ADDED VISCOUS DAMPING IN MICROPERFORATED PLATES WITHIN A NONLINEAR ACOUSTIC REGIME

Lucie Gallerand*[1], Mathias Legrand †[2], Thomas Dupont ‡[1], Raymond Panneton §[3], and Philippe Leclaire ¶[4]
[1]Department of Mechanical Engineering, École de Technologie Supérieure, Montréal, Canada
[2]Department of Mechanical Engineering, McGill University, Montréal, Canada
[3]Department of Mechanical Engineering, Université de Sherbrooke, Sherbrooke, Canada
[4]DRIVE EA1859, Université de Bourgogne, ISAT, Nevers, France

1 Introduction

Microperforated plates (MPPs) are simple systems conventionally used as sound absorbers. In the linear acoustic regime, models characterizing MPP acoustic absorption based on (i) the work by Champoux and Stinson [1] and the Johnson-Champoux-Allard model applied to perforated plates [2] along with (ii) the Kirchhoff equations [3] have been proposed. Extensions to a nonlinear acoustic regime were performed using the Forchheimer law [4] and showed a maximum acoustic absorption for a specific value of fluid velocity. Recent works by the authors in the context of linear dynamics have demonstrated MPP added damping capacities in the low frequency range [5]. The added damping results from the dissipation of energy by viscous friction mechanisms in the boundary layers of the microperforations. MPPs are also susceptible to be implemented in hostile environments with strong fluid displacements in the perforations. The present contribution proposes to extend the MPP linear *vibration* model to the nonlinear framework involving the nonlinear acoustic Forchheimer law.

2 Linear MPP model

2.1 Governing equations

The investigated MPP of dimension $L_x \times L_y \times h$ is oriented in the xy plane and excited by an external periodic driving force $f_{\text{ext}}(x, y, t)$. Using an alternative form of Biot's theory, the model developed in the framework of porous plates in [6] is adapted to an MPP. An *ad hoc* homogenization procedure is performed, leading to two coupled partial differential equations (PDEs) presented in Equation (1) that govern the dynamics of a structural plate and a virtual fluid plate. Obtained by identifying the MPP with a porous plate [2], they account for the vibratory behavior of the MPP and read:

$$h(\rho \ddot{w}_{\text{s}} + \rho_{\text{f}} \ddot{w}) + D\nabla^4 w_{\text{s}} = f_{\text{ext}}, \tag{1a}$$

$$\rho_{\text{f}} \ddot{w}_{\text{s}} + \frac{\rho_{\text{f}} \alpha_\infty}{\phi} \ddot{w} + \sigma_0 \dot{w} + \alpha M_{\text{f}} \nabla^2 w_{\text{s}} = 0, \tag{1b}$$

where $w_{\text{s}}(x, y, t)$ is the solid motion and $w(x, y, t)$ corresponds to the relative fluid-solid motion. The density of the fluid-solid mixture $\rho = (1-\phi)\rho_{\text{s}} + \phi\rho_{\text{f}}$ where ρ_{s} and ρ_{f} are the densities of solid and fluid, respectively, depends on the perforation ratio ϕ. Equation (1a) represents the elastic response of the homogeneous solid plate while Equation (1b) describes the relative fluid-solid motion. The parameters α and M_{f} are elastic coefficients defined by Biot [7]. The coefficient D is the bending stiffness. In order to consider the influence of the microperforations in the MPP stiffness, it becomes

$$D = \frac{Eh^3}{12(1-\nu^2)} \frac{(1-\phi)^2}{1 + (2 - 3\nu)\phi}. \tag{2}$$

All Johnson-Champoux-Allard (JCA) parameters defined for a porous medium can be rewritten for an MPP as functions depending on ϕ and the diameter of the perforations d. For instance, the resistivity and the tortuosity are defined by

$$\sigma_0 = \frac{32\mu_{\text{f}}}{\phi d^2} \quad \text{and} \quad \alpha_\infty = 1 + \frac{2\epsilon}{h} \tag{3}$$

with μ_{f} the fluid dynamic viscosity and $\epsilon = 0.24\sqrt{\pi d^2}(1 - 1.14\sqrt{\phi})$ [2] is a correction factor used to consider the interaction between the perforations and the distortion of the flow at the perforation orifices.

