transfer matrices between the following distribution points: 1 and 2; 3 and 5; 5 and 6; 6 and 7 are calculated, below is an example between points 1 and 2:

$$\begin{pmatrix} p_1 \\ \rho_0 c_0 u_1 \end{pmatrix} = e^{-j \frac{j M_1 k L_{12}}{(1-M_1^2)}} \begin{bmatrix} \cos\left(\frac{k L_{12}}{1-M_1^2}\right) & j \sin\left(\frac{k L_{12}}{1-M_1^2}\right) \\ j \sin\left(\frac{k L_{12}}{1-M_1^2}\right) & \cos\left(\frac{k L_{12}}{1-M_1^2}\right) \end{bmatrix} \begin{pmatrix} p_2 \\ \rho_0 c_0 u_2 \end{pmatrix}; \quad (2)$$

where  $\rho_0$ ,  $k$ ,  $M$  and  $c_0$  represent respectively the air density, wave number, Mach number and wave celerity.

For the Extended Tube, the equation of mass continuity between point 2 and point 4 is used with mean flow [5-11], the transfer matrix is illustrated as:

$$\begin{pmatrix} p_2 \\ \rho_0 c_0 u_2 \end{pmatrix} = \begin{bmatrix} t_{1,1} & t_{1,2} \\ t_{2,1} & t_{2,2} \end{bmatrix} \begin{pmatrix} p_4 \\ \rho_0 c_0 u_4 \end{pmatrix}. \quad (3)$$

Assembling the individual matrices for all points, we can write:

$$\begin{pmatrix} p_1 \\ \rho_0 c_0 u_1 \end{pmatrix} = \begin{bmatrix} T_{1,1} & T_{1,2} \\ T_{2,1} & T_{2,2} \end{bmatrix} \begin{pmatrix} p_7 \\ \rho_0 c_0 u_7 \end{pmatrix}. \quad (4)$$

The STL of a muffler calculated as [5]

$$STL(f, Q, R_1, R_2, R_3, R_4) = 20 \log \left( \frac{T_{1,1} + T_{1,2} + T_{2,1} + T_{2,2}}{2} \right) + 10 \log \left( \frac{S_1}{S_2} \right); \quad (5)$$

where  $f$  is the wave frequency,  $S_1$ ,  $S_2$  the sections and  $Q$  represent the sound flow.

$$R_1 = \frac{D_1}{D_0}; R_2 = \frac{D_2}{D_0}; R_3 = \frac{L_3}{L_0}$$

$$R_4 = \frac{L_2}{L_0} \quad L_1 = \frac{1}{2}(L_0 - L_3) \quad L_4 = \frac{1}{2}(L_0 - L_3)$$

$$L_0 = L_1 + L_3 + L_2$$

## 2.4 Simulated annealing algorithm

Simulated annealing is a generalization of a Monte Carlo method for examining the equations of state and frozen states of n-body systems [Metropolis et al. 1953] [17]. The concept is based on the way liquids freeze or metals recrystallize in the process of annealing. In the annealing process of a melt, initially at high temperature and disordered, the metal is slowly cooled so that the system at any time is approximately in thermodynamic equilibrium. As cooling proceeds, the system becomes more ordered and approaches a "frozen" ground state at  $T=0$ . Hence, the process is considered as an adiabatic approach to the lowest energy state. If the initial temperature of the system is too low or cooling is done insufficiently slowly, the system may become quenched forming defects or freezing out in metastable states (means trapped in a local minimum energy state) [18].

To imitate the evolution of the SA algorithm, a new random solution ( $X'$ ) is chosen from the neighborhood of the current solution ( $X$ ). If the change in objective function (or energy) is negative (ie.  $\Delta F \leq 0$ ), the new solution will be acknowledged as the new current solution with transition property ( $pb(X')$ ) of 1.

