

EFFECT OF MEAN FLOW AND DAMPING
ON THE PERFORMANCE OF REACTIVE MUFFLERS

P. T. Thawani* and A. G. Doige
Department of Mechanical Engineering
The University of Calgary
Calgary, Alberta T2N 1N4, Canada

ABSTRACT

Inclusion of mean flow effects in the mathematical modeling of exhaust mufflers has been found to be essential for achieving realistic results. Two mathematical models are presented which apply to plane wave propagation through a pipe or an expansion chamber including the effect of mean flow and damping. The solutions of the governing equations are represented by 2X2 transfer matrices. In evaluating the performance of exhaust mufflers, noise reduction (NR) is calculated using an open end termination. The measured NR characteristics of the tested laboratory mufflers are in excellent agreement with those predicted by the transfer matrix approach. The method of NR calculation is quite general in its application and can be extended further to optimize muffler configuration at the design stage.

SOMMAIRE

Il est apparu essentiel de tenir compte des effets de l'écoulement moyen dans la modélisation mathématique des silencieux afin d'obtenir des données réalistes. Dans cette communication, on présente deux modèles mathématiques qui décrivent la propagation d'ondes planes dans une conduite ou un détendeur en tenant compte de l'effet de l'écoulement moyen et de l'amortissement. Les solutions des équations du mouvement sont représentées par des matrices de transfert 2 x 2. Dans l'évaluation du rendement de silencieux, on calcule la réduction du bruit (RB) pour un silencieux à extrémité ouverte. Les valeurs de RB mesurées en laboratoire sur des silencieux concordent très bien avec les valeurs prévues par les matrices de transfert. La méthode de calcul de la RB se prête à des applications de caractère assez général, et il est possible d'étendre son utilisation à l'optimisation de la configuration des silencieux, lors de la conception.

*Currently with Nelson Industries, Inc., Stoughton,
Wisconsin 53589, U.S.A.

NOMENCLATURE

All, A12, A21, A22	Transfer Matrix terms
A,B	Pressure amplitudes
c	Speed of wave propagation
f	Frequency of wave propagation
i	Imaginary quantity ($=\sqrt{-1}$)
k	Wave number
ℓ	Length of element
M	Mach number
N	Area ratio
NR	Noise reduction
S	Cross section area of element
t	Time variable
[T]	Transfer Matrix notation
U	Steady mean flow velocity
v	Acoustic mass velocity
x	Space variable in axial direction
Y	Characteristic impedance
Z _L	Load impedance
Z _S	Source impedance
ω	Circular frequency ($=2\pi f$)
ζ	Friction coefficient
Subscript r	Refers to rth element or at point r

1. Introduction

Muffler manufacturers are being confronted with the problem of designing mufflers with low restriction as well as low radiated exhaust noise. Imposed noise regulations for the eighties have intensified the need for development of both experimental and theoretical modeling techniques to assist in the overall design effort. The analytical methods developed to predict the performance of exhaust mufflers in the past have been seriously handicapped due to the difficulties involved in the accurate measurement and proper modeling of the exhaust source impedance [1-3]. Apart from the lack of proper characterization of the source impedance, neglect of mean flow effects has been found to cause a considerable discrepancy between experiments and analytical predictions. Effect of damping due to friction and shear flow on the internal surfaces of the mufflers also has been neglected, which often has a considerable effect around resonant or antiresonant frequencies. The sources of energy loss in an acoustical system may be divided into two categories: those due to dissipation of acoustic energy in the transmitting medium and those associated with the boundaries of the medium [6].

In this paper two simple mathematical models have been suggested for the inclusion of damping along with the effect of mean flow in the system. The first model takes into account the damping due to the friction between the flowing

medium and the walls of the system. The second model accounts for the shear viscosity effects between the layers of the medium. The solutions to these mathematical models are then transformed into the familiar two-by-two transfer matrix form which can be conveniently used in the evaluation of the performance of mufflers. The computed predictions using these transfer matrices have been compared with those measured experimentally by simulating a mean flow of air and a pure tone in the systems tested. Agreement appears to be very good.

2. Mathematical Modeling for Sound Propagation Through a Uniform Duct.

The transfer matrix formulation of a uniform pipe or expansion chamber of certain length with/without mean flow is presented here briefly in Sections 2.1 and 2.2. Thereafter, the models including the damping effects are presented in Sections 2.3 and 2.4.

2.1 No Flow or Stationary Medium

In the classical theory of sound transmission [6] the plane wave equation is given as

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2} \quad (1)$$

The steady state harmonic solution to (1) in terms of acoustic pressure p and acoustic mass velocity v can be expressed as

$$\left. \begin{aligned} p(x) &= A \exp(-ikx) + B \exp(ikx) \\ \text{and } v(x) &= \frac{A}{Y} \exp(-ikx) - \frac{B}{Y} \exp(ikx) \end{aligned} \right] \quad (2)$$

where $k = \omega/c$ is the wave number,

$\omega = 2\pi f$; f is the frequency of wave propagation,

c = speed of wave propagation,

$Y = c/S$ is the characteristic impedance,

and S = cross sectional area of the pipe.

A and B are the amplitudes of the forward and backward waves.

Substituting the boundary conditions at $x=0$ and $x=l$ for a uniform pipe [7], (2) can be transformed into a transfer matrix form

$$\begin{bmatrix} p \\ v \end{bmatrix}_0 = \begin{bmatrix} \cos(kl) & iY \sin(kl) \\ \frac{i}{Y} \sin(kl) & \cos(kl) \end{bmatrix} \begin{bmatrix} p \\ v \end{bmatrix}_l \quad (3)$$

T.M.

Expression (3) gives the desired transfer matrix (T.M.) for the plane wave sound propagation through a pipe of uniform cross section without mean flow in the medium. The subscripted column matrices with subscripts 0 and l represent the acoustic pressure p and the mass velocity v at $x=0$ and $x=l$ respectively.

2.2 Mean Flow or Moving Flow

Replacing the time derivative ' $\frac{\partial}{\partial t}$ ' in (1) by its substantial derivative ' $\frac{D}{Dt}$ ' = $\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}$, mean flow is introduced in the plane wave equation resulting in [7]:

$$\frac{\partial^2 p}{\partial t^2} + 2U\frac{\partial^2 p}{\partial x \partial t} + (U^2 - c^2)\frac{\partial^2 p}{\partial x^2} = 0 \quad (4)$$

The corresponding solution to (4) will be

$$p(x) = A \exp\left[-i\frac{\omega}{c+U}x\right] + B \exp\left[i\frac{\omega}{c-U}x\right] \quad (5)$$

Applying the boundary conditions the transfer matrix form is

$$\begin{bmatrix} p_f \\ v_f \end{bmatrix}_0 = e^{-iM(k_f l)} \begin{bmatrix} \cos(k_f l) & iY \sin(k_f l) \\ \frac{i}{Y} \sin(k_f l) & \cos(k_f l) \end{bmatrix} \begin{bmatrix} p_f \\ v_f \end{bmatrix}_l \quad (6)$$

T.M.

where $k_f = \frac{k}{1-M^2}$ and the subscript f stands for the variables with mean flow included.

