STATISTICAL ENERGY ANALYSIS

A BRIEF INTRODUCTION

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ABSTRACT

Statistical energy analysis (SEA) in a powerful tool in the study of high frequency vibration. A brief review is given of the basic ideas of SEA by describing some sample calculations and listing some areas where SEA has been applied successfully.

SOMMAIRE

L'analyse statistique d'énergie est un puissant instrument de travail pour l'étude des vibrations à haute fréquence. Une brève revue des idées de base de cette analyse est présentée à l'aide d'exemple de calculs et d'applications couronnées de succès.

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INTRODUCTION

Statistical energy analysis (SEA) is a way of studying dynamical systems. The method was developed in the 1960's by R. H. Lyon and others at the research and consulting firm Bolt, Beranek and Newman. A description of the basic theory and of typical methods of application of SEA is given in the text by Lyon [1]. SEA can provide reliable estimates of vibration levels on complicated systems by using very simple models. Its use is most appropriate at high frequencies in complicated structures with very many degrees of freedom. Despite its power, however, the technique has not been adopted as wholeheartedly as one might expect by design engineers [2].

The intent of this paper is to draw the attention of readers of Canadian Acoustics/Acoustique Canadienne to SEA by describing the basic ideas of SEA through a sample application, and by listing some typical areas where SEA has been used very successfully.

BASIC SEA

As a sample application we shall discuss the transmission of vibration from the turbulent-boundary-layer excited skin of a small aerospace vehicle to an internal instrument shelf (Figure 1).

SEA uses power and energy as the basic variables to describe systems. A system is divided into two or more subsystems each characterized by an appropriate energy variable. A basic SEA model that describes the skin and shelf vibration of the aerospace vehicle is shown in Figure 2. We use two energy variables, one each for the skin and shelf, respectively. The skin and shelf can also each be represented by groups of resonant modes of vibration. One of the major advantages of using SEA is that detailed information on the modes of vibration is not always necessary. SEA is usually applied at high frequencies to obtain vibration information in octave or third octave bounds. In order to discuss the energy of a subsystem we use only the total mass of the subsystem, m, an average mean-square velocity, $\langle v^2 \rangle$, and the number, N, of vibration modes that have resonant frequencies in the frequency band of interest. (This information is available in the literature for a wide variety of structures.) The relation $E_{tot} = m\langle v^2 \rangle$ relates the total energy to the mean square vibration velocity. Also $E_{tot} = NE$, where each of the N modes characterizing the subsystem has vibrational energy E. Usually it is the $\langle v^2 \rangle$ for each part of the system that we need to know, and are the answers we hope to find from the SEA calculations.

Power describes the rate of flow of energy into or out of a system and from one part of a system to another. In our model (Figure 2) the power flows of interest are the total input power from the turbulent boundary layer, the power dissipated by damping each of the subsystems, and the power transmitted from one subsystem to the other. In steady state conditions, the net power in to each subsystem must equal the net power out. The basic power balance equations for each subsystem are:

$$P_1 = P_{1diss} + P_{12} \tag{1a}$$

$$P_{12} = P_{2diss} \tag{1b}$$

Dissipated power in each subsystem is related to the energy by the damping loss factor, so that

$$P_{1diss} = \omega\eta_1 N_1 E_1 \tag{1c}$$

$$P_{2diss} = \omega\eta_2 N_2E_2 \tag{1d}$$

Finally, and by analogy with (1c) and (1d), we introduce a coupling loss factor η_{12} and write

$$P_{12} = \omega\eta_{12} N_1(E_1 - E_2)$$

We assume that the input power P_1 to the skin and the loss factors η_1 and η_2 can be measured or otherwise estimated. Independent power input to the shelf P_2 could easily be included if it were present. The numbers of resonant modes $N = n\Delta\omega$ where n is the modal density and $\Delta\omega$ is the frequency bandwidth (usually octave or third octave) must also be measured or calculated. P_{12} represents the power transmitted from the skin to the shelf. By analogy with the dissipated power P_{diss}, P_{12} is written in terms of a coupling loss factor η_{12} which satisfies a symmetry relation $N_1\eta_{12} = N_2\eta_{21}$. Calculations of the parameters required have been made for a number of structural and acoustic couplings [1]. These calculations have formed an important part of the development of SEA.