If not (ie.  $\Delta F > 0$ ), the new transition property ( $pb(X')$ ) varied from 0~1 will be first calculated by the Boltzmann's factor ( $pb(X') = \exp(-\frac{\Delta F}{CT})$ ) as shown in mufflers [16].

$$pb(X') = \begin{cases} 1, \Delta F \leq 0 \\ \exp(-\frac{\Delta F}{CT}), \Delta F > 0 \end{cases}; \quad (6)$$

$$\Delta F = F(X') - F(X)$$

where  $C$  and  $T$  are the Boltzmann constant and current temperature respectively; moreover, compared with the new random probability of  $\text{rand}(0, 1)$ . If the transition property ( $pb(X')$ ) is greater than a random number of  $\text{rand}(0, 1)$ , the new worse solution which results in a higher energy condition will then be accepted. Otherwise, it is rejected. The algorithm repeats the perturbation of the current solution and the measurement of the change in the objective function. To reach an initial transition probability of 0.5, the initial temperature ( $T_0$ ) is selected as 0.2 [16]. Each successful substitution of the new current solution will lead to the decay of the current temperature as:

$$T_{new} = kk * T_{old}; \quad (7)$$

where  $kk$  is the cooling rate.

The process is repeated until the predetermined number (iter) of the outer loop is reached.

## 2.5 Threshold acceptance algorithm

The Threshold Accepting metaheuristic algorithm (TA) is a modification of the Simulated Annealing metaheuristic [19]. TA algorithm uses a predetermined deterministic sequence to decide whether a new point is selected or not (if worse than the current point), whereas the simulated annealing algorithm probabilistically determines in every iteration. Dueck and Scheurer [20] simplified the Simulated Annealing procedure by leaving out the probabilistic element in accepting worse solutions. Instead, they introduced a deterministic threshold,  $\tau$  and a worse solution is accepted if its difference to the incumbent solution is smaller or equal to the threshold. The new procedure is named Threshold Accepting algorithm.

The key components of TA are the function  $g(t)$  that determines the lowering of the threshold during the procedure, stopping criteria as well as the methods used to create initial and neighboring solutions. The main advantages of TA are its conceptual simplicity and its excellent performance on different combinatorial optimization problems [21].

## 2.6 FEM analysis method

In this paper, the finite element analysis method is used to analyze the acoustic performance of the shape optimized expansion-chamber muffler by using the available COMSOL Multiphysics tool including 3D linear acoustic codes. The analyze is programmed with and without mean flow, where the most important effect of flow, without considering the mean flow, is included by altering the boundary conditions of the muffler [22].

The FEM model solves the problem in the frequency domain using the time-harmonic pressure acoustics mode [23]. The equation of the model is a slightly modified version of the Helmholtz equation for the acoustic pressure,  $p$ :

$$\nabla \cdot \left( -\frac{\nabla p}{\rho} \right) - \frac{\omega^2 p}{c_s^2 \rho} = 0 ; \quad (8)$$

where  $\omega = 2\pi f$ , and where  $\omega$ ,  $\rho$ ,  $c_s$  are respectively the angular frequency, the fluid density, and the speed of sound.

The software computes integrals in the power expressions using boundary integration coupling variables, and it plots the resulting attenuation versus frequency.

The following equation defines the attenuation of the acoustic energy  $d_w$ (dB):

$$d_w = 10 \log \left( \frac{w_o}{w_i} \right). \quad (9)$$

Here  $w_o$  and  $w_i$  denote the outgoing power at the outlet and the incoming power at the inlet, respectively. Each of these quantities calculated as an integral over the corresponding surface  $A$ :

$$w_o = \int_{\partial\Omega} \frac{|p|^2}{2\rho c_s} dA. \quad (10)$$

$$w_i = \int_{\partial\Omega} \frac{p_0^2}{2\rho c_s} dA. \quad (11)$$

The sound transmission loss is calculated directly in COMSOL tool using the acoustic power at the inlet and the outlet of the acoustic system. The shape-optimized muffler is simulated using a three-dimensional model and meshed by using the Lagrange-quadratic elements. A harmonic pressure of 1Pa is specified at the inlet's muffler and a radiation boundary condition is applied at the inlet and outlet. Then a material with default values of air is created with density of 1.2 kg/m<sup>3</sup>, and the sound speed is 340 m/s. using the default values of air, the acoustic damping of air is not considered.