2.3 Mean Flow With Damping Due to Friction

The governing equation for this case can be written [8] as

$$\frac{\partial^2 p}{\partial t^2} + 2U\frac{\partial^2 p}{\partial x \partial t} + (U^2 - c^2)\frac{\partial^2 p}{\partial x^2} + 2\zeta U\frac{\partial p}{\partial t} + 2\zeta U^2\frac{\partial p}{\partial x} = 0 \quad (7)$$

where ζ is the friction coefficient. It should be recognized that in (7) there are now two additional terms, $2\zeta U\frac{\partial p}{\partial t}$ and $2\zeta U^2\frac{\partial p}{\partial x}$. These terms represent the damping due to friction on the walls of the system. The approximate solution to equation (7) can be written in the following form:

$$p(x) = A \exp\frac{-(is+r/\omega)\omega x}{c+U} + B \exp\frac{(is+r/\omega)\omega x}{c-U} \quad (8)$$

where $r = \zeta U$ and $s \approx 1$.

Applying the boundary conditions for (8),

$$\begin{bmatrix} p_d \\ v_d \end{bmatrix}_0 = e^{-\gamma M \ell} \begin{bmatrix} \cosh(\gamma \ell) & Y \sinh(\gamma \ell) \\ \frac{1}{Y} \sinh(\gamma \ell) & \cosh(\gamma \ell) \end{bmatrix} \begin{bmatrix} p_d \\ v_d \end{bmatrix}_\ell \quad (9)$$

T.M.

where $\gamma = (is+r/\omega)k_f$, $k_f = \frac{k}{1-M^2}$, $s \approx 1$, and $r = \zeta U$.

2.4 Mean Flow With Damping Due to Shear Viscosity

The governing equation for wave propagation for the case of no flow but with shear viscosity present in the system [9, 10] is

$$\frac{\partial^2 p}{\partial t^2} = c'^2 \frac{\partial^2 p}{\partial x^2} \quad (10)$$

where c' = complex velocity of wave propagation defined as

$$\equiv c(1-i\alpha/k)$$

$$\alpha \equiv [(c+U)/(c-U)]^2 [\eta_e \omega / (2\rho)]^{1/2} / (cr_1)$$

η_e \equiv is the effective shear viscosity, and

r_1 \equiv the radius of the pipe.

Now, replacing the derivative $\frac{\partial}{\partial t}$ by its substantial derivative

$\frac{D}{Dt} = \frac{\partial}{\partial t} + U \frac{\partial}{\partial x}$, mean flow can be introduced in the system and the resulting transfer matrix form is,

$$\begin{bmatrix} p_s \\ v_s \end{bmatrix}_0 = e^{-iM'k'_c \ell} \begin{bmatrix} \cos(k'_c \ell) & iY' \sin(k'_c \ell) \\ \frac{i}{Y'} \sin(k'_c \ell) & \cos(k'_c \ell) \end{bmatrix} \begin{bmatrix} p_s \\ v_s \end{bmatrix}_\ell \quad (11)$$

T.M.

where $k'_c = \frac{\omega/c'}{1-(U/c')^2}$ and $Y' = \frac{c'}{S}$. S is the cross sectional area of the uniform pipe.

The transfer matrices derived in the above four cases are quite general in their application and should be used depending on the physical medium of propagation existing in the system. Friction damping should be considered when higher mean flow is present or when friction losses for the pipe surface are considerable. Similarly, damping due to shear viscosity should be considered when viscosity of the propagating medium is significant.

3. Noise Reduction

There are several performance criteria that have been suggested by many researchers in the past. However, some authors use them with different meanings. There are three most commonly used terms: insertion loss, transmission loss, and noise reduction. Insertion loss (IL) is mostly preferred by manufacturers because it is easier to measure; however, its prediction also requires the knowledge of the source impedance. Transmission loss (TL) is easier to calculate and, therefore, is used by researchers. Noise reduction (NR), on the other hand, can be easily predicted as well as measured. By definition NR is defined as the difference between the sound pressure levels measured at the input of a muffler and its output [11]. Experimentally, NR is measured simply by the difference in pressure levels of two microphones - one at the inlet and the other at the outlet of the muffler, whereas the expression for NR can be easily derived starting from the open outlet end. This criterion of muffler performance has been used quite frequently by many investigators presenting experimental work [12-16].

4. Analytical Expression for NR by the Transfer Matrix Approach

Consider a muffler system as shown in Figure 1a. Stations I and II contain the muffler which is terminated by the source and load impedance respectively. Figure 1b shows the equivalent block diagram with source and load impedances connected as lumped elements. The product of transfer matrices from the load end L to station I can be written as follows:

$$\begin{bmatrix} p_I \\ v_I \end{bmatrix} = \begin{bmatrix} T \\ \end{bmatrix}_n \begin{bmatrix} T \\ \end{bmatrix}_{n-1} \dots \begin{bmatrix} T \\ \end{bmatrix}_1 \begin{bmatrix} 1 & Z_L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ v_L \end{bmatrix} \quad (12)$$

where subscripted [T]'s represent the transfer matrices corresponding to the number of elements n present in the muffler system and Z_L represents the load impedance of an unflanged open end. Also in the above equation, p_L has been taken to be zero because of the load end being open to the atmosphere [1].

Equation (12) can also be written as

$$\begin{bmatrix} p_I \\ v_I \end{bmatrix} = \underbrace{\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}}_A \begin{bmatrix} 0 \\ v_L \end{bmatrix} \quad \begin{matrix} A \text{ is the overall} \\ \text{product matrix} \end{matrix} \quad (13)$$

giving,

$$p_I = A_{12} \cdot v_L \quad (14)$$

Similarly, considering the tail pipe, as shown in Figure 1c, one can write,

$$\begin{bmatrix} p_{II} \\ v_{II} \end{bmatrix} = \begin{bmatrix} T \\ \ell \end{bmatrix} \begin{bmatrix} 1 & Z_L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ v_L \end{bmatrix} \quad (15)$$

where $[T]_\ell$ stands for the transfer matrix of a pipe corresponding to the tailpipe of length ℓ .