Equation (1e) has a straightforward thermodynamic analog. In thermodynamic terms the energy E of the individual modes of oscillation represents temperature. What equation (1e) states for vibration is thus analogous to the statement that the heat transfer between two bodies is proportional to the difference in their temperatures.

The basic steps in SEA consist of (i) formulating a model in terms of two or more groups of resonant modes of vibration, noting that occasionally non-resonant motion must be included, and also that more than one group of modes, for example both flexural and torsional modes, may be needed to describe the motion of part of a structure, (ii) evaluating the parameters required and (iii) solving the power flow equations (1) to obtain the energies and hence the mean square vibration levels on various parts of the structure. Other vibration parameters such as stress can subsequently be obtained as required.

SAMPLE APPLICATION

Sample calculations made by H. G. Davies are included as a chapter in the text by Lyon [1]. These calculations draw heavily on earlier work and data taken J. E. Manning. The example is discussed briefly here.

Figure 1 shows a schematic of part of a vehicle designed to gather data during reentry into the earth's atmosphere. The skin of the vehicle is in essence a conical shell of length about 12 ft. and diameter varying from 6.4 to 39 in. A stiff ring attached to the skin supports an instrument shelf. The shelf itself is a fairly involved structure having several flanges. The skin is subjected to very intense turbulent boundary layer excitition as it reenters the earth's atmosphere. We try to estimate the vibration level on the instrument shelf during this part of the flight. This is by no means a trivial example. But we shall see that a very simple SEA model gives quite adequate agreement with experimental data.

Below about 50 Hz the vibration is dominated by large scale flexural modes of the whole vehicle. The usual analytical or numerical techniques of vibration analysis may be used in this frequency range and no advantage is gained by using SEA. However, in the third octave band centered for example at 2000 Hz we estimate that there are 64 resonant modes of vibration. It is in this type of situation that SEA can be particularly useful.

We first formulate a suitable SEA model. We could treat the skin, stiffeners, ring connector, various parts of the instrument shelf and the interior acoustic spaces as separate interacting systems each described by one or more groups of similar modes. Very many parameters are required for this complicated model. Experience suggests that the simple two system model shown in Figure 2 will give adequate results. One of the advantages of SEA is that groups of modes that are treated separately in complicated models often show up merely as additional dissipative mechanisms in a simpler model, and this added dissipation is included implicitly in any experimentally determined loss factor for the simple model. Thus a simple model will often suffice.

We next estimate the values of the parameters needed for our SEA model. A combination of calculated and measured values is usually needed. As SEA is a statistical approach detailed information about the structure is not always necessary, and may in fact be redundant. This aspect of SEA modeling is one of the advantages of SEA particularly when it is used during the early design stages of a structure or vehicle. Ways of estimating or measuring the parameters involved are described in detail in reference [1]. Typical values are given below for the 2000 Hz third-octave band to provide a feeling for the sorts of numbers involved in the calculations.

Modal densities

Information on the modal densities (numbers of resonant modes per unit frequency) of simple structures is to be found for example in references [1] and [3]. In our example no analytic solution exists for the modal densities of a cylindrical, let alone conical, shell. Values of N_1 can be obtained by using an average diameter of the conical shell and empirical results from [4] for cylindrical shells. A better estimate is obtained by dividing the conical shell into a number of cylindrical shells and adding the modal densities for each section. Manning obtains, for example, 64 modes in the 2000 Hz third octave band. Well above the ring frequency $c_\ell/2\pi a$ (in this example about 1000 Hz) the modal density asymptotes to that of a flat plate of the same area, $n(\omega) = (\text{area})/(4\pi h c_\ell/\sqrt{12})$. Suitable values for use in our example are diameter = 0.95 ft., length = 12 ft., c_ℓ = 10,000 ft/sec (an experimental value for the sandwich panel type of construction used), and h = 3/8 in.

The instrument shelf is a small fairly stiff structure and a detailed analysis does not seem feasible. One approximate estimate at high frequencies is obtained by treating the shelf as an equivalent flat plate, adding the areas of each flange. Since modal densities of coupled structures are additive this approach is reasonable. Suitable values are area = 4 ft^2 and $(hc_\ell/\sqrt{12})$ = 200 ft^2/sec, giving $n(\omega) = (200\pi)^{-1}$ and hence 4.6 modes in the 2000 Hz third-octave band.