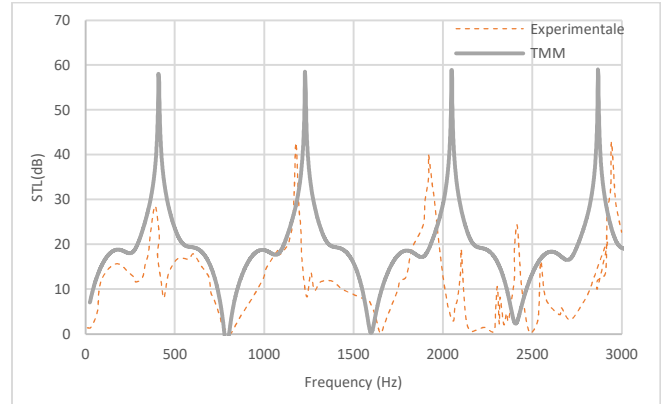
## 3 Case studies

To check the transmission loss model on the basic single-chamber muffler a comparison between theoretical and experimental data is performed in this paper [3]. As shown in figure 3, there is a coherence between the theoretical results based on transmission matrix method (TMM) and

experimental data. Hence, the transmission loss model is acceptable and can be used to the studied models.

The available space selected for the muffler is 0.3 m in width, 0.3 m in height, and 1.5 m in length. To obtain the best acoustical performances of the muffler within a fixed space; a pure tone noise of 300 Hz is introduced as a numerical case.

A maximization of the sound transmission loss is performed with respect to the shape-optimized muffler obtained by simulated annealing and threshold acceptance algorithms. For the purpose of an accuracy check, various targeted pure tones (250, 500, 700, 800Hz) are applied.



**Figure 3:** Performance curves of Sound transmission loss (STL), comparison between TMM theoretical model and experimental values of simple expansion chamber muffler [3].

In the second part of this work, we examine the sound transmission properties of the idealized expansion-chamber muffler with extended inlet tube by finite element method with a data tool COMSOL Multiphysics. 3D simulation analysis is used in this step with parametric solver providing results for a range of frequencies. The software computes integrals in the power expressions using boundary integration coupling variables, and it plots the resulting attenuation versus frequency.

## Objective function

As per formula (5) and under the assumption of the symmetric design ( $L1 = L2 = (L0-L3)/2$ ), the objective function maximizing the sound transmission loss at a pure tone ( $f$ ) [11] is

$$Obj = STL(f, Q, R_1, R_2, R_3, R_4). \quad (12)$$

The related ranges of parameters are  $Q = 0.01$  (m<sup>3</sup>/s);  $R_1: [0.2, 0.8]$ ;  $R_2: [0.2, 0.8]$ ;  $R_3: [0.5, 0.9]$ ;  $R_4: [0.5, 0.9]$ .

As mentioned, the optimization process done by simulated annealing and threshold acceptance algorithms with respect to the objective function  $Obj$  is performed by varying the control parameters: cooling rate ( $kk$ ) and the number of iteration ( $iter_{max}$ ).

## 4 Results and discussion

One of the important parameters for the optimization accuracy used by the SA and TA algorithms are the cooling

rate ( $kk$ ) and the number of iteration ( $iter_{max}$ ) (cave et al., 2002). The Shape optimization of the muffler is performed with Matlab tool by varying the two parameters gradually during optimization process, then the numerical prediction is compared with the finite element's solution.

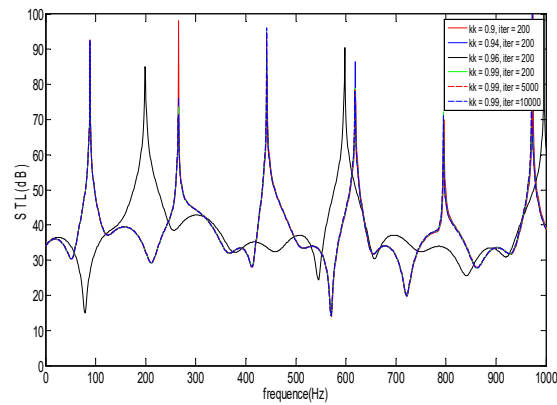
The sound control of pure tone noise with 300 Hz is introduced as the numerical case.

Various sets of parameters are tested during optimal process; Table 1 and 2 show the simulated result optimized with respect to the pure tone of 300Hz. The optimal design data is obtained at the cooling rate  $kk = 0.99$  and iteration number  $iter_{max} = 5000$  for the Simulated Annealing (SA) and  $kk = 0.94$  and iteration number  $iter_{max} = 5000$  for Threshold Acceptance (TA).