2.2 Added damping

For an MPP, the existence of a substantial added damping in the low frequency range could be exhibited [5]. The added damping reaches a maximum at the characteristic frequency

$$f_{\text{c}}(d) = \frac{32\mu_{\text{f}}}{2\pi\alpha_\infty \rho_{\text{f}} d^2} \tag{4}$$

defined from the Biot frequency for porous materials [6]. In Equation (4), $f_{\text{c}}(d)$ depends on fluid parameter, μ_{f} and ρ_{f} and on d, which can be tuned to induce maximum added damping at the resonance frequency, i.e. to make $f_{\text{c}}(d)$ coincide with a resonance frequency of the microperforated plate.

3 Nonlinear MPP model

3.1 Governing equations

When the fluid velocity in the microperforations becomes sufficiently large, the inertial effects occurring in the microperforations become significant. It is then necessary to consider them by using the Forchheimer law $\sigma = \sigma_0(1 + \varepsilon|\dot{w}_{\text{f}}|)$, where ε is the Forchheimer coefficient and $|\dot{w}_{\text{f}}(x, y, t)|$ is the absolute value of the fluid velocity. This law was used for rigid MPPs with a high fluid flow in microperforations [4]. In the context of a vibrating microperforated plate, the relative fluid-solid velocity corresponds to the fluid velocity for a rigid

*lucie.gallerand.1@ens.etsmtl.ca
†mathias.legrand@mcgill.ca
‡thomas.dupont@etsmtl.ca
§Raymond.Panneton@USherbrooke.ca
¶Philippe.Leclaire@u-bourgogne.fr

MPP. Accordingly, the resistivity is expressed as a function of $\dot{w}(x,y,t) = \phi(\dot{w}_f(x,y,t) - \dot{w}_s(x,y,t))$

$$\sigma(\dot{w}(x,y,t)) = \sigma_0(1 + \varepsilon|\dot{w}(x,y,t)|). \quad (5)$$

Inserting Equation (5) in Equation (1) provides the following nonlinear system with a quadratic damping term:

$$h(\rho\ddot{w}_s + \rho_f\ddot{w}) + D\nabla^4 w_s = f_{ext}, \quad (6a)$$

$$\rho_f\ddot{w}_s + \frac{\rho_f\alpha_\infty}{\phi}\ddot{w} + \sigma_0\dot{w} + \sigma_0\varepsilon\dot{w}|\dot{w}| + \alpha M_f\nabla^2 w_s = 0. \quad (6b)$$

The resulting nonlinear governing equations are space semi-discretized and projected onto the non-perforated plate mode. The solid motion is expanded as

$$w_s(x,y,t) = \sum_i^N w_i^s(t)\Psi_i(x,y), \quad (7)$$

where $w_i^s(t)$ represents the generalized coordinate of eigenmode i of shape $\Psi_i(x,y)$; N is the number of degrees-of-freedom in the plate spacial discretization. A similar formulation is used for $w(x,y,t)$. After spacial discretization, Equation (6) is solved in the steady state regime by using the Harmonic Balance Method, which consists in the use of truncated Fourier series to describe the problem's unknowns.

3.2 Results

The model is used to study the vibratory behavior of a simply supported MPP in the nonlinear acoustic regime. An MPP of dimension $490\,\text{mm} \times 570\,\text{mm} \times 1\,\text{mm}$ with $d = 2.2\,\text{mm}$ and $\phi = 10\,\%$ is excited at one point by an external force of magnitude F_{ext}. The corresponding mobility of the plate, *i.e.* displacement divided by F_{ext}, is plotted as a function of dimensionless forcing frequency for three values of F_{ext} in Figure 1. The frequencies f_0 and f correspond respectively to

