A METHODOLOGY FOR DETERMINING ERRORS IN AIRCRAFT NOISE EXPOSURE MODELS

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ABSTRACT
This paper describes a technique for estimating the error in predicting noise exposure indexes due to inaccuracy in the input data required in their calculation.

1. INTRODUCTION
The extent of aircraft noise exposure is generally expressed by the noise index derived from a noise exposure model. A number of aircraft noise exposure models have been developed over the years - namely the CNR, NEF, $L_{DN}$, CNEL and $L_{eq}$ system. The accuracy of these models is determined by (1) the underlying assumptions used in the development of these models and (2) the accuracy of the input data. Some aspects of the first category, more specifically, the effects of tone and duration corrections, temperature and relative humidity on the size and shape of the noise exposure contours, have been determined in two recent sensitivity studies (1 and 2). However, little or no effort has been made on the effects of errors in aircraft noise levels and operations on the size and shape of the contour. The intention here is to develop a methodology to determine the magnitude of the noise index error given the inaccuracies of the input data.

2. FORMULATION
The total noise exposure at a point due to a cumulative sum of each flight's contribution is given as:

$$NE = 10 \log \left\{ \sum_{i=1}^{n} (aD_i + bE_i + cN_i)10^{EL_i/10} \right\} - A \quad \ldots \ldots \ (1)$$

where
$n$ = Number of distinct flights (unique combinations of track, profile and aircraft type)
$D, E, N$ = The actual number of Day, Evening and Night operations for the flight in question.
$a, b, c$ = Respectively; the Day, Evening and Night weighting factors
EL = The single event exposure level (i.e. EPNL for NEF, SEL for \( L_{\text{DN}} \), CNEL or \( L_{\text{eq}} \)) for exactly one such operation as the flight in question with all correction factors applied.

\[ A = \begin{cases} \text{(88.0 for NEF)} \\ \text{(49.4 for LDN, CNEL and LEq)} \end{cases} \]

To obtain the error of the model, Equation (1), due to inaccuracies of \( D_0, E_1, N_1 \) and \( E_1 \), consider the general expression \( Y \) as a function of \( m \) independent variables, such that:

\[ Y = \sum_i g_i \left( x_1, x_2, x_3, \ldots, x_m \right) \]

where \( x_j, j = 1, m \) are the independent variables and \( i \) subscript denotes the \( i \) similar items to be summed.

Define the function:

\[ Y_i = g_i \left( x_1, x_2, x_3, \ldots, x_m \right) \]

such that: \( Y_i + \epsilon_i = g_i \left( x_1 + \delta, x_2 + \delta, \ldots, x_m + \delta \right) \)

where \( \epsilon_i \) is the error of \( Y_i \) due to errors \( \delta \) of the independent variables.

By taking the Taylor Series expansion around the mean, \( x_i \), we have

\[ g_i \left( x_1, x_2, \ldots, x_m \right) = g_i + \left( x_1 - x_i \right) \frac{\partial g_i}{\partial x_1} + \left( x_2 - x_i \right) \frac{\partial g_i}{\partial x_2} + \ldots \]

\[ \ldots \left( x_1 - x_i \right)^2 \frac{\partial^2 g_i}{\partial x_1^2} + \left( x_2 - x_i \right)^2 \frac{\partial^2 g_i}{\partial x_2^2} + \ldots + R_n \]

If the second and higher order terms and remainder are small, then the first approximation gives:

\[ \epsilon_i = \sum_{l=1}^{m} \left\{ \frac{\partial g_i}{\partial x_l} \right\} \delta_l \]
The total error $e_i$ in the aggregate $Y$ is then given by: $\varepsilon = \sum_i e_i$

Equation (1) can be expressed in another form, namely:

$$NE_i = 10 \log \sum_i 10$$

when $NE_i = EL_i + 10 \log (aD_i + bE_i + cN_i) - A$ ........ (4)

Now let $g_i = NE_i$ and $n_i = aD_i + bE_i + cN_i$

$$\frac{\partial g_i}{\partial NE_i} = 1, \quad \frac{\partial^2 g_i}{\partial NE_i^2} = 0$$

$$\frac{\partial g_i}{\partial n_i} = \frac{10 \log e}{n_i}, \quad \frac{\partial^2 g_i}{\partial n_i^2} = -\frac{10 \log e}{n_i^2}$$