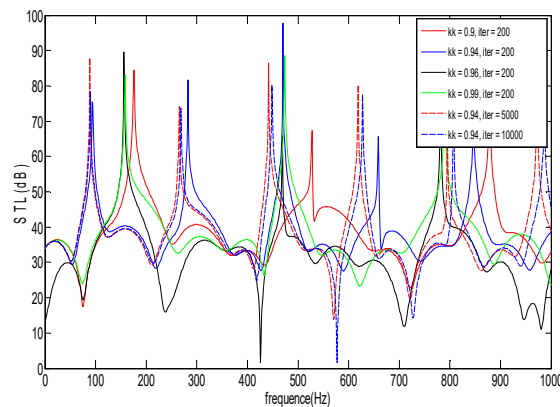
The acoustic performances of sound transmission loss, with respect to frequency in various design case, are plotted in figures 4 and 5, for the SA and TA methods respectively, we notice that at frequencies higher than approximately 750 Hz, which represent the cut-off frequency, there is generally less damping.

The plots reveals also that the highest values of the parameters ( $kk$ ,  $iter_{max}$ ) gave the highest STL and these attenuations are roughly maximized at the desired frequencies, therefore,  $kk$  and  $iter_{max}$  parameters variation play essential roles in SA and TA optimizations and using these optimization methods in finding the optimal design solution is reliable.

By using the optimal design in a theoretical calculation of the expansion-chamber muffler, the muffler's optimal sizes with respect to various pure tones are shown in Tables 3 and 4, for the SA and TA methods respectively.



**Figure 4:** Performance curves of STL with respect to various maximal iterations ( $iter_{max}$ ) by Simulated annealing ( $To = 0.2$ ).



**Figure 5:** Performance curves of STL with respect to various maximal iterations ( $iter_{max}$ ) by Threshold acceptance ( $To = 0.2$ ).

**Table 1:** Optimal STL by SA method with targeted tone of 300 Hz and various  $kk$  and  $iter_{max}$  (iter).

Case	SA parameters	Results				
		R1	R2	R3	R4	STL (dB)
1	$kk = 0.9$ , $iter_{max} = 250$	0,100000000	0,100005595	0,799180631	0,799289484	34,239752
2	$kk = 0.93$ , $iter_{max} = 250$	0,100001448	0,100243207	0,79995194	0,799371918	34,232275
3	$kk = 0.96$ , $iter_{max} = 250$	0,100000008	0,100010386	0,799902935	0,35483868	33,049906
4	$kk = 0.99$ , $iter_{max} = 250$	0,100000008	0,100000000	0,799773512	0,799833439	34,245198
5	$kk = 0.99$ , $iter_{max} = 500$	0,100000002	0,100000003	0,799999629	0,798960195	34,241627
6	$kk = 0.99$ , $iter_{max} = 1000$	0,100000001	0,100000000	0,79999991	0,799998591	34,246961
7	$kk = 0.99$ , $iter_{max} = 1500$	0,100000000	0,100000000	0,799999964	0,799999797	34,246967
8	$kk = 0.99$ , $iter_{max} = 2000$	0,100000000	0,100000039	0,79998701	0,799998671	34,246908
9	$kk = 0.99$ , $iter_{max} = 5000$	0,100000000	0,100000000	0,799999989	0,799999981	34,246968
10	$kk = 0.99$ , $iter_{max} = 10000$	0,100000000	0,100000000	0,799980814	0,799998117	34,246882

**Table 2:** Optimal STL by TA method with various  $kk$  and  $iter_{max}$  (Targeted tone of 300 Hz).

Case	TA parameters	Results				
		R1	R2	R3	R4	STL (dB)
1	$kk = 0.9,$ $iter_{max} = 250$	0,100001665	0,100003713	0,799999117	0,402021553	33,1111
2	$kk = 0.94,$ $iter_{max} = 250$	0,100049917	0,100455321	0,79788546	0,753029291	33,9849
3	$kk = 0.96,$ $iter_{max} = 250$	0,343493076	0,100000031	0,767402627	0,471203302	30,6808
4	$kk = 0.99,$ $iter_{max} = 250$	0,100394942	0,100003418	0,743257284	0,481595607	32,8829
5	$kk = 0.94,$ $iter_{max} = 500$	0,10000002	0,100377302	0,798539371	0,799987046	34,2238
6	$kk = 0.94,$ $iter_{max} = 1000$	0,101163199	0,100004002	0,799895123	0,470203404	33,1370
7	$kk = 0.94,$ $iter_{max} = 1500$	0,100060575	0,102657842	0,767058504	0,795454055	33,9477
8	$kk = 0.94,$ $iter_{max} = 2000$	0,107972696	0,100286787	0,788587376	0,608442604	32,7582
9	$kk = 0.94,$ $iter_{max} = 5000$	0,10000466	0,100007901	0,799544694	0,799988801	34,2441
10	$kk = 0.94,$ $iter_{max} = 10000$	0,100000893	0,10059617	0,788518539	0,799312242	34,1680

**Table 3:** Optimal STLs by SA for expansion-chamber muffler with extended inlet tube with respect to various targeted frequencies ( $kk=0.99$ ,  $iter_{max} = 10000$ ).