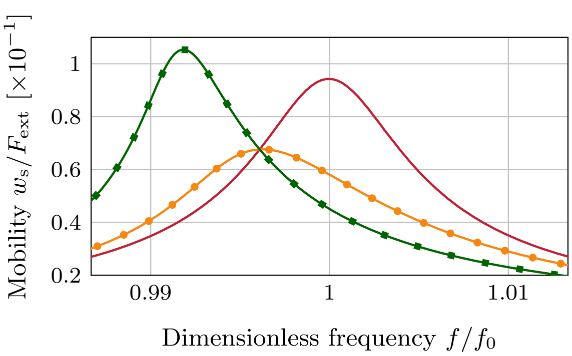

Figure 1: Mobility of the first linear microperforated plate mode for three magnitudes of external excitation: (——) $F_{ext} = 0.5\,\text{mN}$; (–●–) $F_{ext} = 20\,\text{mN}$; (–■–) $F_{ext} = 110\,\text{mN}$.

the resonance frequency obtained in the linear case and to the forcing frequency. The Forchheimer nonlinearity parameter $\varepsilon = 0.8\,\text{s/m}$ is obtained by experimental measurement. It can be observed that increasing the nonlinearity softens the system and that the added damping reaches a maximum for a specific value of F_{ext}, *i.e.* for a characteristic value of the magnitude

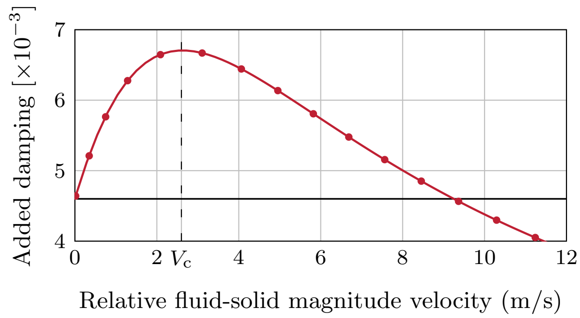

Figure 2: Added damping for the first linear MPP mode: (–●–) the nonlinear case; (——) the constant value of the linear case.

of the relative fluid-solid velocity as shown in Figure 2. A theoretical development provides an expression for a critical relative fluid-solid velocity

$$V_c = \frac{I_1^{(3)}}{I_1^{(4)}} \frac{\omega\rho_f\alpha_\infty - \sigma_0\phi}{\sigma_0\phi\varepsilon} \quad (8)$$

for which the added damping is maximum, where $I_1^{(3)}$ and $I_1^{(4)}$ are spacial integrals associated with the beam functions in the spacial projection of the coupled equations for the first linear plate mode. $I_1^{(4)}$ carries the Forchheimer acoustic nonlinear physical mechanism.

4 Conclusions

The vibratory response of a fluid-saturated MPP in a nonlinear acoustic regime is presented. The inertia effect due to the high velocity of the airflow in the microperforations is captured analytically by the Forchheimer law, which yields a system of coupled PDEs with a nonlinear damping term. The results show that the nonlinear damping introduced by the Forchheimer law softens the system. In addition, a critical relative fluid-solid velocity is found for which the added damping reaches a maximum.

References

[1] Y. Champoux and M.R. Stinson. On acoustical models for sound propagation in rigid frame porous materials and the influence of shape factors. *Journal of the Acoustical Society of America*, 92:1120–1131, 1992. DOI, OA.

[2] N. Atalla and F. Sgard. Modeling of perforated plates and screens using rigid frame porous models. *Journal of Sound and Vibration*, 303:195–208, 2007. DOI, OA.

[3] D.-Y. Maa. Potential of microperforated panel absorber. *Journal of the Acoustical Society of America*, 104:2861, 1997. DOI, OA.

[4] R. Tayong, T. Dupont, and P. Leclaire. On the variations of acoustic absorption peak with particle velocity in micro-perforated panels at high level of excitation. *The Journal of the Acoustical Society of America*, 127(5):2875–2882, 2010. DOI, OA.

[5] L. Gallerand, M. Legrand, T. Dupont, and P. Leclaire. Vibration and damping analysis of a thin finite-size microperforated plate. *Journal of Sound and Vibration*, 541:117295, 2022. DOI, OA.

[6] P. Leclaire, K.V. Horoshenkov, and A. Cummings. Transverse vibration of a thin rectangular porous plate saturated by a fluid. *Journal of Sound and Vibration*, 247(1):1–18, 2001. DOI, OA.

[7] M.A. Biot and D.G. Willis. The elastic coefficients of the theory of consolidation. *Journal of Applied Mechanics*, pages 594–604, 1957. DOI, OA.