THE ANALYSIS OF ARRAYS USING STARPAK

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ABSTRACT

Some of the main features and a typical application of STARPAK (Simulation for Testing Array Response) is presented. STARPAK is a package of Fortran subroutines designed to study the performance of an arbitrary planar array in a variety of Gaussian signal-noise environments. Array data with the appropriate statistics are simulated and then processed using the conventional, optimal and Bienvenu techniques.

SOMMAIRE

Quelques uns des traits principaux ainsi qu'une application pratique de "STARPAK" sont présentés. "STARPAK" est un ensemble de sous-routines en language "Fortran" qui a pour but d'étudier le fonctionnement d'un groupe quelconque de détecteurs en présence d'un signal et d'un bruit gaussien. Les signaux recueillis par les détecteurs de même que les données statistiques sont simulés, et, par la suite, analysés en se sevant des techniques: conventionnel, optimal et "Bienvenu".

1. Introduction

The detection of sinusoidal signals in ocean ambient noise is an important problem in underwater acoustics[1]. The acoustic energy radiated by ships, submarines and torpedoes are examples of such sinusoidal signals. The data available upon which to make a decision (signal or no signal present) are often those received at an array of hydrophones which is located in the ocean. The detection of signals using array data is a very complex problem[2] and many questions can only be answered using numerical simulation during the analysis. STARPAK[3] is a package of Fortran programs useful in such a study.

Herein we describe the user friendly input-output features of STARPAK and indicate how the program package may be employed to generate random array data for an important class of real world sinusoidal signal-ocean noise scenarios. STARPAK contains efficient algorithms for three methods often suggested for use in the detection of signals using arrays. Finally some comparisons are made of these processing techniques.

2. Theory

a) Statement of the Problem

We assume we have a planar array of n hydrophones located at arbitrary positions in the plane. The signals are sinusoidal and hence a first step in the analysis is to transform the data to the frequency domain. In this domain signals and noise are assumed to be complex Gaussian variables. The noise is modell d as the sum of white, cylindrical and spherical noises with arbitrary powers. Spherical noise is noise generated by a large number of discrete sources uniformly distributed on a sphere whose radius is much larger than the dimensions of the array. Cylindrical noise is noise generated by a large number of discrete sources uniformly distributed over the surface of a cylinder whose radius is much larger than the dimensions of the array and whose axis is normal to the array plane. White noise is due to a large number of discrete sources which are located close to each sensor, and is independent from sensor to sensor. The signals are plane waves of a specific power and direction arriving in the plane of the array.

Our aim is to simulate data received at an array for the above signal-noise scenario and then process the data with algorithms suitable for signal detection in such situations.

b) <u>Data Simulation</u>

At each hydrophone the data are zero mean Gaussian. Hence the data received at the array are completely decribed by its covariance matrix Q. The functional form of Q depends on the signal-noise scenario[3,4]. Let E be an n dimensional vector of independent complex zero mean Gaussian random variables. Subroutines are available to simulate such data. Also suppose Q = U*U is a Cholesky decomposition[5] of the covariance matrix. Here * denotes complex conjugate transpose. In such a decomposition U is a lower triangular matrix. Then X = U*E is a vector sample with the required statistics. An important quantity in the following section is the sampled covariance matrix, $\hat{Q}$. It is defined as the average of a number of XX* samples.

c) <u>Array Processing</u>

The advantages of an array over a single sensor are numerous. One of the most important features is its directional property - which enables it to discriminate between signals arriving from different directions. The direction we are interested in at a particular instant is called the "look direction". The central task of array processing is to investigate techniques which reduce the effect due to noise from "non-look" directions. STARPAK examines three such methods. All require knowledge of the sampled covariance matrix and one, the single frequency version of Bienvenu's detection test technique[3,6] assumes a priori knowledge of the noise only covariance matrix, Q_N. The conventional[2] (Bc), optimal[2] (Bo) and Bienvenu (Bb) beam outputs for direction Θ are:

$$Bc(\Theta) = C^* D_\Theta^* \hat{Q} D_\Theta C \qquad (1)a$$

$$Bo(\Theta) = n^2/(C^* D_\Theta^* \hat{Q}^{-1} D_\Theta C) \qquad (2)a$$

$$Bb(\Theta) = \frac{C^* D_\Theta^* \hat{Q}^{-1} D_\Theta C}{C^* D_\Theta^* \hat{Q}^{*-1} Q_N \hat{Q}^{-1} D_\Theta C} \qquad (3)a$$

where C = [1,...1]* and D_Θ is the diagonal steering matrix in the look direction.