Case	Target frequency	Results				
		R1	R2	R3	R4	STL (dB)
1	250Hz	0,100000000	0,100000000	0,799999993	0,799999989	33,4685
2	500Hz	0,100000000	0,100000040	0,800000000	0,800000000	37,9647
3	700Hz	0,181190078	0,350935552	0,792555241	0,640175448	113,1414
4	800Hz	0,115139572	0,155598287	0,719181217	0,61730614	118,0296

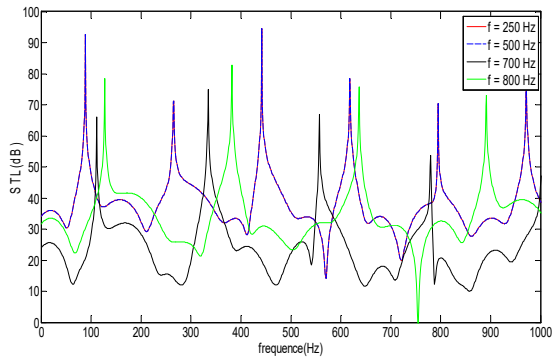
**Table 4:** Optimal STLs by TA for expansion-chamber muffler with extended inlet tube with respect to various targeted frequencies ( $kk=0.99$ ,  $iter_{max} = 10000$ ).

Case	Target frequency	Results				
		R1	R2	R3	R4	STL (dB)
1	250Hz	0,100507767	0,100843896	0,79543887	0,790742762	33,2806
2	500Hz	0,1	0,1	0,8	0,8	37,9647
3	700Hz	0,180830576	0,131903453	0,678432782	0,747856159	122,4975
4	800Hz	0,184036953	0,496377168	0,767474176	0,578452178	131,5955

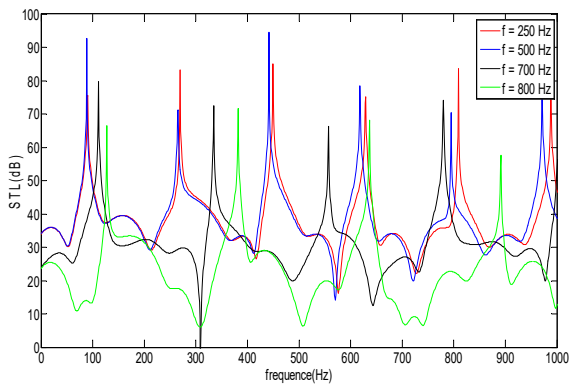
The optimal STL curves with respect to targeted frequencies are plotted in figure 6 and 7 according to SA and TA methods respectively. The case studies show that increasing the pure tone expands the frequency bandwidth and the STLs are precisely maximized at the desired

frequencies and TA method improves the acoustical performance of the muffler better than SA method.

The second part of this study, concern a 3D analysis by FEM of the propagation modes of the shape optimized expansion-chamber muffler with extended tube with target tone of 800Hz.

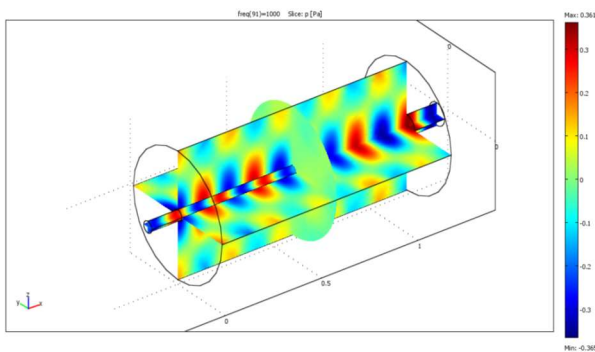


**Figure 6:** STL with respect to frequencies of the muffler for various pure tones of SA (Targeted frequency: 250, 500, 700 and 800 Hz)