Equation (15) can be rewritten in a product matrix form:

$$\begin{bmatrix} p_{II} \\ v_{II} \end{bmatrix} = \underbrace{\begin{bmatrix} A_{11}' & A_{12}' \\ A_{21}' & A_{22}' \end{bmatrix}}_{A'} \begin{bmatrix} 0 \\ v_L \end{bmatrix}; \quad A' \text{ is the product matrix} \quad (16)$$

giving,

$$p_{II} = A_{12}' \cdot v_L \quad (17)$$

From relations (14) and (17) one can define noise reduction (NR) as,

$$NR = 20 \log_{10} \left| \frac{p_I}{p_{II}} \right|$$

or

$$NR = 20 \log_{10} \left| \frac{A_{12}}{A_{12}'} \right| \text{dB} \quad (18)$$

It should be recognized here that the two product matrix terms A_{12} and A_{12}' are functions of the physical parameters of the muffler system, the load impedance, and also the frequency of wave propagation. The source impedance does not appear in the expression (18) for noise reduction.

A computer program incorporating the transfer matrices for various muffler elements shown in Figure 2 has been written to predict noise reduction for a particular muffler system consisting of these elements. Recognizing the various elements in a given muffler, one can quite readily predict the noise reduction characteristics. Various transfer matrices with no flow and including flow effects [5,15] have been given in Appendix A.

5. Experimental Set-Up and Measurement Procedure

The schematic diagram of the experimental set-up is shown in Figure 3. The noise reduction due to the muffler system 'I II' can be found by measuring the difference in sound pressure levels at stations I and II.

A sinusoidal acoustic signal was fed into the system by means of a horn driver connected to a power amplifier and the B & K (1022) beat frequency oscillator. The frequency range was controlled by the oscillator between 150 and 1000 Hz. Mean flow in the system was simulated by introducing an air flow of the order of 0.15 Mach number. The velocity of the mean flow was measured at the open end of the system by means of a Pitot tube.

To reduce the inherent fluctuating flow noise due to air turbulence, a silencing tank was connected in-line with the pipe introducing air into the system. A pressure regulator was connected before the silencing tank to maintain a steady pressure and flow of air. Two heterodyne slave filters were used along with measuring amplifiers to filter the signals from the two condenser microphones measuring the sound pressure levels at the input and output of the muffler. A level recorder was mechanically connected to the oscillator to get a continuous record of the sound level variation at sections I and II. All the muffler models were made from 4 mm thick mild steel pipes to avoid any significant vibrations or yielding of the walls.

The five basic types of laboratory mufflers that were tested experimentally are shown in Figure 4. It should be noted that these mufflers are essentially made up of the basic reactive muffler elements shown in Figure 2. Wooden supports were used to hold the mufflers rigidly on the test table.

6. Discussion of Results

6.1 Expansion Chamber Mufflers

The expansion chamber muffler is the simplest possible muffler configuration. Basically, it is made up of an inlet tail pipe, a sudden expansion, an expansion chamber, a sudden contraction, and an outlet tail pipe connected in series. Two expansion chamber type mufflers were fabricated: one with a small expansion chamber of 79.8 mm diameter and 500 mm length and the other with a large expansion chamber of 142.8 mm diameter but of the same length. The two sizes were chosen to determine how well the mathematical model represents a range of diameters.

Before introducing flow into the system, it was decided to verify experimentally the performance of these mufflers without mean flow. The solid line in Figures 5 and 6 shows

the NR versus frequency plot for the small and large expansion chamber mufflers respectively, with the effect of viscous damping included. Experimentally measured values of noise reduction have been shown by small circles, whereas triangles represent the predicted NR when there was no damping in the system. These triangles have been shown only at the resonant and antiresonant frequencies where the effect of damping is most pronounced, since for other frequencies the NR remains essentially the same as those represented by the solid line. Generally, the experimental results are in very good agreement with the predicted results except at sharp peaks and valleys. The small discrepancies may be in part due to the inability of the measuring system to record accurately the suddenly changing pressure levels as the frequency of the input is swept over the frequency range. It may be observed that in the NR spectra there are none of the usual expansion chamber humps as are observed in the case of typical TL spectra. The main reason for this characteristic of NR curves is that both the inlet and the outlet tail pipes are also taken into account when predicting the NR of a muffler system.

By the introduction of mean flow into the system, the sharp resonant and antiresonant peaks were dramatically flattened, although the general shape of the attenuation curves still remained the same. Figures 7 and 8 show the predicted and measured results. The general observation in these cases is that damping flattens the peaks. The experimental results are in closer agreement with the case of friction damping with mean flow than with the case of only mean flow. The overall conclusion is that analytical and experimental results agree consistently.

6.2 Muffler With Extended Inlet and Outlet

Extended inlet and extended outlet are, in general, branched types of elements. Figure 9 shows the predicted and measured performance of a muffler using an extended inlet and an extended outlet. The configuration of this muffler has been shown in Figure 4c. Again, the agreement between theory and experiment is reasonably good even though there was a mean flow of 0.15 M through the system. The sharp NR peak observed around 700 Hz is due to the extended outlet elements with a length of 125 mm. The cavity in the extended outlet element acts as a resonator and produces the sharp peak of attenuation at a frequency dependent on the length of the cavity.

6.3 Muffler With a Hole-Cavity Resonator

Hole-cavity type of resonators are meant to generate a high noise reduction peak corresponding to their resonating frequency. This resonating frequency is proportional to the impedance of the holes and volume of the cavity. Unfortunately, by introducing flow into the system, the sharp peak

due to the resonant frequency of the resonator disappears. This behavior is verified by the comparison of predicted and measured results for a hole-cavity type of muffler (Figure 4d) presented in Figure 10. Nevertheless, it is a common practice of muffler manufacturers to use fully perforated resonator elements in mufflers to cut down the high frequency content flow noise generated due to various other elements, without excessively restricting the flow of exhaust gases.

6.4 Muffler With Two Flow Reversals

Flow reversal elements form an integral part of present day mufflers used in most North American cars. The usual practice is to use a combination of flow reversals along with expansion chambers, resonators, and branched elements to arrive at the desired muffler configuration. The only disadvantage of reversal elements is that they cause heavy back pressure in the system [7]. Therefore, they attenuate noise at the cost of engine performance. Nevertheless, their high attenuation characteristics are made use of in many automobile mufflers. Figure 11 shows the predicted and measured performance of a muffler using two reversals - an expansion reversal and a contraction reversal (Figure 4e). In general, the agreement is very good except at the resonant peak around 350 Hz.