For large values of $n_i = n_i^*$, $\frac{\partial^2 g_i}{\partial n_i^2}$ is small

$$R_n = (NE_i - NE_i^*)^n \frac{\partial^n NE_i}{\partial n_i^n} = (NE_i - NE_i^*)^n (-1)^{n+1} (n-1) \frac{10}{n!}$$

Again for large $n$ and $n_i^*$, $R_n$ is small. Therefore, the error in noise index $NE_i$ is:

$$\delta NE_i = \delta EL_i + \log e \cdot \frac{\delta n_i}{n_i^*}$$
Now, define \( z_i = 10^{NE_i/10} \) and \( Z = \sum z_i \), then \( \frac{\partial z_i}{\partial NE_i} = \frac{\ln 10 \cdot 10^{NE_i/10}}{10} \). The error in \( z_i \) is \( \delta z_i = \left( \frac{\ln 10 \cdot 10^{NE_i/10}}{10} \right) \delta NE_i \) and the error in the aggregate \( Z \) is \( \delta Z = \sum \delta z_i \), i.e. \( \delta Z = \sum \left( \frac{\ln 10 \cdot 10^{NE_i/10}}{10} \right) \delta NE_i \).

Using the new definitions, the total noise exposure is then given as \( NE = 10 \log Z \). Now \( \frac{\partial NE}{\partial Z} = \frac{10 \log e}{Z} \) and

\[
\delta NE = 10 \log e \cdot \sum \left( \frac{\ln 10 \cdot 10^{NE_i/10}}{10} \delta NE_i \right) \sum_{i=1}^{10^{NE_i/10}}
\]

The above expression can be further simplified to:

\[
\delta NE = \frac{\sum \left( 10^{NE_i/10} \delta NE_i \right)}{\sum_{i=1}^{10^{NE_i/10}}} \quad \ldots \quad (6)
\]

This error equation can be further simplified if the deviations in noise level and aircraft operations for each aircraft type are the same, i.e.

\( \delta EL = \delta EL_i, \quad \delta n = \delta n_i \). Equation (5), under these conditions, becomes

\( \delta NE = \delta EL + 10 \log e \cdot \frac{\delta n}{n^*} \) and Equation (6) is simplified to

\[
\delta NE = \frac{\sum \left( 10^{NE_i/10} \right) \delta NE_i}{\sum_{i=1}^{10^{NE_i/10}}} \quad \text{or} \quad \delta NE = \delta NE = \delta EL + 10 \log e \cdot \frac{\delta n}{n^*}.
\]

3. APPLICATION

The derived error equation was used to determine the resultant NEF error due to errors in aircraft noise levels and forecasted movements. The airport being studied has two runways and 356 aircraft movements with ten aircraft types using four different flight paths. The aircraft noise level is assumed
to be accurate to within ± 3 EPNdB and the inaccuracy in aircraft movement
is assumed to be within ± 10%. The total deviation in NEF at any ground
position obtained from the NEF computer program is ± 3.4 units of NEF. The
total inaccuracy as computed from equation (6) is ± 3.434 units of NEF.
Within the accuracy of computations, the results compare very well.

4. REFERENCES

1.0 D.E. Bishop and T.C. Dunderdale, 1976, Army Medical Research Laboratory
    AMRL-TR-75-115, "Sensitivity Studies of Community - Aircraft Noise
    Exposure".

2.0 D.E. Bishop and T.C. Dunderdale, April 1977, Army Medical Research
    Laboratory AMRL-TR-76-116, "Further Sensitivity Studies of Community
    - Aircraft Noise Exposure".
To obtain an absolute measurement of noise, it is not enough simply to measure the sound pressure level at a specified distance from the source because the acoustic environment adds its own parameters to the readings. The B & K Sound Power Calculator determines the total acoustic power output of the test object in pico-Watts — an absolute figure that is not affected by the acoustic environment.

The Sound Power Calculator determines octave, third octave and A-Weighted sound power levels, and provides digital display of centre frequency and level (sound pressure, sound power, or room corrections). For a brochure containing complete technical information, and some typical set-ups, contact any Bruel & Kjaer office.