A METHODOLOGY FOR DETERMINING ERRORS IN AIRCRAFT NOISE EXPOSURE MODELS

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ABSTRACT

This paper describes a technique for estimating the error in predicting noise exposure indexes due to inaccuracy in the input data required in their calculation.

1. INTRODUCTION

The extent of aircraft noise exposure is generally expressed by the noise index derived from a noise exposure model. A number of aircraft noise exposure models have been developed over the years - namely the CNR, NEF, L_{DN}, CNEL and L_{eq} system. The accuracy of these models is determined by (1) the underlying assumptions used in the development of these models and (2) the accuracy of the input data. Some aspects of the first category, more specifically, the effects of tone and duration corrections, temperature and relative humidity on the size and shape of the noise exposure contours, have been determined in two recent sensitivity studies (1 and 2). However, little or no effort has been made on the effects of errors in aircraft noise levels and operations on the size and shape of the contour. The intention here is to develop a methodology to determine the magnitude of the noise index error given the inaccuracies of the input data.

2. FORMULATION

The total noise exposure at a point due to a cumulative sum of each flight's contribution is given as:

$$NE = 10 \log \left\{ \sum_{i=1}^{n} (aD_i + bE_i + cN_i) 10^{EL_i/10} \right\} - A \quad \ldots\ldots \quad (1)$$

where n = Number of distinct flights (unique combinations of track, profile and aircraft type)

a, b, c = Respectively; the Day, Evening and Night weighting factors

D, E, N = The actual number of Day, Evening and Night (respectively) operations for the flight in question.

EL = The single event exposure level (i.e. EPNL for NEF, SEL for L_{DN}, CNEL or L_{eq}) for exactly one such operation as the flight in question with all correction factors applied.

A = (88.0 for NEF
(49.4 for L_{DN}, CNEL and L_{eq}

To obtain the error of the model, Equation (1), due to inaccuracies of D_i, E_i, N_i and EL_i, consider the general expression Y as a function of m independent variables, such that:

$$Y = \sum_i g_i \left({}^1X_i, {}^2X_i, {}^3X_i, \ldots\ldots\ldots\ldots\ldots {}^mX_i \right)$$

where ${}^jX_i$, j = 1, m are the independent variables and i subscript denotes the i similar items to be summed.

Define the function:

$$Y_i = g_i \left({}^1X_i, {}^2X_i, {}^3X_i, \ldots\ldots\ldots {}^mX_i \right)$$

such that: $Y_i + \varepsilon_i = g_i \left({}^1X_i + {}^1\delta_i, {}^2X_i + {}^2\delta_i, \ldots\ldots {}^mX_i + {}^m\delta_i \right)$

where ε_i is the error of Y_i due to errors ${}^j\delta_i$ of the independent variables. By taking the Taylor Series expansion around the mean, ${}^jX_i{}^*$, we have

$$g_i \left({}^1X_i, {}^2X_i, \ldots\ldots {}^mX_i \right) = g_i + \left({}^1X_i - {}^1X_i{}^* \right) \frac{\partial g_i}{\partial {}^1X_i} + \left({}^2X_i - {}^2X_i{}^* \right) \frac{\partial g_i}{\partial {}^2X_i} + \ldots\ldots$$

$$\ldots\ldots \frac{\left({}^1X_i - {}^1X_i{}^* \right)^2}{2\,!} \frac{\partial^2 g_i}{\partial {}^1X_i{}^2} + \frac{\left({}^2X_i - {}^2X_i{}^* \right)}{2\,!} \frac{\partial^2 g_i}{\partial {}^2X_i{}^2} + R_n$$

If the second and higher order terms and remainder are small, then the first approximation gives:

$$\varepsilon_i = \sum_{\ell=1}^{m} \left\{ \left. \frac{\partial g_i}{\partial {}^\ell X_i} \right|_{\ell\ *} \right\} {}^\ell\delta_i$$

The total error ε_i n the aggregate Y is then given by: $\varepsilon = \sum_i \varepsilon_i$

Equation (1) can be expressed in another form, namely:

$$NE = 10 \log \sum_i 10^{NE_i/10} \qquad \ldots\ldots (3)$$

when $NE_i = EL_i + 10 \log (aD_i + bE_i + cN_i) - A$ ..... (4)

Now let $g_i = NE_i$ and $n_i = aD_i + bE_i + cN_i$

$$\frac{\partial g_i}{\partial NE_i} = 1 \qquad \frac{\partial^2 g_i}{\partial NE_i^2} = 0$$

$$\frac{\partial g_i}{\partial n_i} = \frac{10 \log e}{n_i} \quad , \quad \frac{\partial^2 g_i}{\partial n_i^2} = - \frac{10 \log e}{n_i^2}$$