7. Conclusions

The following conclusions can be drawn from the results presented:

- 1) Including damping in the no flow case, very good agreement can be obtained between theory and experiment. Damping does not have much effect in the case of mean flow except at the resonant and antiresonant frequencies.
- 2) Knowledge or characterization of the source impedance is not a prerequisite in determining noise reduction using the transfer matrix approach.
- 3) Due to the absence of flow in complex elements like extended inlets, extended outlets, hole cavity resonators, and flow reversals, there would not be any additional terms due to friction damping accompanied with the mean flow. Therefore, the transfer matrices developed and used for these elements for the no flow case are satisfactory for the analysis of exhaust mufflers when flow is considered.
- 4) Perforated tube elements can be successfully modeled as a lumped hole-cavity element only if all holes are located in a single grouping along the tube.
- 5) Flow reversal elements, which are commonly used in most commercial exhaust mufflers, are effective in generating a large noise reduction in the lower frequency range, but the

fact that they can produce considerable back pressure on the engine should not be overlooked. Instead, extended outlet elements could be introduced in the mufflers to get a similar effect if space permits.

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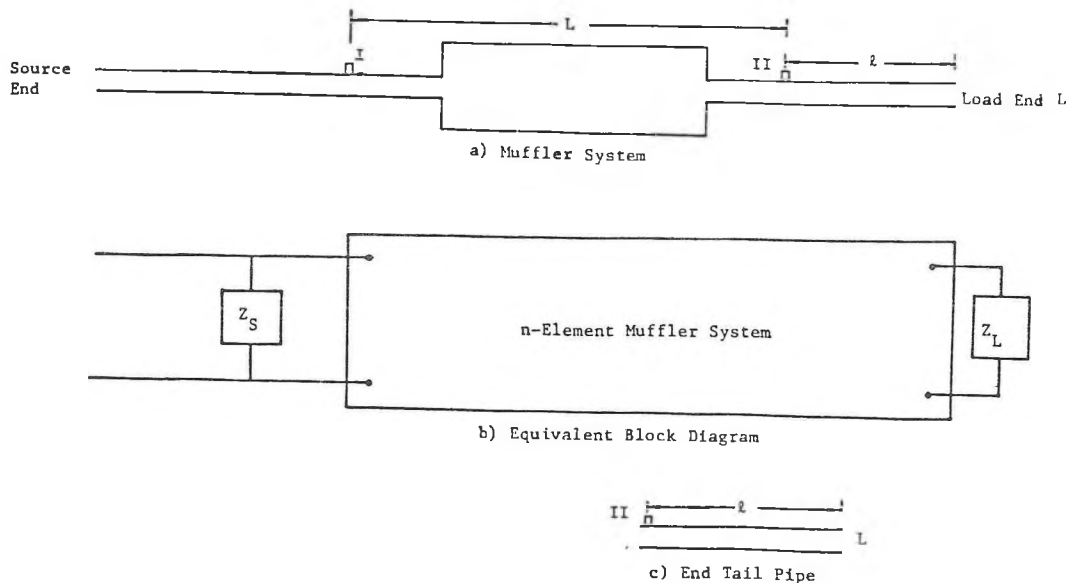


Figure 1. ANALOGOUS MUFFLER SYSTEM TO PREDICT NOISE REDUCTION.

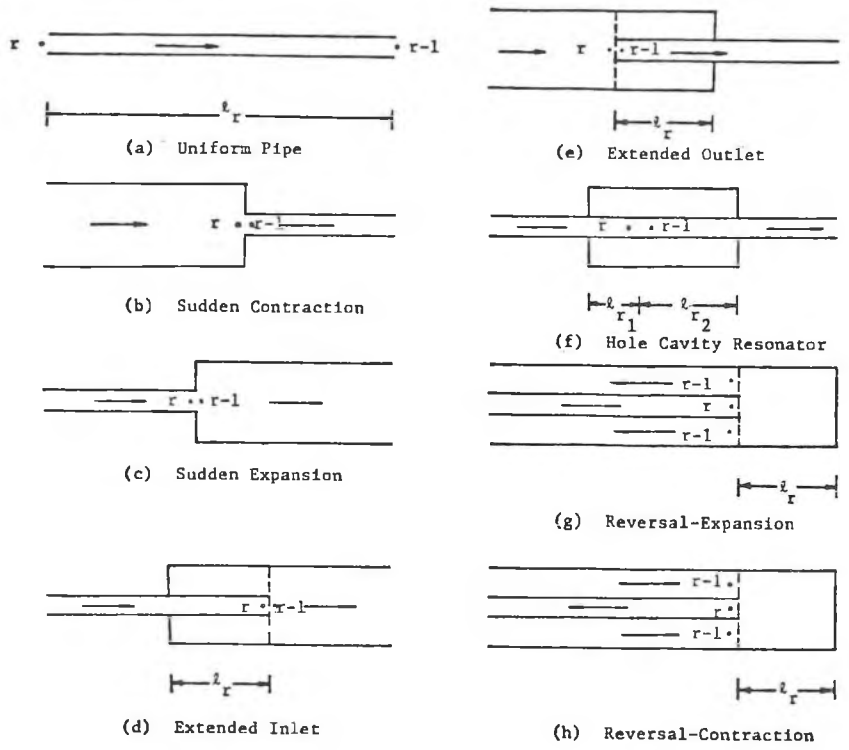


Figure 2. VARIOUS ELEMENTS CONSTITUTING A REACTIVE MUFFLER.