A complete description of these processors and their properties is beyond the scope of this paper. However some of their characteristics are noted here. In conventional beamforming the phases of the sensor inputs are adjusted so that a signal from the look direction adds coherently. An optimal beamformer results when we process the data so that a constant signal response is maintained in the look direction and the power from non-look directions is minimized. An optimal beamformer is not in general optimum for the detection question posed here. The Bienvenu statistic is based on the theory of hypothesis testing[7]. If the data are noise only $\hat{Q}$ will in general be close to Q_N and the beam output close to one. In the case where a signal is present $\hat{Q}$ will in general be different than Q_N and the Bienvenu beam output greater than one for the look directions containing signals.

It is easily verified that the above formulae can be written as:

$$Bc(\Theta) = |\hat{U}D_\Theta C|^2 \qquad (1)b$$

$$Bo(\Theta) = n^2/(Y^*Y) \text{ where } \hat{U}^*Y = D_\Theta C \qquad (2)b$$

$$Bb(\Theta) = (Y^*Y)/(Z^*Z) \text{ where } Z = BX \text{ and } \hat{Q}X = D_\Theta C \qquad (3)b$$

and where $Q_N = B^*B$ and $\hat{Q} = \hat{U}^*\hat{U}$ are Cholesky decompositions.

Using the second set of equations we are able to obtain the beam outputs without evaluating the inverse of $\hat{Q}$, a very difficult numerical problem. In our implementation once $\hat{Q}$ and Q_N have been Cholesky decomposed the beam outputs are calculated by carrying out forward and backward substitution in two systems of linear equations and performing a number of matrix multiplications. The result is a fast and accurate method to obtain the required quantities.

3. Implementation and Examples

The input to STARPAK consists of an array geometry and signal-noise scenario to be investigated. As well, an a priori noise matrix is required for Bienvenu's method. Finally, the number of samples to be averaged in the sampled covariance matrix is set. This number will be referred to as the number of samples averaged in the following. A menu type format is used to input these parameters. After execution we obtain the processor beam outputs. See Figure 1.

