THE TRANSMISSION OF SOUND THROUGH WALLS, WINDOWS, AND PANELS: A ONE DIMENSIONAL TEACHING MODEL

By:
R.W. Guy
Centre for Building Studies, Concordia University
Montreal, Quebec, Canada H3G 1M8

ABSTRACT

A One dimensional mathematical teaching model which incorporates many of the sound transmission characteristics found in real panels, has been developed.

The model enables the ready formulation of panel velocities and sound pressure level ratios for simple and multiple panel systems. The analysis is ideally suited for micro computer application.

SOMMAIRE

Un modèle mathématique à une dimension, incorporant plusieurs caractéristiques de transmission sonore propres à des panneaux réels a été développé pour des applications pédagogiques.

Le modèle permet de calculer les vélocités et le niveau de pression sonore pour des systèmes de panneaux simples et multiples. L'analyse s'adapte parfaitement à des applications sur micro-ordinateur.

LIST OF SYMBOLS

A	Constant of equations 5 and 6
B	Constant of equations 5 and 6
B	Plate bending stiffness defined after equation (36)
B'	Complex Plate stiffness
b	Panel length
C	Panel damping
c	Panel width
d	Depth or axial distance
E	Youngs Modulus
e	Exponential
F	Forces
h	Panel thickness
j	$\sqrt{-1}$
K	Panel spring stiffness constant
k	Wave number
M	Mass
P	Pressure
q	Modal integer
R	Real or resistive component
r	Modal integer
SPR	Sound pressure ratio
t	Time
t'	Thickness of absorbent material, employed at equation (30)
V	Velocity
X	Imaginary or reactive component
x	Spatial ordinate
Z	Impedance
Z'	Normalised impedance
α	Resistive component of complex propogation constant
β	Reactive component of complex propogation constant
η	Material internal damping factor
θ	Angle of incidence
Π	Successive products
ρ	Density
ψ	Velocity potential
ω	Angular frequency

Suffix

A	Designator employed after equation (20)
B	Designator employed after equation (20)
i	Variable integer relating to panel number
i'	Incident
L	Left of
m	Mechanical (in vacuo)
n	Relating to the last panel
o	Material or propogating medium property
p	Panel
q	Panel mode in y direction
R	Right of
r	Panel mode in Z direction
r'	Reflected
rr	Rereflected

1. INTRODUCTION

The teaching of sound transmission through windows, walls, and panels is typically achieved by presenting a series of simple models each of which attempts to describe a trend or phenomenon observed in practice.

Such models lead, for example to a qualitative understanding of the 'Stiffness Controled Region', the 'Damping Controled Region', the 'Mass Controled Region', the 'Coincidence Region, as applied to single leaf systems, and the 'Mass Spring Mass Resonance' encountered in double leaf systems

The models generally involve infinite panels and consider the influence of incidence angle upon various resonant frequencies, whilst in practice the most prominent resonance - 'Mass Spring', 'Coincidence', and 'Mass Spring Mass', have been found independent of angle; meanwhile the models do not readily consider the influence of 'Room Effects', 'Multiple Panels', 'Multiple Resonance', 'Absorbent layers', since their inclusion is usually accompanied by complicated analysis and interpretation, however each of these factors may drastically change the manifestation of a 'Region', or 'Resonance Conditions'. In consequence a need exists to expand the 'repertoire' of model types whilst avoiding the complexities of model formulation and analytical intractability.

In an attempt to satisfy these requirements an analytical structure and procedure based upon the work of Nestrov [1] is developed which the student may employ to generate formulations in a simple manner, for models ranging from the most rudimentary to complicated multi-faceted systems; the analytical procedure developed is capable of implementation on micro computer systems and applied in this way will assist in relieving problems associated with analytical intractability.

The present analysis is generally confined to one dimensional considerations particularly with respect to incident air born waves, thus complexity of analysis is eased, however the concept and influence of two dimensional panel vibration is introduced for illustrative purposes.

