AN ANALYTICAL MODEL FOR INPUT SPECTRAL DENSITIES FOR RESPONSE ESTIMATION

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ABSTRACT

A mathematical approach is proposed to describe any arbitrarily varying power spectral density of a random process in order to obtain analytically the response of mechanical systems to such inputs. The proposed model optimally envelopes the input power spectral density with linear segments having rising and falling slopes as well as flat portions in dB scale and can be used to describe any power spectral density distribution to the required degree of accuracy. Response of a mechanical system to an arbitrary random loading is obtained by describing the input using this model and the result is compared against experimentally measured response for the system under same loading. The computed mean square response agreed well with the experimentally measured value, thereby validating the usefulness of the proposed model in evaluating the response of any general dynamic systems under random loading.

RESUMÉ

Une nouvelle approche est proposée pour la description d'une densité spectrale arbitrairement variable d'un processus accidentel pour obtenir, d'une façon analytique, la réponse d'un système mécanique, pour cette entrée. Le modèle proposé enveloppe d'une façon optimale la densité spectrale d'entrée de la puissance avec des segments ayant des pentes montantes et descendantes et aussi des portions plates dans une échelle de dB. Ils peuvent être utilisés pour décrire n'importe quelle distribution de la densité spectrale avec le degré de précision requis. La réponse d'un système mécanique à une charge accidentelle arbitraire est obtenue par la description d'une entrée en utilisant ce modèle et le résultat est comparé avec la réponse obtenue expérimentallement pour le système avec une charge identique. La réponse moyenne carrée calculée se compare bien avec les résultats mesurés en confirmant l'utilité du modèle proposé pour l'évaluation de la réponse d'un système dynamique général sous une charge accidentelle.

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The spectral densities of many random processes, which excite structural systems, are often not amenable to an exact mathematical description. In order to design structures subjected to such random loadings, it is essential to describe these spectral densities of excitations analytically in a functional form. In some practical situations, it is sufficient to describe them approximately with simple mathematical functions, such as a white noise [1], a band limited white noise [2] or as a product of an algebraic and exponential function [3]. However, such models cannot describe any spectral densities in general, which may possess several peaks of varying magnitudes.

The present paper proposes a simple analytical model to describe any general spectral density of random processes, which may be used to estimate the responses of structural systems excited by such random processes. This model envelopes the spectral density curve with a number of linear segments having rising and falling slopes as well as flat portions in the decibel versus octave frequency scale. The mean square response of the structure subjected to that excitation can be evaluated for such individual segments analytically and the total response can be obtained by summing the responses to individual segments for a linear analysis. For illustration, a spectral density having three linear segments having a rising slope, a flat region and a falling slope in the decibel versus frequency scale is considered as shown in Figure 1. The mean square acceleration response of a typical dynamic system to this excitation is evaluated analytically. The mean square acceleration response is also measured experimentally on the same system by synthesizing a spectral density of loading comparable to that considered in the analytical study. A judicious comparison of the analytical and experimental mean square responses showed a good agreement thus validating the usefulness of the proposed model in estimating the random responses of structural systems.

ANALYSIS OVERVIEW

Any shape of spectral density can be described approximately by enveloping the curve with lines of rising and falling slopes and flat segments in the decibel versus frequency scale. The relationship between the slope in the dB/octave, spectral density and frequency, along such linear segments is given by

$$S_x(f) = S_x(f_\ell)\ (f/f_\ell)^{N/3} \tag{1}$$

where

N = slope of the line in dB/octave

f = frequency at any point along the line

$S_x(f)$ = spectral density at frequency f

f_ℓ = lowest frequency on the line

$S_x(f_\ell)$ = spectral density at f_ℓ

The response spectral density for a structure subjected to random excitations is given by the standard relation

$$S_y(f) = |H(f)|^2\ S_x(f) \tag{2}$$

where

$S_y(f)$ = output spectral density

$H(f)$ = complex frequency response relating the input to the output.

The mean square response in the frequency band covered by each segment can be obtained by the relation

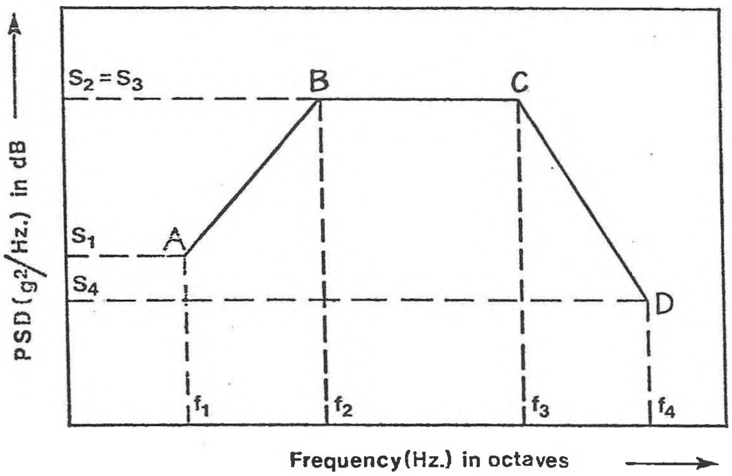

Fig. 1: Profile of a Power Spectral Density of Excitation

$$\sigma^2 = \int_{f_\ell}^{f_u} S_y(f)\, df \tag{3}$$

where

σ^2 = mean square value

f_u = upper frequency limit on the line.

RESULTS

The above analysis is used to estimate the mean square acceleration response of a cantilever beam structure of length 30 cm and a cross section of 2.5 x 1.25 cm. Only the first two modes of the structure were considered in the analysis. The damping ratios of the structure corresponding to the first two normal modes were estimated from the frequency response of the structure under harmonic excitation, and were found to be 0.00625, respectively. An excitation signal having a spectral density profile as shown in Figure 1 was employed for the analytical estimation and for the experimental verification of the mean square response. The excitation signal was chosen in such a manner that the two regions with the non-zero slopes contained the first and second natural frequencies of the structure.

The root mean square acceleration estimated at the tip of the structure, when the root of the cantilever was subjected to excitation, was 61.3 g rms using the present approach. The experimentally measured root mean square acceleration was 62 g rms at the tip showing a good agreement with the analysis.

REFERENCES

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