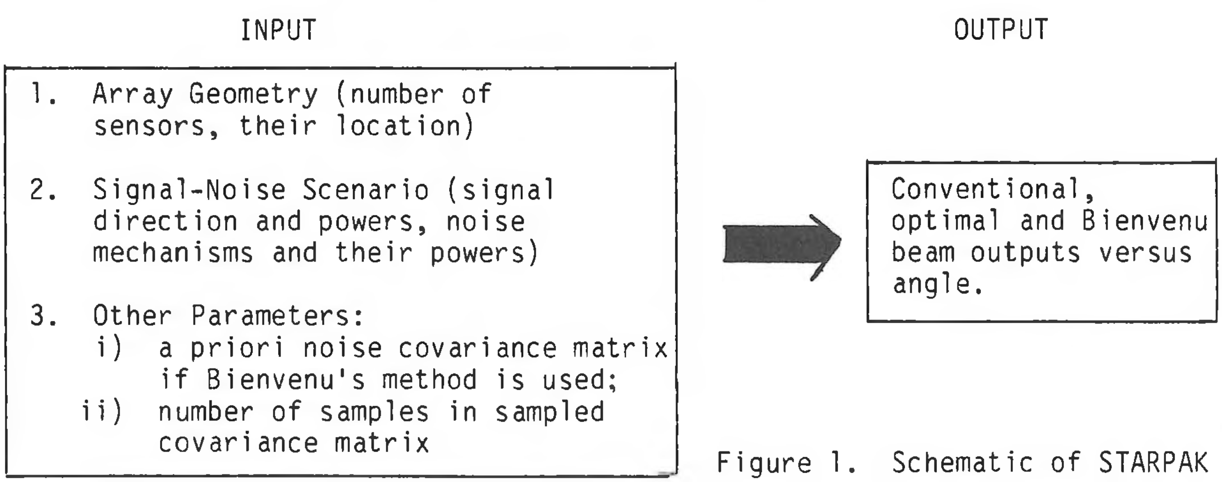


Figure 1. Schematic of STARPAK

We present one example to illustrate some of the main features of STARPAK. The reader can envision many more. Consider a 16-element equispaced linear array with interelement spacing $d = .4\lambda$ where λ is the wavelength of the signal. Imbedded in cylindrical noise of power 1. and white noise power .1 is one signal at 60° of power .4 (signal-to-noise ratio -4.41dB). We have assumed Q_N is white and cylindrical of the appropriate powers. Typical results are shown in Figure 2 where the beam output for the three methods is plotted versus angle. All curves have been normalized to have a maximum value of one. Figures 2(b), (c) and (d) present examples when 32, 64 and 128 samples respectively have been averaged. We note the left-right ambiguity of linear arrays, that is the signal also appears at 300°.

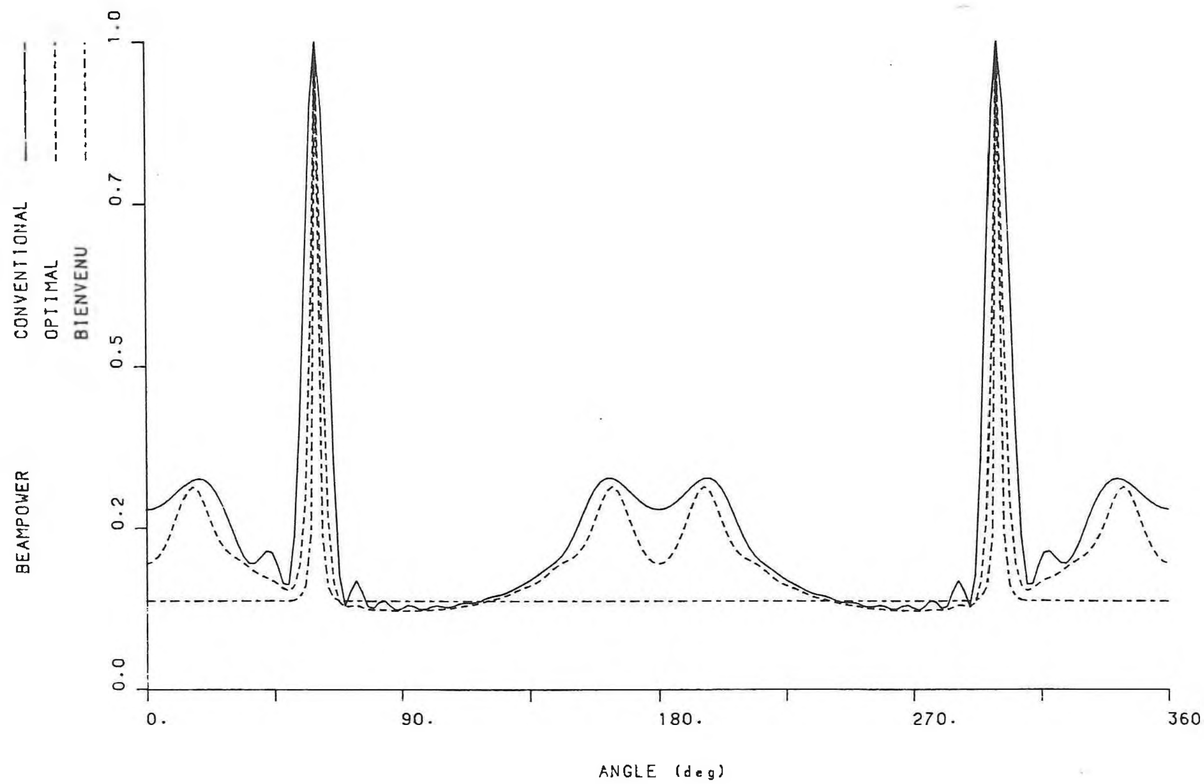
Two measures are useful when comparing processing methods when various numbers of samples are averaged. Signal beamwidth, BW, is defined as the width of the signal at its -3dB point and signal-to-background noise ratio, SBN, as the peak signal level divided by the background noise. Signals processed by an ideal technique would have a BW of zero and SBN proportional to their signal-to-noise ratios. In practice we expect BW to decrease and SBN increase as the number of samples averaged increases. Table I illustrates these trends for our data. As the number of samples averaged increases $\hat{Q}$ approaches Q and best performance is attained. This is referred to as the deterministic case. See Figure 2(a) and Table I for these values in our example.