At low frequencies (100 to 1000 Hz) Manning's experiments suggested that some third-octave bands contain resonant modes of the shelf, while others do not. In this case, when the modal density is very low a better model of the shelf is a one degree of freedom system attached by massless moment arms to the shelf. We take $N_2 = 1$ for this range of frequencies. This model is discussed below.

Coupling loss factor

This is usually the most difficult parameter to evaluate. Typical values and expressions are given in references [1] and [3]. In the present example the ring connector provides a stiff connection that preserves the angle between the shelf and skin. At high frequencies, vibration on the shell is purely in surface bending modes. The power transmission in this case may be modelled by that between two flat plates one attached at right angles to the other (Figure 3). The coupling loss factor for this case is given in reference [1]. For the parameter values already quoted we find $\eta_{21} = 2f^{-\frac{1}{2}}$ and $\eta_{12} = N_2\eta_{21}/N_1$. At 2000 Hz the numerical values are $\eta_{21} = 0.045$ and $\eta_{12} = 0.0032$.

To develop a suitable low frequency model we again have recourse to some of the experimental data of Manning. It was noted that at low frequencies typical vibration levels on the skin were at least 20 dB higher than the level at the ring connector, showing that the connector has a considerable stiffening effect. Power is transmitted to the shelf primarily by a moment at the connector. We are thus justified in considering only axial motion of the shelf. Figure 4 shows our low frequency model. Because of axial symmetry we may replace the shell by a beam of width $2\pi a$. We make the additional simplication of treating the beam as infinitely long. We may then use the input impedance for an infinite beam given, for example, in reference [3] and the result [1]

$$\eta_{21} = \text{Real}(Z)/\omega M_2 \tag{2}$$

where Z is the total impedance of the mass, spring and beam. For our parameters, we find $\eta_{21} = 20\ f^{-\frac{3}{2}}$ and $\eta_{12} = \eta_{21}/N_1$.

Experimental values of coupling loss factors can be obtained in cases where the coupling is too complicated to be modelled accurately analytically. Mean square vibration levels on two coupled structures may be measured and equations (1) used to find η_{21}. Details of these techniques are given by Lyon [1].

Loss factor

Values of loss factors can usually be obtained only by experiment. The

values used in the present calculations are $\eta_1 = 0.025$ and $\eta_2 = 0.1$.

Power input

Calculation of the power input from the turbulent boundary layer to the conical shell is by no means trivial. Various theoretical and empirical methods may be used to estimate either the input power directly or the mean square acceleration of the skin. These values are related by

$$P_{IN} = \omega\eta_1 E_{1tot} = \frac{\eta_1 M_1}{\omega} \langle a_1{}^2\rangle \tag{3}$$

where it is noted that η_1 is usually considerably larger than η_{12} so that to a first approximation at least we may neglect the effect of the coupling when calculating the input power. These calculations, although discussed in reference [1] are outside the scope of this paper. However, what our SEA model provides specifically is the relative amplitudes of the vibration levels on the skin and shelf. From equations (1) we obtain

$$\frac{\langle v_2{}^2\rangle}{\langle v_1{}^2\rangle} = \frac{M_1}{M_2} \frac{\eta_{12}}{\eta_2 + \eta_{21}} \tag{4}$$

Typical third-octave band values of this ratio are shown in Figure 5. Specific values of the shelf vibration level $\langle v_2{}^2\rangle$ can be obtained, of course, once $\langle v_1{}^2\rangle$ is known.

Results of sample calculations

Figure 5 compares theoretical values using equation (2) and experimental values obtained by Manning. Manning's values are for a full-sized shelf inside a cylindrical shell, with the values adjusted theoretically to account for the different modal density of the actual conical shell. Agreement between theory and experiment is seen to be adequate.

APPLICATIONS OF SEA

SEA was developed initially for aerospace applications and has been used very successfully in that area for calculating the vibratory response of complicated structures involving structure to structure vibration transmission as discussed in our sample application, and also for structure/acoustic field interactions for estimating sound radiation and noise levels. In this latter case a simple SEA model might consist of one group of modes representing the vibrating structure and a second group representing the acoustic field with which the structure interacts. SEA models involving both structural and airborne transmission have also been used in noise control on board ships. Similar structure/acoustic interaction models were used in the design stages of a U.K. gas-cooled nuclear reactor to estimate the fatigue life of the reactor shell excited by the very intense internal acoustic field caused by large axial flow compressors.