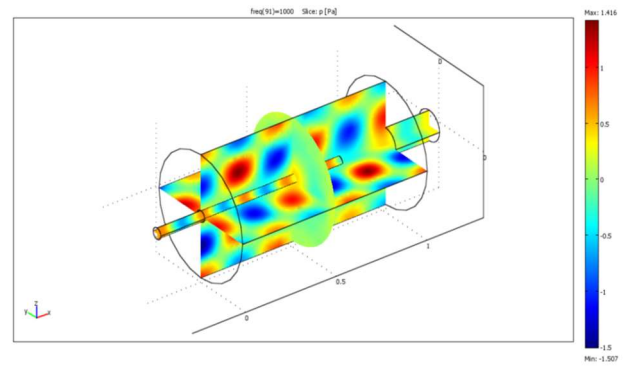
**Figure 7:** STL with respect to frequencies of the muffler for various pure tones of TA (Targeted frequency: 250, 500, 700 and 800 Hz).

This analysis method is performed using COMSOL tool, the optimal shape design of the muffler according to the SA and TA methods are designed, we applied the required boundary conditions and performed the meshing with a coarse predefined mesh sizes with  $0.25 \times$  – direction scale. The internal sound pressure distribution at 1500 Hz of the shape-optimized mufflers using the two methods is displayed in figure 8 and 9.



**Figure 8:** Optimized FEM model of the muffler using SA and internal sound pressure distribution at 1500 Hz (3D View)

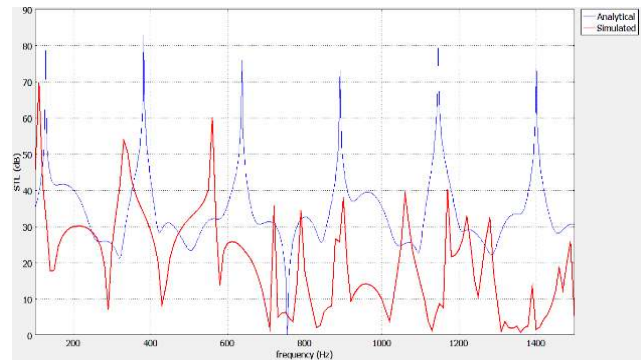
We notice that the attenuation is more important in the shape optimized muffler with TA method than with SA method. In addition, the pressure field varies primarily with



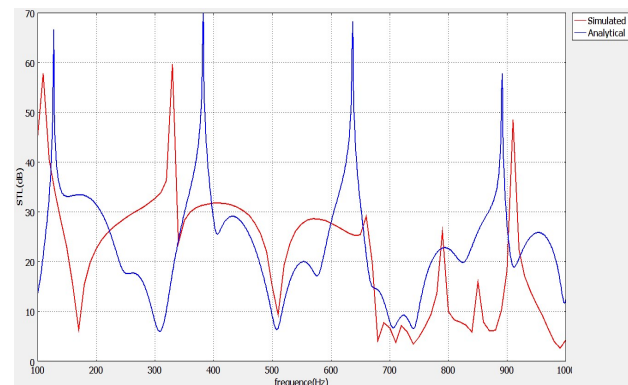
**Figure 9:** Optimized FEM model of the muffler using TA and internal sound pressure distribution at 1500 Hz (3D View)

the y-coordinate, while it is nearly constant in the z direction as the frequency 1500 Hz is just higher than the cut-off frequency for the first symmetric propagating mode excited by the incoming wave. We can also observe for the selected frequencies how the distributions of the sound pressure level near the muffler extended inlet and outlet tube is important.

Figures 10 and 11, plots the theoretical transmission loss based on the TMM (blue line) and the COMSOL Multiphysics solution (red line) as a function of frequency for the two shape optimized mufflers by SA and TA method.



**Figure 10:** Shape optimization by SA; STL versus Frequency: theoretical solution (blue line) and COMSOL Multiphysics solution (red line) (Targeted frequency: 800Hz).



**Figure 11:** Shape optimization by TA, STL versus Frequency: theoretical solution (blue line) and COMSOL Multiphysics solution (red line) (Targeted frequency: 800Hz).