2. BASIC TERMS AND EXPRESSIONS

2.1 THE WAVE EQUATION

The one dimensional form of the acoustic wave equation may be written as, [2]:

$$\frac{d^2\psi}{dx^2} = \frac{1}{c_0^2}\frac{d^2\psi}{dt^2} \tag{1}$$

where
- ψ is the velocity potential
- c_0 is the velocity of sound in the medium
- x a spatial coordinate
- t time

Confining attention to the steady state harmonic form, that is

$$\psi \; \alpha \; e^{j\omega t}$$

where ω is the angular forcing frequency.
Equation (1) may now be rewritten as:-

$$d^2\psi/dx^2 + k^2\psi = 0 \quad (2)$$

where k is the wave number ($k = \omega/c_0$).

The wave equation is here expressed in terms of the velocity potential 'ψ' to facilitate the derivation of expressions for pressure and particle velocity via the relationships:-

$$V = d\psi/dx \quad (3)$$

where V is the acoustic particle velocity

and $P = - \rho\, d\psi/dt$

which, for the steady state harmonic case yields:-

$$P = - j\, \rho\omega\psi \quad (4)$$

where P is the excess or acoustic pressure
ρ is the density of the propogating medium

Non trivial solutions of the wave equation may be expressed in several ways: one common form of solution to equation (2) being:-

$$\psi = A\, e^{j(\omega t - kx)} + Be^{j(\omega t + kx)} \quad (5)$$

where A and B are the wave amplitudes of two distinct plane waves travelling in opposite direction. the wave amplitudes are determined by having equation (5) satisfy the prevalent boundary conditions.

An alternative form of solution to equation (2) may be written as:-

$$\psi = [A \cos(kx) + jB \sin(kx)]e^{j\omega t} \quad (6)$$

where A and B are also determined from the applied boundary conditions.

The common time dependence $e^{j\omega t}$ will subsequently be ommitted for brevity.

2.2 IMPEDANCE

Specific Acoustic Impedance Z

Z is defined as the ratio of excess pressure 'P' at a point to the acoustic particle velocity 'V' at that point,

thus $$Z(x) = P(x)/V(x) \quad (7)$$

For example, in the case of a positive going plane wave as described by the first term on the right hand side of equation (5) one can, by utilising the relationships shown in equations (3) and (4), show that

$$Z_{(+)} = \rho c_0 \qquad (8)$$

where the index (+) indicates a positive travelling wave.

Similarly, for the negative travelling wave described by the second term on the right hand side of equation (5),

$$Z_{(-)} = - \rho c_0 \qquad (9)$$

where the index '(-)' indicates a negative travelling wave

Normalised Impedance Z'

Z' is defined as the ratio of the specific impedance to that of a standard impedance.

A common 'standard impedance' is the 'characteristic impedance' of air 'ρc_0'.

Mechanical Impedance Zm

Zm is defined here as the ratio of the vibratory force 'F' per unit area acting on a surface to the vibratory velocity 'V' of the surface caused by that force

$$\text{thus. } Zm = F/V \qquad (10)$$

In the case of a panel caused to vibrate by acoustic pressures acting on either side of it, one may write:-

$$Z_P = (P_L - P_R)/V_P \qquad (11)$$

where
- Z_P is the panel impedance
- P_L is the pressure acting on the left side of the panel
- P_R is the pressure acting on the right side of the panel
- V_P is the panel velocity

3. BASIC MODEL AND ANALYSIS

The basic model consists of the i^{th} and (i+1) vibrating panel separated from each other by a distance d_i and forming part of 'n' vibrating panels in series as shown in Figure (1).