Perhaps the best recent overview of the scope of SEA and its applications was given at the 100th meeting of the Acoustical Society of America in 1979. One session at that meeting was developed entirely to SEA and included among other papers invited review papers on the application of SEA to building acoustics, vibration of internal combustion engines, shipbuilding, and wave excited motion of off-shore structures.

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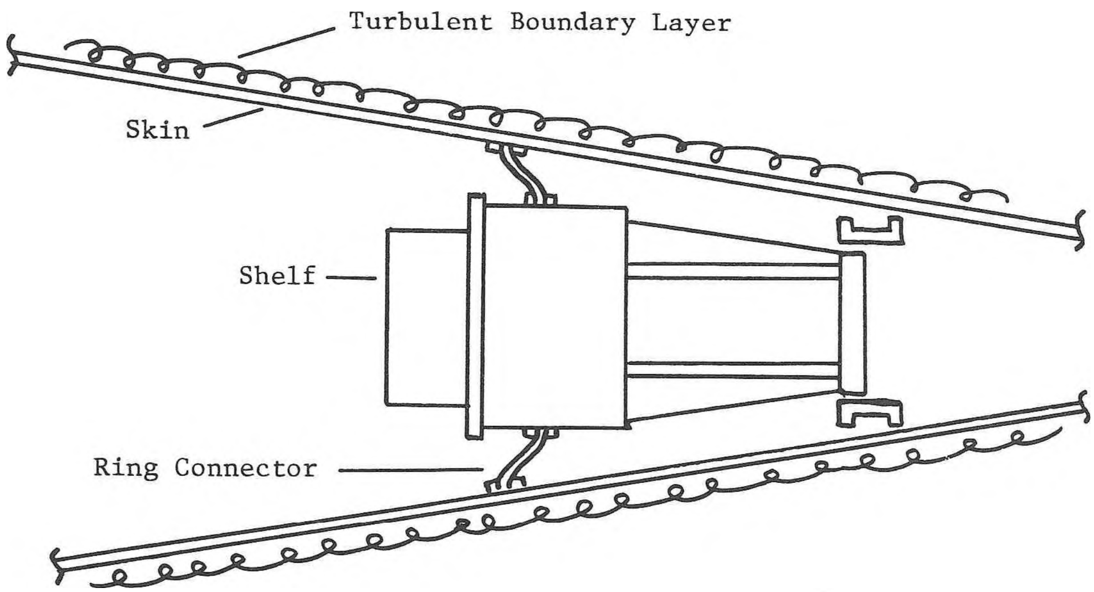