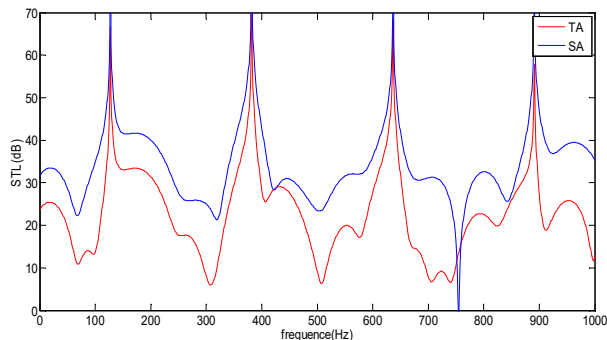
The FEM solution has an upper frequency limit for its validity known as the cut-off frequency, which defines the frequency range where only plane waves can propagate; above this frequency, also higher modes can propagate.

At frequencies higher than approximately 750 Hz, the plot's behavior is more complicated and there is generally less damping. This is because, for such frequencies, the tube supports not only longitudinal resonances but also cross-sectional propagation modes.

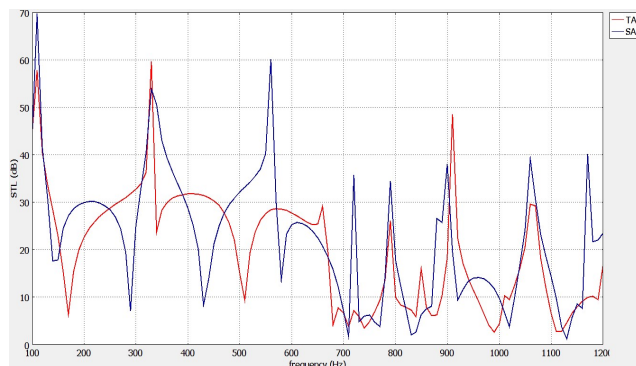
We notice from figures 10 and 11 that a discrepancy exists between the theoretical and the FEM solution. Even below the cut-on frequency, this discrepancy is due to those the elementary transfer matrices depend on the element that is modelled. The transmission loss (STL) calculated by TMM is independent of the source and does not involve neither the source nor the radiation impedance of the termination, whereas for the sound acoustic power method, it depends only on the sound source and does not allow the transfer matrices of the acoustic system to be obtained.

The differences obtained in the results plot may also be attributed to finite element formulation used in COMSOL tool that is Lagrange elements or to the computational applied meshes (density and refinement).

In figure 12, we compare the theoretical results between SA and TA optimization methods, as shown in the figure, the attenuation discrepancy between the TA and SA method is about 10dB. Same comparison and result is done for the attenuation obtained by the FEM solutions between the SA and TA optimization methods, as shown in Figure 13.



**Figure 12:** Comparison of the theoretical solution based on the TMM between SA and TA (Targeted frequency: 800Hz).



**Figure 13:** Comparison of the FEM solution between SA and TA (Targeted frequency: 800Hz).

## 5 Conclusion

The shape optimization of Expansion chamber Muffler with inlet extended tube under space constraints is applied in this paper by using two novel schemes Simulated annealing and Threshold Acceptance coupled with 3D Finite Element analysis,

The SA and TA optimizers are based on the Transfer Matrices Method applying the plane wave theory as well as four-pole transfer matrices. The optimization process of the reactive muffler showed the importance of the parameters ( $kk$ ,  $iter_{max}$ ), also it reveals that this method is valid when the influence of high order modes can be neglected.

This numerical analysis using SA and TA optimizers has shown to be an interesting method to optimize reactive mufflers under space constraints.

The comparison between the numerical prediction based on the TMM and the FEM solution based on the sound acoustic power has shown discrepancies in the curves. This is due to that the first method depends only on the element, which is modelled, and not on the sound source. Whereas for second method depend on the sound source and does not allow the transfer matrices of the acoustic system to be obtained, also it depends on the finite element formulation and the computational meshes used in COMSOL tools.

The optimization method coupled with the FEM analysis revealed that the TA has a better acoustic performance than SA. Consequently, the approach used for the optimal design of the STL proposed in this study is interesting in dealing with the reactive muffler within a space-constrained situation.