Number of Samples in Covariance Matrix	Beamwidth			Signal to Background Noise Ratio		
	Bc	Bo	Bb	Bc	Bo	Bb
32	10°	5°	28°	5.2	7	2.04
64	9°	4°	4°	5.4	5.6	2.75
128	10°	3.5°	2°	5.3	5.62	4.18
∞ (deterministic case)	10°	5°	< 1°	5.76	6.98	7.31

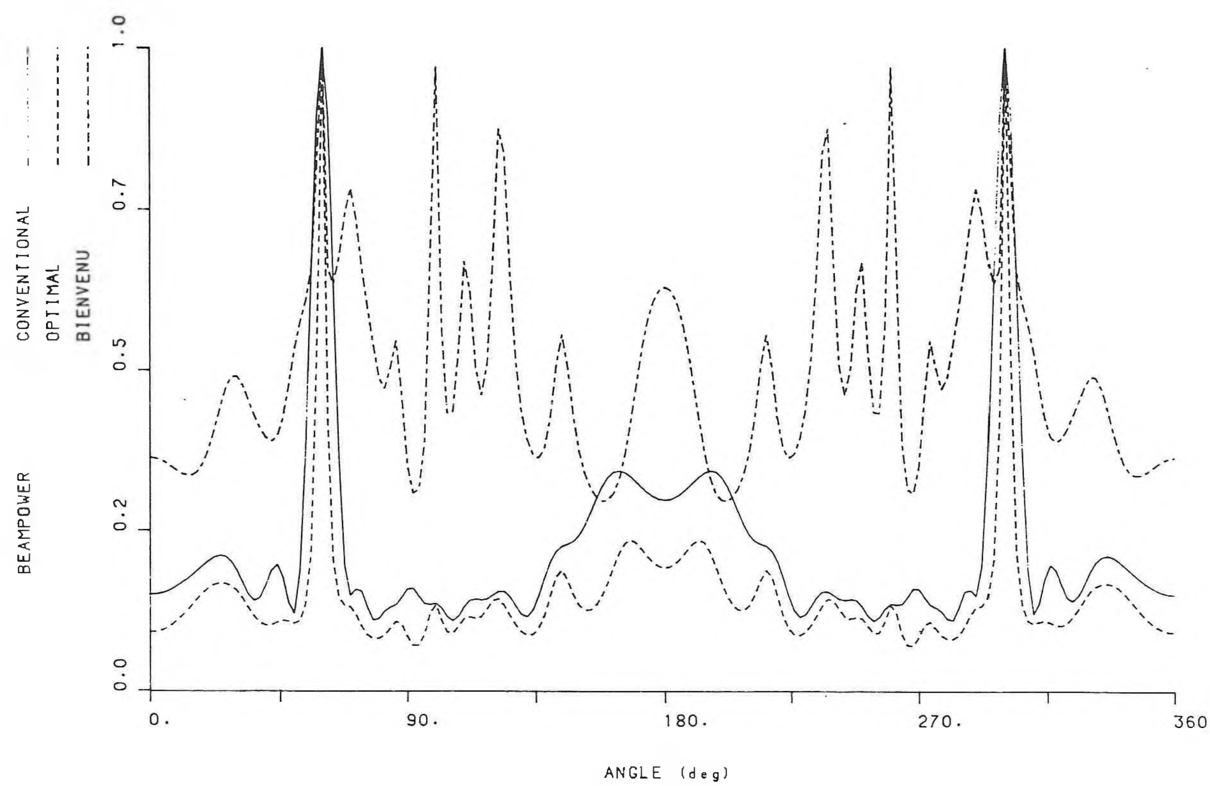
Bc is Conventional, Bo is Optimal, and Bb is Bienvenu Processing

Table I. Signal beamwidth and signal-to-background noise ratio for the cases illustrated in Figure 2.