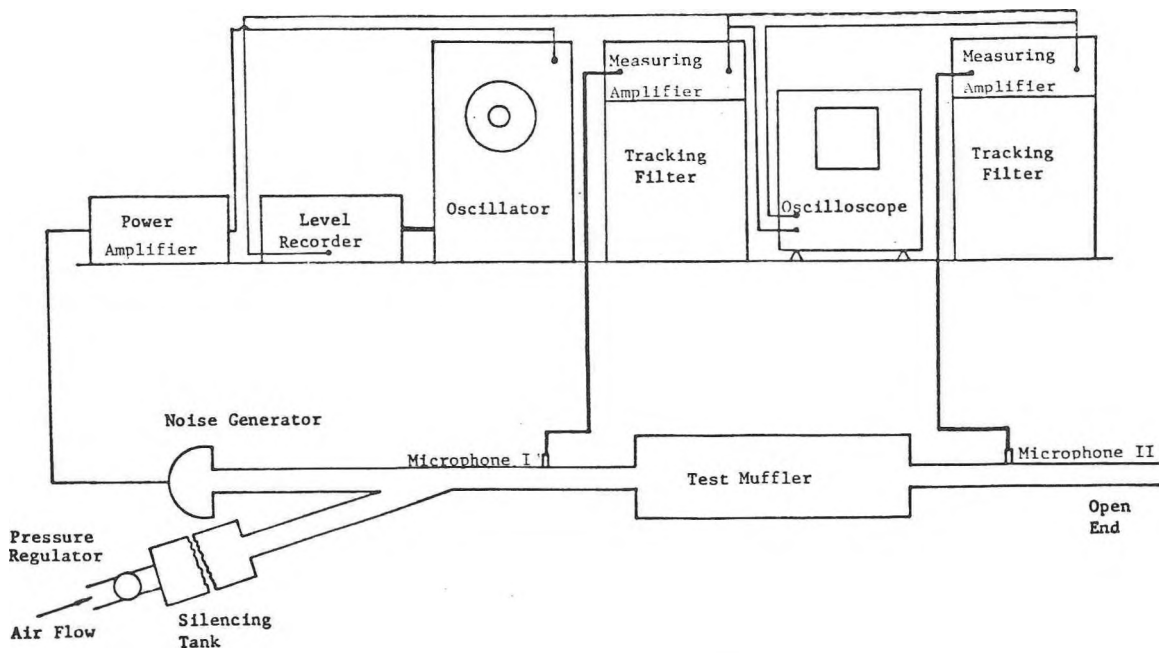


Figure 3. SCHEMATIC DIAGRAM OF EXPERIMENTAL APPARATUS FOR NOISE REDUCTION MEASUREMENTS.

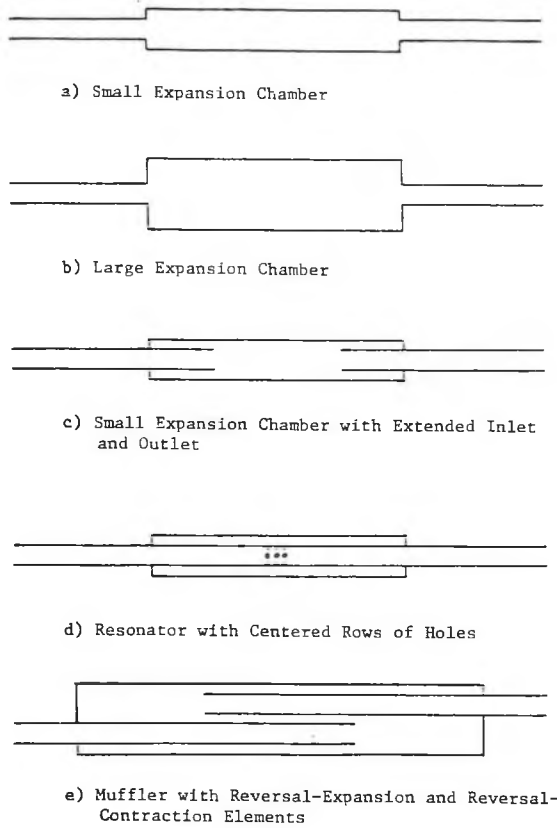


Figure 4. LABORATORY MUFFLERS TESTED

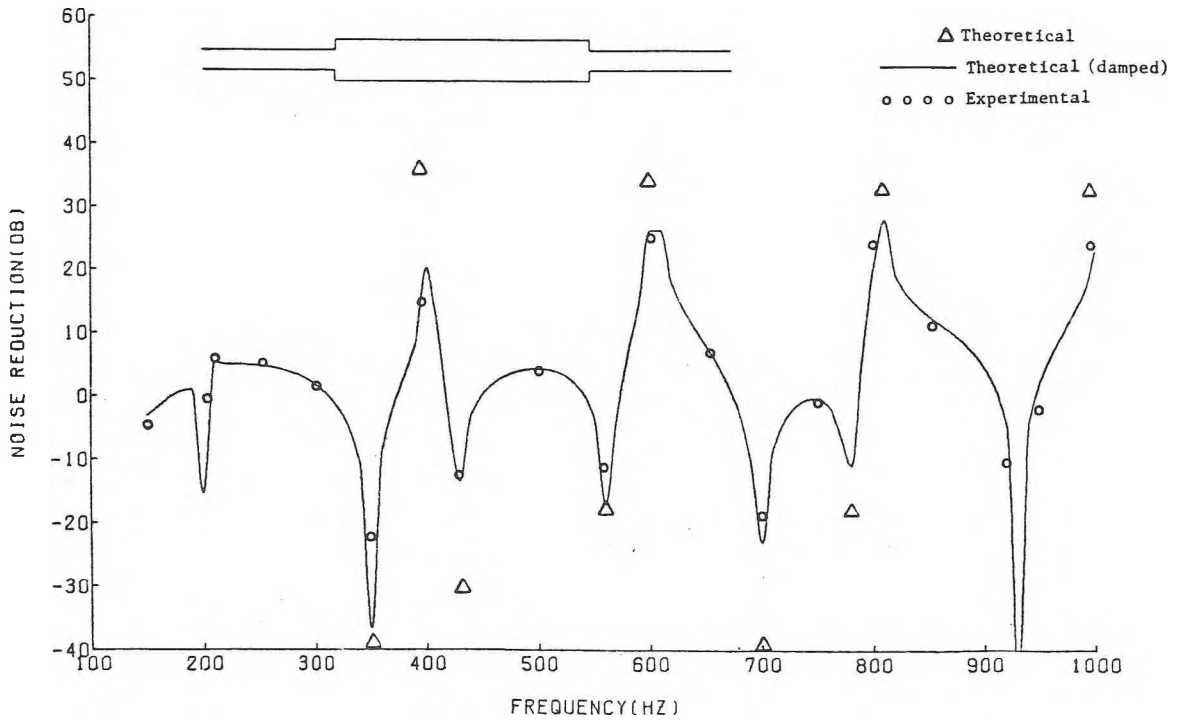


Figure 5. NOISE REDUCTION CHARACTERISTICS OF SMALL EXPANSION CHAMBER MUFFLER WITH NO FLOW.