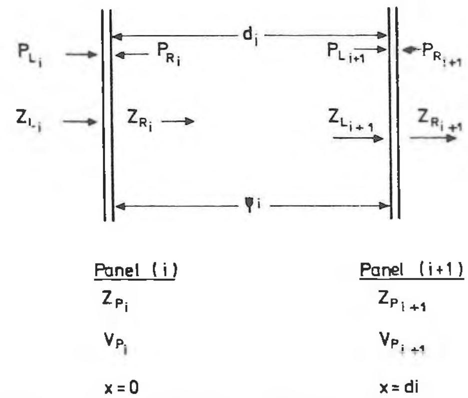

Figure 1 - Model describing parameters for the interspace between vibrating panels

With respect to Figure (1) the indices 'i' and 'i+1' denote 'with respect to given panel', and for each panel

P_L is the acoustic pressure developed on the left
P_R is the acoustic pressure developed on the right
Z_L is the total impedance of the system to the right of and including the given panel
Z_R is the total impedance of the system to the right of the given panel
Z_P is the panel's mechanical impedance
V_P is the panel's velocity
and ψ is the velocity potential between panels

3.1 THE ANALYSIS

Consider in the first instance the forces acting on the i+1 panel.
Recalling equation (11) and rewriting :-

$$V_{P_{i+1}} = (P_{L_{i+1}} - P_{R_{i+1}})/Z_{P_{i+1}} \tag{12}$$

Recalling also equation (7) and considering the right hand surface of the i+1 panel:-

$$P_{R_{i+1}} = V_{P_{i+1}} \cdot Z_{R_{i+1}} \tag{13}$$

Substituting equations (13) into equation (12) yields

$$P_{L_{i+1}} = V_{P_{i+1}} \cdot (Z_{P_{i+1}} + Z_{R_{i+1}})$$

or, again noting equation (7)

$$Z_{L_{i+1}} = Z_{P_{i+1}} + Z_{R_{i+1}} \tag{14}$$

Since no assumption has been made concerning the nature of $Z_{R_{i+1}}$ one may generalize the result shown in equation (14) by writing

$$Z_{L_i} = Z_{P_i} + Z_{R_i} \quad ;i=1,2,3 \ldots n \tag{15}$$

Consider now the velocity potential developed between the i^{th} and (i+1) panels.

Recalling equation (6), one may write the velocity potential as

$$\psi_i = A_i \cos(kx) + jB_i \sin(kx) \text{ for } 0 \leqslant x \leqslant d_i \tag{16}$$

Noting the relationship between velocity potential and particle velocity shown in equation (3), one may form the boundary conditions:-

$$\left(\frac{d\psi}{dx}\right)_{x=0} = V_{P_i}, \text{ and } \left(\frac{d\psi}{dx}\right)_{x=d_i} = V_{P_{i+1}} \tag{17}$$

Applying the boundary conditions of equations (17) to equation (16) yields

$$\psi = V_{P_i} \cdot \frac{\cos(k[di-x])}{k \sin(kd_i)} - V_{P_{i+1}} \cdot \frac{\cos(kx)}{k \sin(kd_i)} \tag{18}$$

Applying the relationship of equation (4) to equation (18), one may form an expression for the pressure at any point 'x' between the panels as:-

$$P_{i(x)} = -V_{P_i} \cdot j\rho c_o \cdot \frac{\cos(k[d_i-x])}{\sin(kd_i)} + V_{P_{i+1}} \cdot j\rho c_o \cdot \frac{\cos(kx)}{\sin(kd_i)} \tag{19}$$

or, setting $x = d_i$ in equation (19)

$$P_i\ (x=d_i) = P_{L_{i+1}} = V_{P_i} \cdot Z_{B_i} - V_{P_{i+1}} \cdot Z_{A_i} \tag{20}$$

where $Z_{A_i} = -j\ \rho c_o \cot(kd_i)$ and $Z_{B_i} = -j\ \rho c_o \operatorname{cosec}(kd_i)$