For large values of $n_i = n_i^*$, $\left. \frac{\partial^2 g_i}{\partial n_i^2} \right|_{n_i^*}$ is small

$$R_n = \frac{(NE_i - NE_i^*)^n}{n\,!} \cdot \frac{\partial^n NE_i}{\partial n_i^n} = \frac{(NE_i - NE_i^*)^n}{n\,!} (-1)^{n+1} (n-1) \frac{10}{n_i^{*n}}$$

Again for large n and $n_{i_j}^*$, R_n is small. Therefore, the error in noise index NE_i is:

$$\delta NE_i = \delta EL_i + \log e. \frac{\delta n_i}{n_i^*}$$

Now, define $z_i = 10^{NE_i/10}$ and $Z = \sum_i z_i$, then $\frac{\partial z_i}{\partial NE_i} = \frac{\ell n\ 10}{10} \cdot 10^{NE_i/10}$. The error in z_i is $\delta z_i = \left\{ \frac{\ell n\ 10}{10} \cdot 10^{NE_i/10} \cdot \delta NE_i \right\}$ and the error in the aggregate Z is $\delta Z = \sum_i \delta z_i$, i.e. $\delta Z = \sum_i \left\{ \frac{\ell n 10}{10} \cdot 10^{NE_i/10} \cdot \delta NE_i \right\}$

Using the new definitions, the total noise exposure is then given as $NE = 10 \log Z$. Now $\frac{\partial NE}{\partial Z} = \frac{10 \log e}{Z}$ and

$$\delta NE = \frac{10 \log e}{\sum_i 10^{NE_i/10}} \cdot \sum_i \left\{ \frac{\ell n 10}{10} \cdot 10^{NE_i/10}\ \delta NE_i \right\}$$

The above expression can be further simplified to:

$$\delta NE = \frac{\sum_i \{10^{NE_i/10} \cdot \delta NE_i\}}{\sum_i 10^{NE_i/10}} \qquad \ldots\ldots \quad (6)$$

This error equation can be further simplified if the deviations in noise level and aircraft operations for each aircraft type are the same, i.e. $\delta EL = \delta EL_i$, $\frac{\delta n}{n^*} = \frac{\delta n_i}{n_i^*}$. Equation (5), under these conditions, becomes

$\delta NE = \delta EL + 10 \log e. \frac{\delta n}{n^*}$ and Equation (6) is simplified to

$$\delta NE = \frac{\sum_i \{10^{NE_i/10}\} \cdot \delta NE_i}{\sum_i \{10^{NE_i/10}\}} \quad \text{or } \delta NE = \delta NE = \delta EL + 10 \log e. \frac{\delta n}{n^*}.$$

3. APPLICATION

The derived error equation was used to determine the resultant NEF error due to errors in aircraft noise levels and forecasted movements. The airport being studied has two runways and 356 aircraft movements with ten aircraft types using four different flight paths. The aircraft noise level is assumed to be accurate to within ± 3 EPNdB and the inaccuracy in aircraft movement is assumed to be within ± 10%. The total deviation in NEF at any ground position obtained from the NEF computer program is ± 3.4 units of NEF. The total inaccuracy as computed from equation (6) is ± 3.434 units of NEF. Within the accuracy of computations, the results compare very well.

4. REFERENCES

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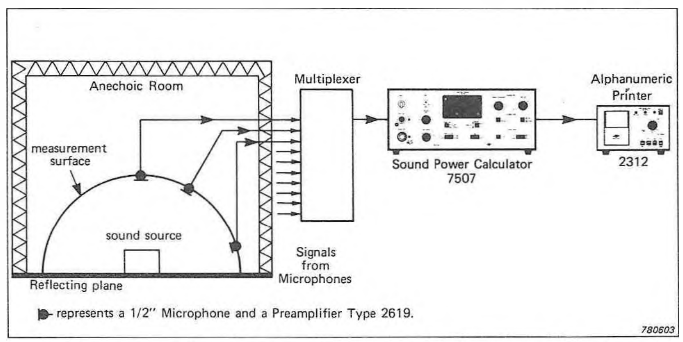

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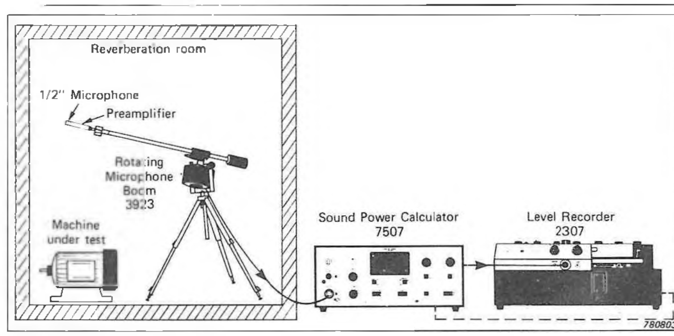

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