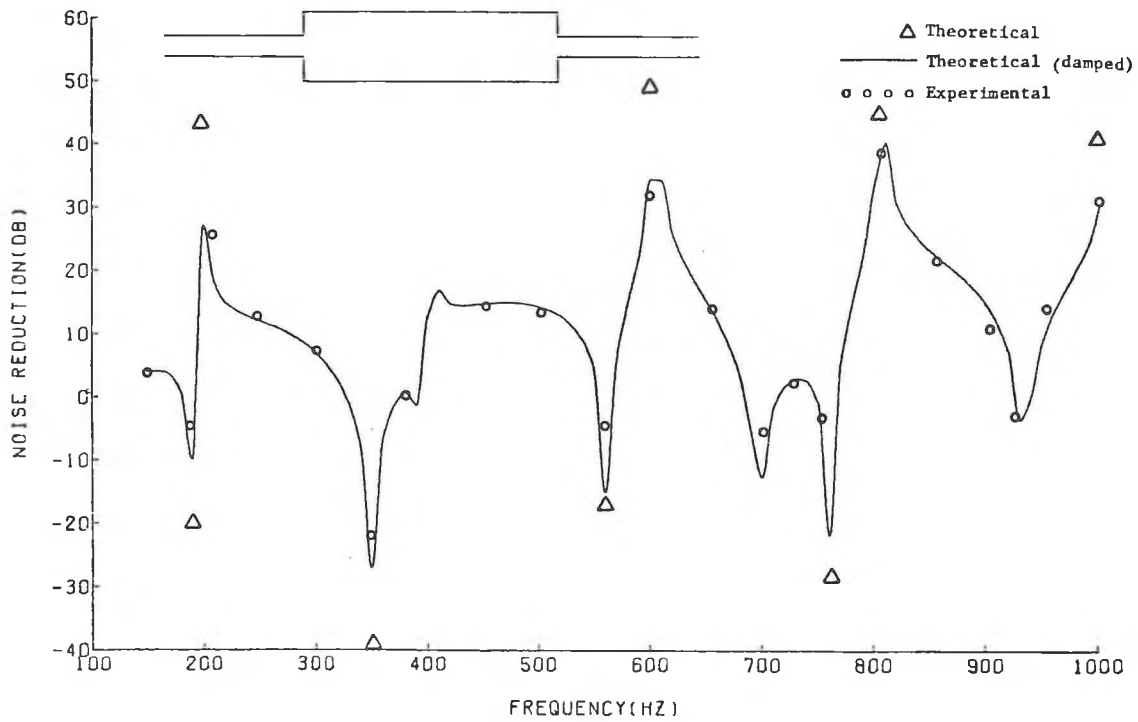


Figure 6. NOISE REDUCTION CHARACTERISTICS OF LARGE EXPANSION CHAMBER MUFFLER WITH NO FLOW.