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## References

- [1] F.B. Randall, Industrial Noise Control and Acoustics, *Marcel Dekker Inc.*, 2001.
- [2] S. Bilawchuk, K. R. Fyfe, Comparison and implementation of the various numerical methods used for calculating transmission loss in silencer systems, *Applied Acoustics*, Vol. 64, 2003, pp. 903-916.
- [3] S.N.Y. Gerges, R. Jordan, F.A. Thime, J.L. Bento Coelho, J.P. Arenas, Muffler Modeling by Transfer Matrix Method and Experimental Verification, *J. Braz. Soc. Mech. Sci.& Eng.*, Vol. 27, No. 2, 2005, pp. 132-140.
- [4] M. L. Munjal, Acoustics of Ducts and Mufflers with Application to Exhaust and Ventilation System Design, *John Wiley & Sons, Inc.*, 1987, pp. 328.
- [5] M. L. Munjal, Advances in the acoustics of flow ducts and mufflers, *Sadhana, Academy Proceeding in Engineering and Science*, Vol. 15, No. 2, 1990, pp. 57-72.
- [6] M.L. Munjal, Plane wave analysis of side inlet/outlet chamber mufflers with mean flow, *Applied Acoustics*, Vol. 52, No. 2, 1997, pp. 165-175.

- [7] K.S. Andersen, Analyzing Muffler performance using the Transfer Matrix Method, *Proceeding of Consol Conference*, 2008.
- [8] E. Bécache, A.S. Bonnet-Bendhia, Numerical simulation of exhaust muffler, An homogenized finite element method, *Cari'06*, Vol. 1, 2006, pp. 1-10.
- [9] L.J. Yeh, M.C. Chiu, Computer aided design on single expansion muffler under space constraints, *Proceeding 19th National Conference in Mechanic and Engineering*, Vol. C7, 2002, pp. 625-633.
- [10] R.J. Bernhard, Shape Optimization of Reactive Mufflers, *Noise Control and Engineering Journal*, Vol. 27, No.1, 1986, pp. 10-17.
- [11] L.J. Yeh, M.C. Chiu, G.J. Lay, Computer-Aided Optimal Design of a Single-Chamber Muffler with Side Inlet/Outlet under Space Constraints, *Journal of Marine Science and Technology*, Vol. 11, No. 4, 2003, pp. 1-8.
- [12] Y.C. Chang, M.C. Chiu, Numerical Optimization of Single-chamber Mufflers Using Neural Networks and Genetic Algorithm, *Turkish Journal of Engineering and Environment*, Vol. 32, 2008, pp. 313-322.
- [13] Y.C. Chang, L.J. Yeh, Shape optimization on constrained single-chamber muffler by using GA method and mathematical gradient method, *International Journal of Acoustics and Vibrations*, Vol. 10, No. 1, 2005, pp. 17-25.
- [14] L.J. Yeh, Y.C. Chang, GA optimization on multi-segments muffler under space constraints, *Applied Acoustics*, Vol. 65, No. 5, 2004, pp. 521-543.
- [15] M.C. Chiu, L.J. Yeh, Shape Optimization of Single-Chamber Mufflers with Side Inlet/Outlet by Using Boundary Element Method, Mathematic Gradient Method and Genetic Algorithm, *Tamkang Journal of Science and Engineering*, Vol. 12, No. 1, 2009, pp. 85-98.
- [16] L.J. Yeh, Y.C. Chang, M.C. Chiu, Shape Optimal Design on Double-Chamber Mufflers Using Simulated Annealing and a Genetic Algorithm, *Turkish Journal of Engineering and Environment*, Vol. 29, 2005, pp. 207- 224.
- [17] S. Kirkpatrick, C. D. Gelatt, Jr., M. P. Vecchi, Optimization by simulated annealing, *Science*, Vol.220, No.4598, 1983, pp. 671-680.
- [18] M. Yagiura, T. Ibaraki, on metaheuristic algorithms for combinatorial optimization problems, *Systems and Computers in Japan*, Vol.32, 2001, pp. 33-55.
- [19] O. Bräysyean, J. Berger, A Threshold Accepting Metaheuristic for the Vehicle Routing Problem with Time, *Applied Mathematics*, 2003.
- [20] G. Dueck, T. Scheurer, Threshold accepting: A general-purpose optimization algorithm appearing superior to simulated annealing, *Journal of Computational Physics*, Vol. 90, 1990, pp. 161-175.
- [21] D. Givoli, B. Neta, High-order non-reflecting boundary scheme for time-dependent waves, *Journal of computational Physics*, Vol. 186, 2003, pp. 24-46.
- [22] COMSOL 3.4, Acoustics Module Model Library, *COMSOL AB*, 2007, pp. 74-85.
- [23] D. Givoli, B. Neta, High-order non-reflecting boundary scheme for time-dependent waves, *Journal of Computational Physics*, Vol. 186, 2003, pp. 24-46.



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