ASSESSMENT OF EQUIVALENT PROPERTIES FOR MULTILAYERED PANELS

Diego Martin Tuozzo[*1] **and Noureddine Atalla**[†1]
[1]Département de Génie Mécanique, Université de Sherbrooke, Sherbrooke, Québec, Canada

1 Introduction

The use of lightweight complex heterogeneous structures increased during the last years principally in the transportation sector (i.e., aviation and trains). This sector's technology enhancement pursues reducing long-term CO_2 emissions and increasing efficiency. Lightweight structures may have poor vibro-acoustic behavior and in designs with complex shapes and material heterogeneities, its vibro-acoustic modeling brings new challenges in terms of accuracy and computational cost. Techniques such as model order reduction, homogenization, mesh and meshless methods (with and without periodicity conditions) and energy methods are typically employed to tackle this problem. Within homogenization techniques, an equivalent properties strategy can be utilized to equivalently represent complex structures into more simple ones (for example, a single layer panel). The latter is named equivalent structure or carrier and it is responsible for representing certain conserved quantities of the original structure.

This work aims to stress the use and limitations of two equivalent properties strategies applied to multilayered structures. Limitations provide insight into the potential applicability of these strategies in complex structures. The whole numerical procedure is computed utilizing the General Laminate Model (GLM) [1, and related papers] in a wave and forced response context. In the first homogenization strategy, the carrier is represented by a single layer isotropic plate. In the second one, the carrier is a thin orthotropic plate. To verify the effectiveness of both strategies comprehensive examples of multilayered structures are presented.

2 Method

In this section, the necessary theoretical background of the two carriers utilized for the proposed homogenization strategies is presented.

2.1 Theoretical background

The first carrier is a single layer thin isotropic plate. To obtain the equivalent material properties, the dynamic equations which describes the in-plane and out-of-plane motion of every point within the plate in three perpendicular directions ($u(x, y, t)$, $v(x, y, t)$ and $w(x, y, t)$) are utilized (see e.g., [2, chaps. 13 and 15]). From those equations, after applying a Helmholtz decomposition, the three main wave phase velocities, $c_{\text{ph},i}$, can be obtained and are presented as follows,

$$c^2_{\text{ph},1} = \frac{\omega}{\sqrt{\frac{D}{m_s}}}, c^2_{\text{ph},2} = \frac{E}{2\rho(1+\nu)}, c^2_{\text{ph},3} = \frac{E}{\rho(1-\nu^2)}, \quad (1)$$

where E, ν, ρ, D, ω and $m_s = \rho h$ are the material's Elastic modulus, Poisson ratio and material density, the bending stiffness, the angular frequency, the surface mass and h the plate thickness. Utilizing Eq. 1 and the relation between E and G (the shear modulus) for an isotropic material, $G = E/(2(1+\nu))$, it is possible to obtain (after some algebraic manipulation) five equivalent material properties named, $E_{\text{eq}}, G_{\text{eq}}, \nu_{\text{eq}}, h_{\text{eq}}$ and ρ_{eq}. The latter is obtained considering equal mass between the original and equivalent structure.

The second homogenization strategy proposes the use of a single layer thin orthotropic plate. Then, the three governing dynamic equations utilized for this carrier, after assuming a time harmonic motion for the displacements u, v and w on the form $\mathbf{d} = [\hat{u}, \hat{v}, \hat{w}]e^{\mathrm{j}(k_x x + k_y y - \omega t)}$, are presented as follows,

$$k^2(A_{11}c^2 + A_{66}s^2 + (A_{12}+_{66})c\,s) - m_s\omega^2 = 0, \quad (2)$$
$$k^2(A_{66}c^2 + A_{22}s^2 + (A_{12}+_{66})c\,s) - m_s\omega^2 = 0 \quad (3)$$
$$k^4 D(\varphi) - m_s\omega^2 = 0, \quad (4)$$

where $k_x = k\cos(\varphi)$ and $k_y = k\sin(\varphi)$ are the x and y components of the wavenumber k, c and s refer to $\cos(\varphi)$ and $\sin(\varphi)$, φ is the heading angle and A_{ij} and $D(\varphi) = D_{11}c^4 + D_{22}s^4 + 2(D_{12}+2D_{66})c^2\,s^2$ are the in-plane and out-of-plane stiffnesses, respectively. Definitions of A_{ij} and D_{ij} can be found in textbooks such as [2].