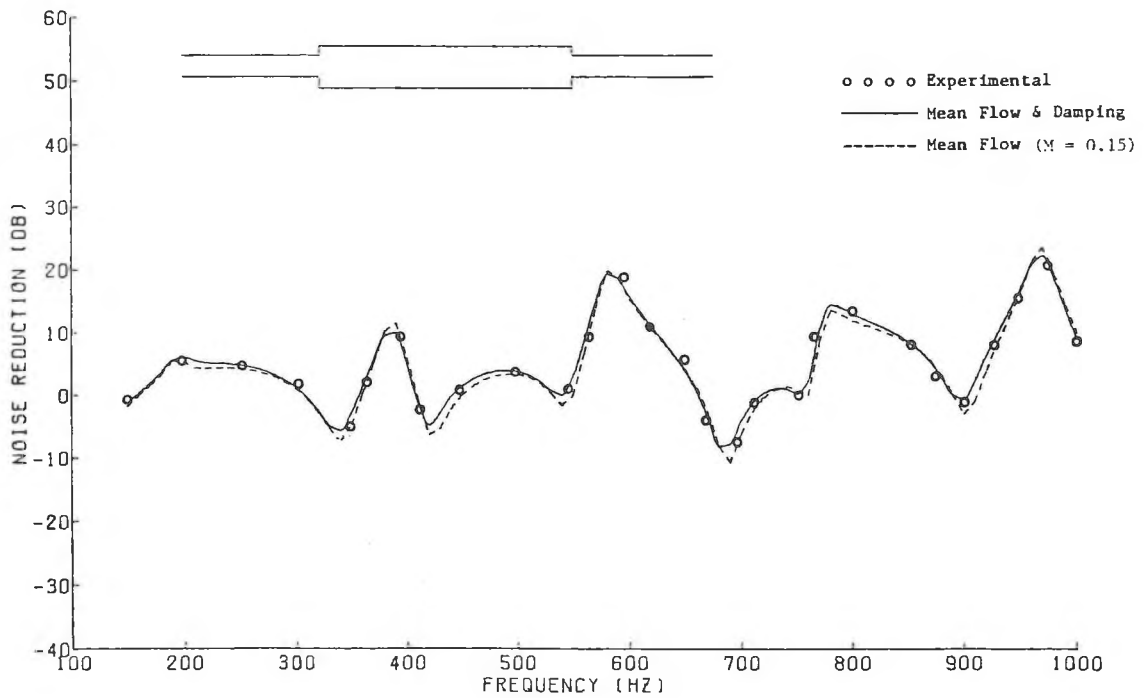


Figure 7. NOISE REDUCTION CHARACTERISTICS OF SMALL EXPANSION CHAMBER MUFFLER

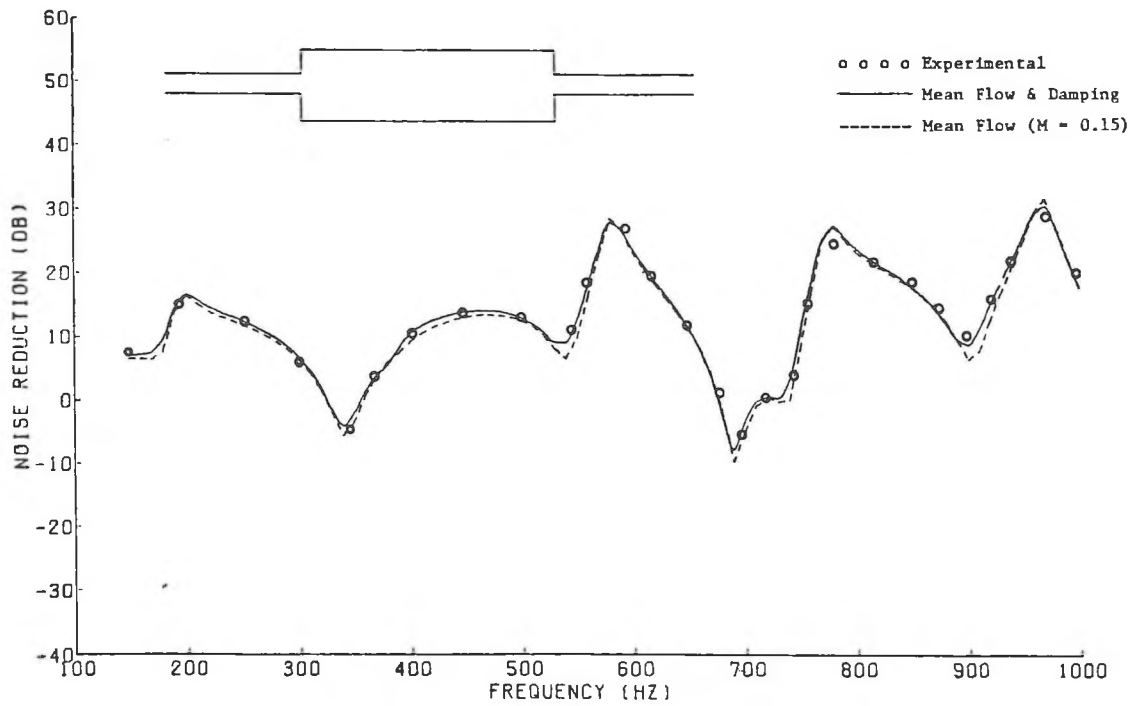


Figure 8. NOISE REDUCTION CHARACTERISTICS OF LARGE EXPANSION CHAMBER MUFFLER.

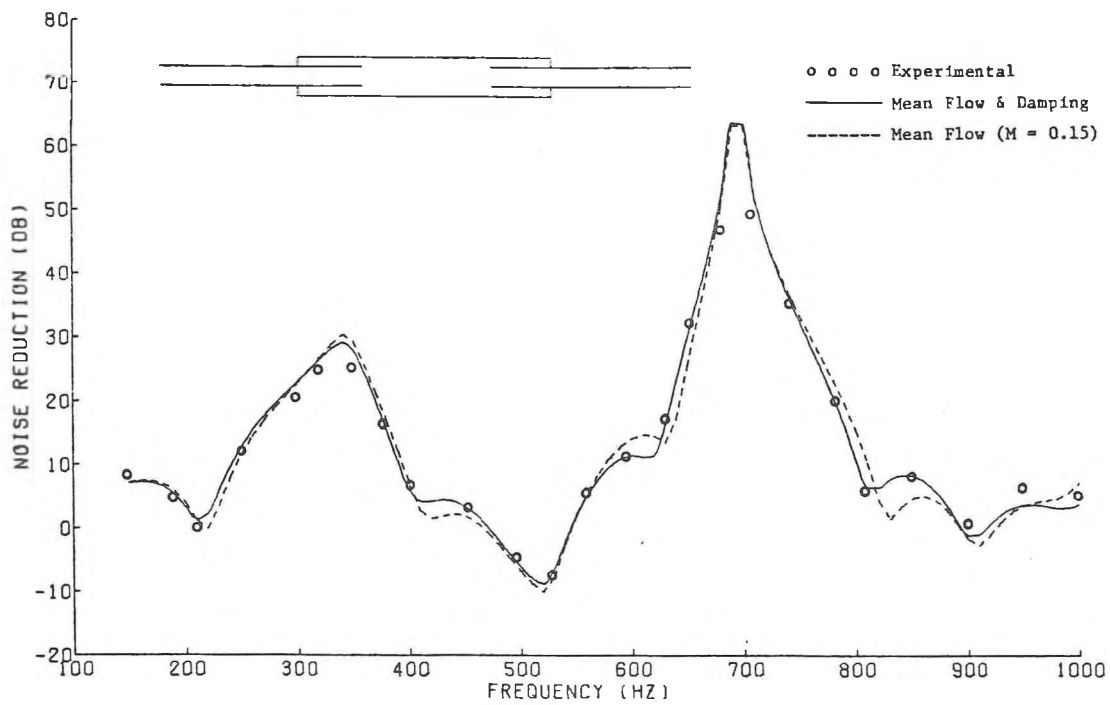


Figure 9. NOISE REDUCTION CHARACTERISTICS OF SMALL EXPANSION CHAMBER WITH EXTENDED INLET AND OUTLET.

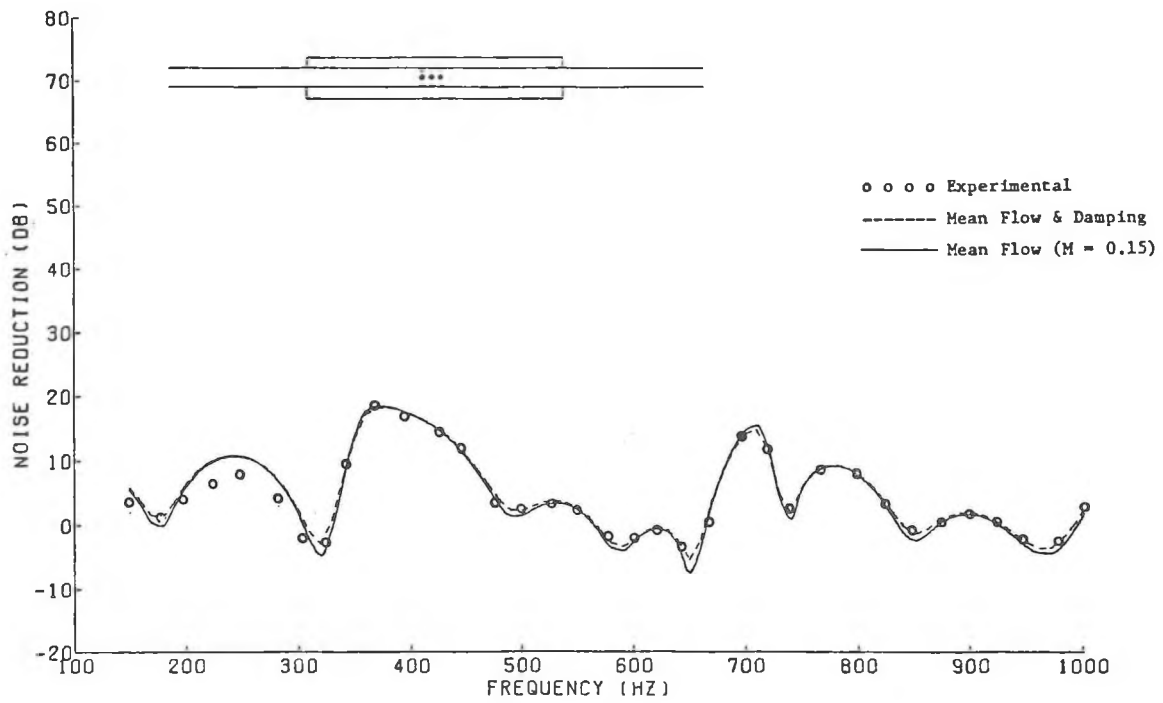


Figure 10. NOISE REDUCTION CHARACTERISTICS OF HOLE-CAVITY RESONATOR MUFFLER.