Using these measures we observe that optimal processing is better than conventional. When the number of samples is at least 64 (at least four times the number of array elements) Bienvenu processing is preferred to optimal. It has an acceptable SBN ratio and superior resolution (smaller BW). Our experience shows "this rule of thumb" holds in a wide variety of cases. When fewer than 64 samples are available for averaging Bienvenu processing is worse than optimal. This occurs due to the form of the Bienvenu statistic which is a quotient of two random variables as given in equation 3(a). Even small fluctuations in both variables about their means cause large fluctuations in the overall statistic.

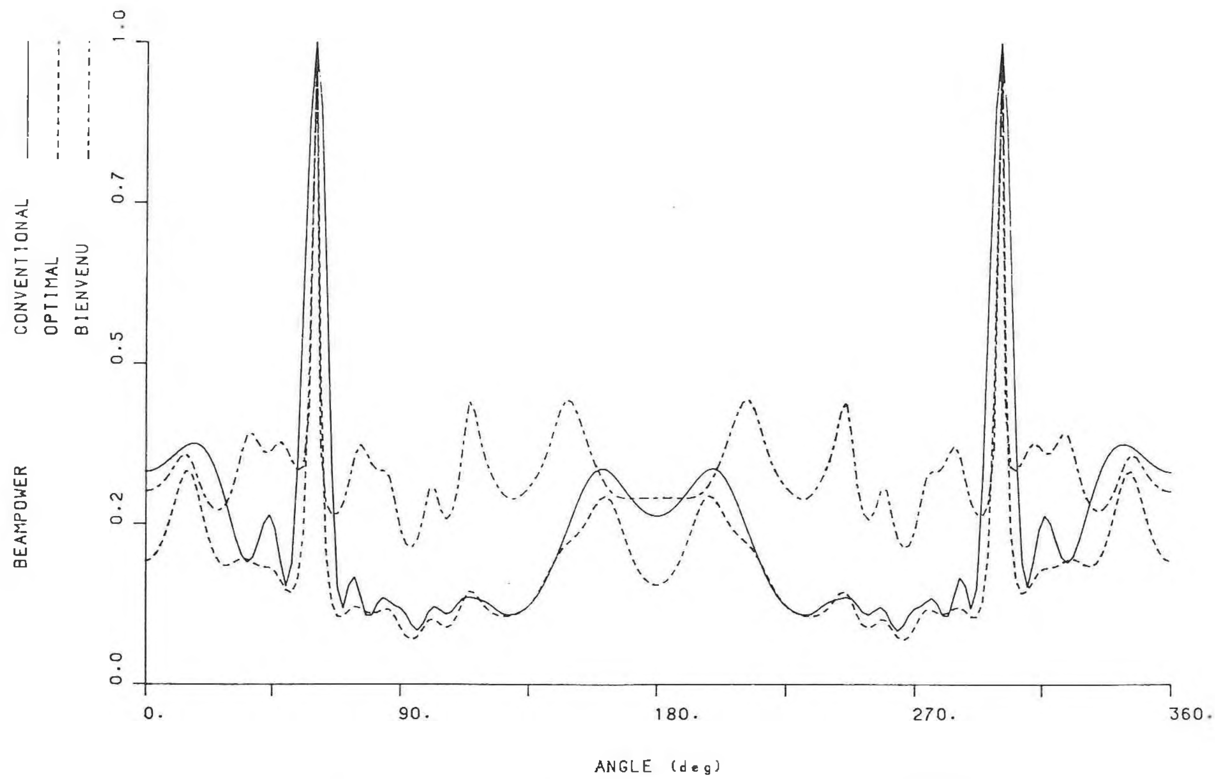


(a) Deterministic covariance matrix.