Figure 1

Schematic of Skin and Shelf

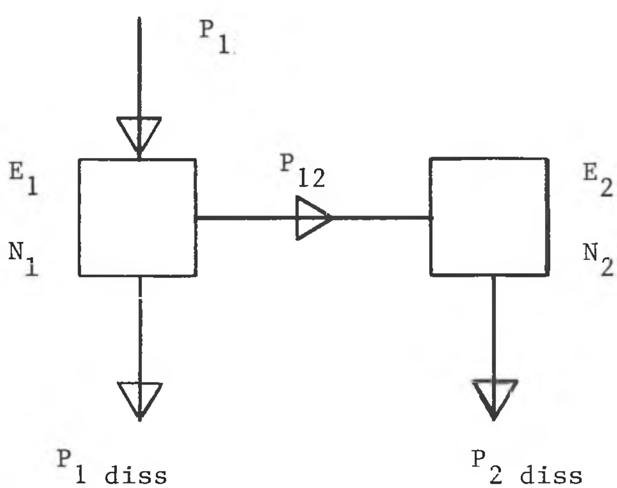

Figure 2

SEA Model of Skin and Shelf

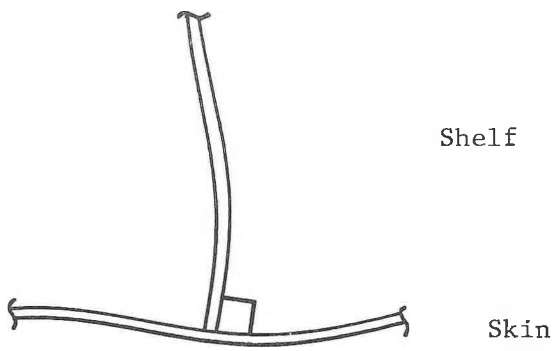

Figure 3

Plate model for high frequency y_{12}

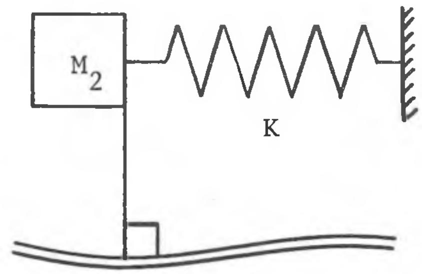

Figure 4

Lumped parameter model for low frequency y_{12}

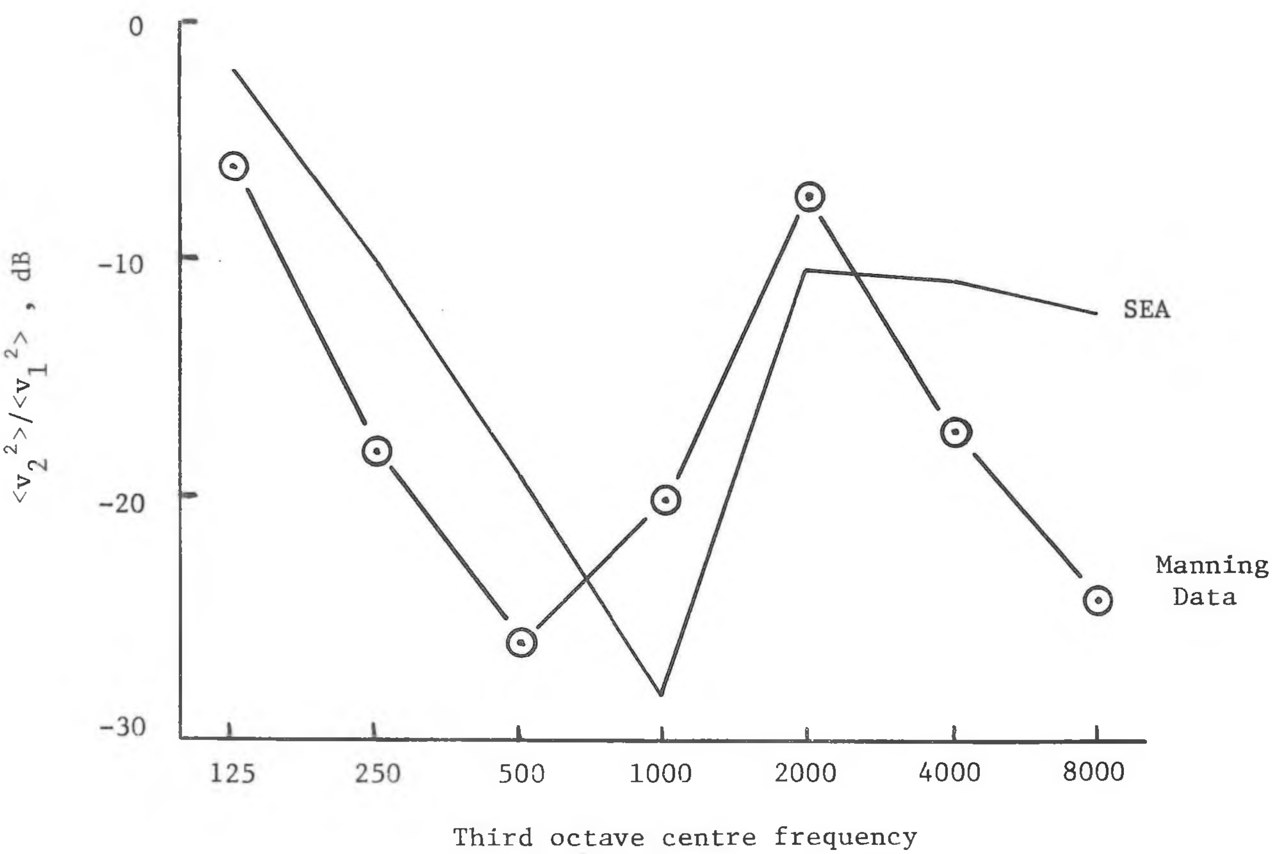

Figure 5