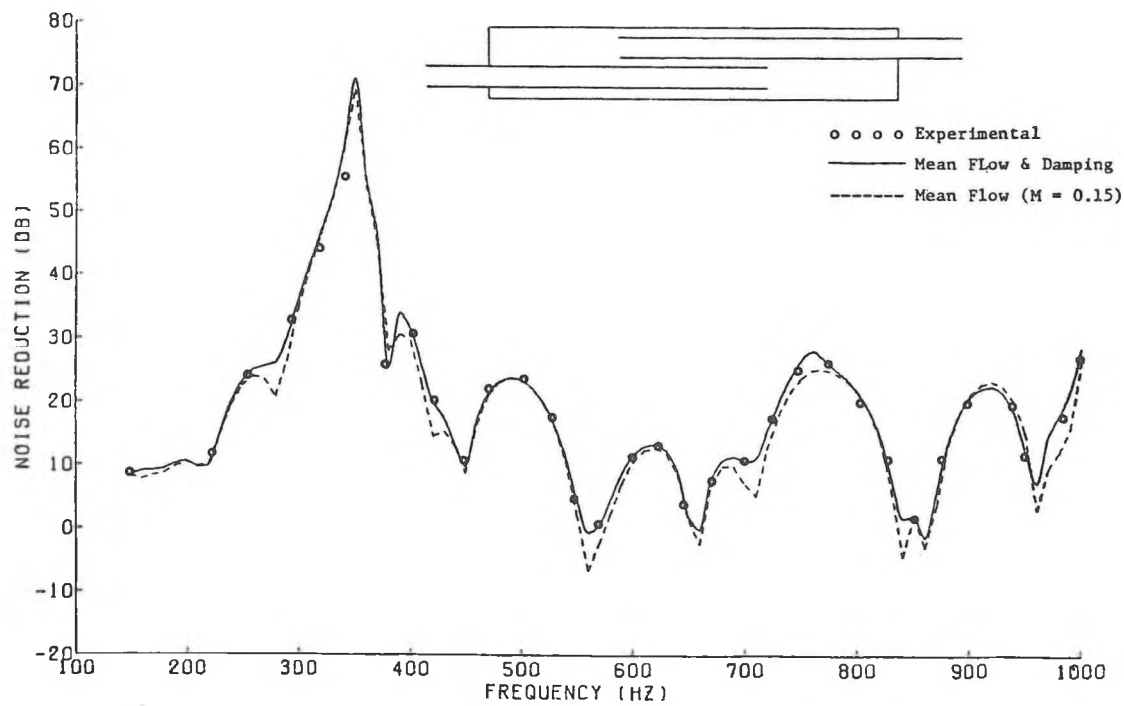


Figure 11. NOISE REDUCTION CHARACTERISTICS OF REVERSAL MUFFLER.

APPENDIX A

TRANSFER MATRICES FOR VARIOUS MUFFLER ELEMENTS

(a) Uniform Pipe

No Flow

$$\begin{bmatrix} \cos(k\ell_r) & iY_r \sin(k\ell_r) \\ \frac{i}{Y_r} \sin(k\ell_r) & \cos(k\ell_r) \end{bmatrix}$$

Mean Flow

$$e^{-iM_r(k_c \ell_r)} \begin{bmatrix} \cos(k_c \ell_r) & iY_r \sin(k_c \ell_r) \\ \frac{i}{Y_r} \sin(k_c \ell_r) & \cos(k_c \ell_r) \end{bmatrix}$$

where $k_c = \frac{k}{1-M^2}$; r is a subscript corresponding to the element.

(b) Sudden Contraction

No Flow

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Mean Flow

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(c) Sudden Expansion

No Flow

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Mean Flow

$$\begin{bmatrix} 1 & M_{r+1} \cdot Y_{r+1} (1-2N+N^2) \\ 0 & 1 \end{bmatrix}$$

where $N = \frac{S_{r+1}}{S_{r-1}}$.

(d) Extended Inlet

No Flow

$$\begin{bmatrix} 1 & 0 \\ \frac{1}{Z_r} & 1 \end{bmatrix}$$

Mean Flow

$$\frac{1}{1+2NA} \begin{bmatrix} 1+A & M_{r+1} \cdot Y_{r+1} (1-2N+N^2+N^2A) \\ \frac{1}{Z_r} & 1+N^2A \end{bmatrix}$$

where $A = M_{r+1} \cdot Y_{r+1} / Z_r$; $Z_r = -iY_r \cot(k\ell_r)$

(e) Extended Outlet

No Flow

$$\begin{bmatrix} 1 & 0 \\ \frac{1}{Z_r} & 1 \end{bmatrix}$$

Mean Flow

$$\begin{bmatrix} 1 & 0 \\ \frac{1}{Z_r} & 1 \end{bmatrix}$$

where $Z_r = iY_r \cot(kl_r)$.

(f) Hole-Cavity Resonator

No Flow

$$\begin{bmatrix} 1 & 0 \\ \frac{1}{Z_r} & 1 \end{bmatrix}$$

Mean Flow

$$\frac{1}{1+2A} \begin{bmatrix} 1+A & M_{r+1} \cdot Y_{r+1}^A \\ \frac{1}{Z_r} & 1+A \end{bmatrix}$$

where Z_r is the impedance of the resonator.

(g) Reversal Expansion

No Flow

$$\begin{bmatrix} 1 & 0 \\ \frac{1}{Z_r} & 1 \end{bmatrix}$$

Mean Flow

$$\frac{1}{1+N-NA(1-a)} \begin{bmatrix} 1+N-NA(1-a) & M_{r+1} \cdot Y_{r+1}^a \\ \frac{1}{Z_r} & 1 \end{bmatrix}$$

where 'a' would vary between 2 and 3; it needs to be determined experimentally.

(h) Reversal Contraction

No Flow

$$\begin{bmatrix} 1 & 0 \\ \frac{1}{Z_r} & 1 \end{bmatrix}$$

Mean Flow

$$\begin{bmatrix} 1 & 0 \\ \frac{1}{Z_r} & 1 \end{bmatrix}$$

where $Z_r = -iY_r \cot(kl_r)$.