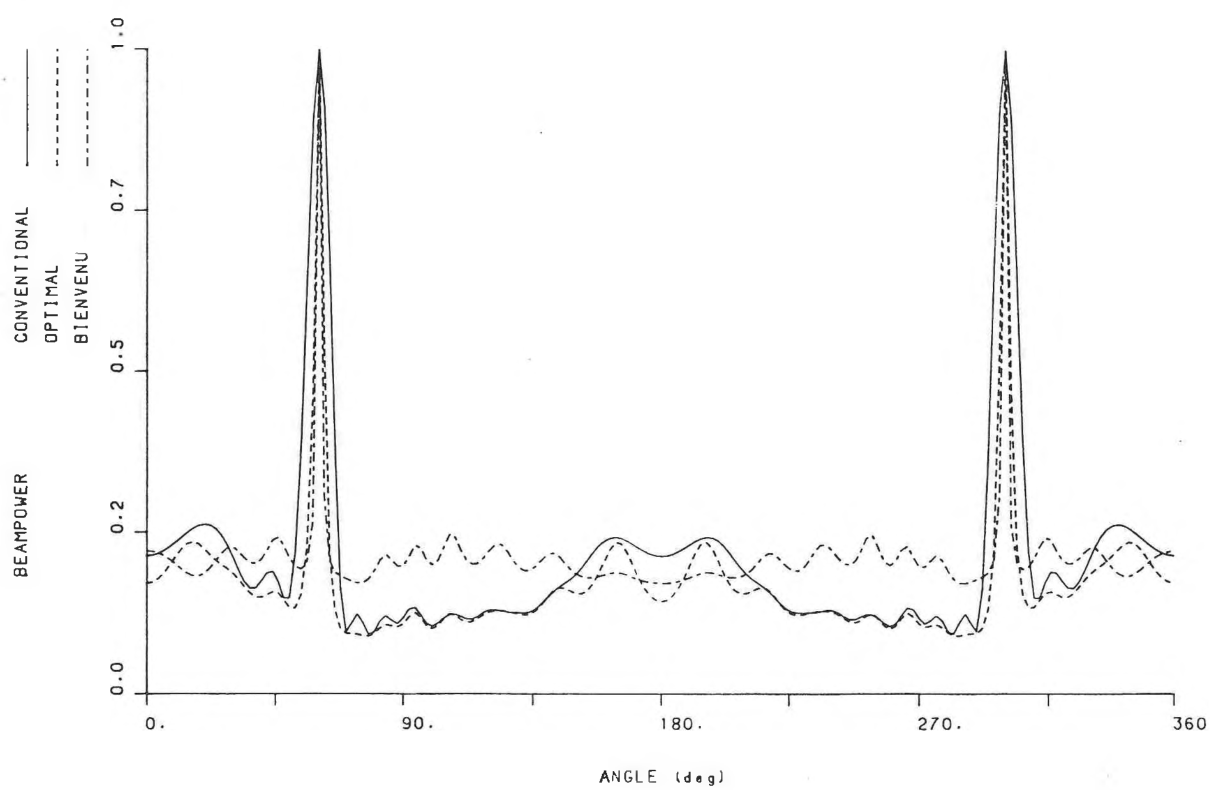


(b) Sampled covariance matrix using 32 samples.

Figure 2. Beam output versus angle for text example.



(c) Sampled covariance matrix using 64 samples.



(d) Sampled covariance matrix using 128 samples.

Figure 2. Beam output versus angle for text example.

4. Summary

STARPAK is a powerful, versatile tool in the study of an important class of real world signal detection problems using arrays. The algorithms used in STARPAK are very efficient. Its user friendly input-output features make it accessible to both the novice and experienced researcher.

Beamwidth and signal-to-background noise ratio are two useful quantities when comparing detection capabilities of array processing methods. Optimal processing is usually better than conventional. Bienvenu processing was superior to optimal when the noise field is "almost" known exactly and a large number of samples are averaged to form the sampled covariance matrix.

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