2.2 Numerical procedure

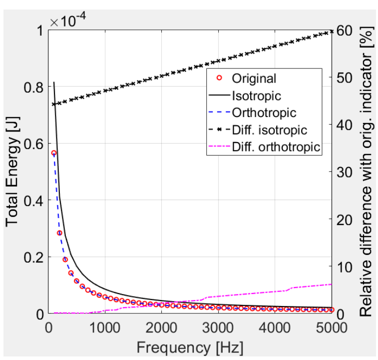
The numerical procedure applied to both homogenization strategies consists basically in finding the equivalent isotropic/orthotropic properties of the equivalent single layer plate solving the eigenvalue problem of the original structure for each heading angle and each frequency of interest utilizing GLM. The first strategy utilizes Eq. 1 to find the equivalent properties and the second calculates them using Eqs. 2 to 4 and three ellipses that represent the best possible fit to the original dispersion curves. With the equivalent properties, the equivalent global stiffness matrix is built and used to calculate the equivalent forced response indicators. The composite Damping Loss Factor (DLF) of the original structure is calculated and added in the global stiffness matrix of the homogenized or equivalent structure.

*diego.martin.tuozzo@usherbrooke.ca
†noureddine.atalla@usherbrooke.ca

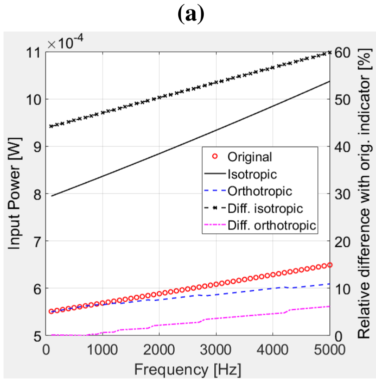
3 Results

To evaluate the effectiveness of the proposed strategies, both homogenization procedures are evaluated with two representative structures, a bilayer thick asymmetric multilaminate (two different orthotropic layers with non-zero angle lamina) and a highly damped sandwich panel with viscoelastic core (complex and frequency dependent core's material properties and aluminum skins). To characterize the bilayer asymmetric laminate, the total energy and input power, obtained for a point normal load, are presented in Fig. 1 for both homogenization strategies. It can be seen that the orthotropic homogenization performs better than the isotropic strategy (lower relative difference with the original structure's indicators). As a general trend, the relative difference increases with frequency.

Total energy and input power are also presented in Fig. 2 for the case of the sandwich panel. It can be seen that again, the orthotropic homogenization performs better than the isotropic strategy (lower relative difference with the original structure's indicators). However, the relative difference values are considerable higher than the previous less damped structure (i.e, structural damping less than 5 $\%$). As a general trend, the relative difference increases with frequency for the isotropic homogenization and decreases for the orthotropic strategy. The same behavior described here was observed in thick isotropic/orthotropic sandwich panels with soft core (having less relative difference at low frequencies).

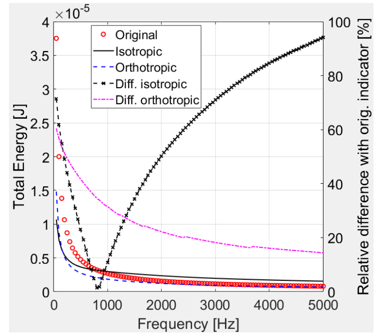


(a)

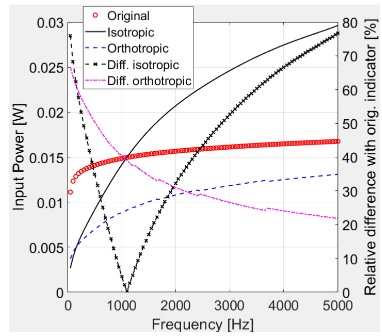


(b)

Figure 1: Total Energy (1a) and Power input (1b) forced indicators of the bilayer asymmetric multilaminate.



(a)



(b)

Figure 2: Total Energy (1a) and Power input (1b) forced indicators of the sandwich with viscoelastic core.

4 Final remarks and discussions

Two different homogenization strategies were presented. The necessary theoretical background utilized by each of these strategies was described. To stress the use and limitations of both strategies, two forced indicators were computed and compared with the original structure's indicators. The results show that the homogenization process (the equivalent properties) affects the equivalent forced quantities and, as a consequence, the composite DLF of the equivalent structure. The better the homogenization approach, the better the agreement between original/equivalent forced indicators such as total energy, injected power and composite DLF. The use of a linear composite damping (equivalently acting only on E and G) probably affects the correct reproduction of the bending behavior of the whole panel in thick or highly damped sandwich type structures. The performance of the strategies in wave (estimation of dispersion curves) and forced analyses is unrelated, highlighting the importance of a forced response assessment. In regards of the potential applicability of the presented strategies to complex structures, the orthotropic homogenization seems to capture the dynamic-anisotropic nature of the original structure (frequency, heading and material spatial properties variations).

References

[1] Sebastian Ghinet and Noureddine Atalla. Modeling thick composite laminate and sandwich structures with linear viscoelastic damping. *Computers & Structures*, 89(15):1547–1561, 2011.

[2] J.-M. Berthelot. *Composite Materials: Mechanical Behavior and Structural Analysis*. Springer New York, NY, 1st.ed., 1999.