Now, $$V_{P_{i+1}} = P_{L_{i+1}} / Z_{L_{i+1}} \tag{21}$$

thus replacing $V_{P_{i+1}}$ in equation (20) by equation (21) one finds

$$P_{L_{i+1}} = V_{P_i} \cdot Z_{B_i} \cdot Z_{L_{i+1}} / (Z_{A_i} + Z_{L_{i+1}}) \tag{22}$$

or replacing $P_{L_{i+1}}$ from equation (22) into equation (21)

$$V_{P_{i+1}} = Z_{B_i} \cdot V_{P_i} / (Z_{A_i} + Z_{L_{i+1}}) \tag{23}$$

setting $x = 0$ in equation (19)

$$P_i\ (x=0) = P_{R_i} = Z_{A_i} \cdot V_{P_i} - Z_{B_i} \cdot V_{P_{i+1}} \tag{24}$$

Replacing $V_{P_{i+1}}$ from equation (23) in equation (24) yields

$$P_{R_i} = V_{P_i} \left[Z_{A_i} - Z_{B_i}^2 / (Z_{A_i} + Z_{L_{i+1}}) \right] \quad (25)$$

and finally, forming the impedance Z_{R_i}, and rearranging, thus

$$Z_{R_i} = P_{R_i}/V_{P_i} = Z_{A_i} - Z_{B_i}^2/(Z_{A_i} + Z_{L_{i+1}}) \quad (26a)$$

where Z_{A_i} and Z_{B_i} are defined after equation (20).

Equation (26a) expresses the impedance relationship in a form suitable for deriving the recurring fraction one may develop to express the impedance of a multi layered system, Nestrov [1].

By re arranging trigonometric terms and utilising normalised impedance one may express equation (26a) as:-

$$Z'_{R_i} = (Z'_{L_{i+1}} \cdot \cos(k \cdot di) + j\sin(k \cdot di))/(\cos(k \cdot di) + jZ'_{L_{i+1}} \cdot \sin(k \cdot di)) \quad (26b)$$

where Z' represents the normalised impedance $Z/\rho c_0$

Equation (26b) may be recognised as a generalised input impedance, similar to the case of a pipe terminated by an impedance Z_L, Kinsler and Frey [2, Chapter 8.7]; also this mathematical form eliminates 'removable singularities' and thus is better suited for computational purposes.

In summary, the input impedance to the left of any panel surface may be expressed via equation (15) as the sum of the mechanical 'in vacuo' impedance of the panel and the prevailing impedance to the right of the panel surface; in addition, the impedance to the right of the panel surface may be expressed via equations (26) in terms of the input impedance to the next panel on the right and the characteristics defining their separation, namely the distance between panels and the characteristic impedance of the separating medium.

Thus, by progressing panel index 'i' and by successive cross substitution for Z_{R_i} or $Z_{L_{i+1}}$ between equations (15) and (26), one may develop comprehensive formulations for the impedance on either side of any panel within the vibrating system.
In may instances it will be found expedient to develop overall equations in reverse order, that is set the panel index to $i = n$ and determine Z_{R_n} from equation (26); substitute Z_{R_n} into equation (15) and determine Z_{L_n}; repeat the procedure successively for $i=n-1$, $n-2$, etc. until the whole system has been analysed. Naturally this procedure requires a knowledge of or assumption concerning the termination impedance $Z_{L_{n+1}}$ to begin and successive panel impedance terms Z_{p_i} to continue.

3.2 CAVITY TERMINATION IMPEDANCE - $Z_{L_{n+1}}$

The cavity's termination impedance '$Z_{L_{n+1}}$' will be found in practice to lie between conditions given by surface n+1 being non existant, and, being acoustically hard.

a) Surface n+1 does not exist (completely transmitting)

Under this condition, $Z_{L_{n+1}} = \rho c_o$ which upon substitution into equation (26a) yields

$$Z_{R_n} = \rho c_o \tag{27}$$

b) Surface n+1, acoustically hard (completely reflecting)

Since $V_{n+1} = 0$, $Z_{L_{n+1}} = \infty$ hence by substitution into equation (26a)

$$Z_{R_n} = Z_{A_n} = - j \rho c_o \cot (kd_n) \tag{28}$$

c) Surface n+1, reflecting and absorbing or transmitting

In general, $$Z_{L_{n+1}} = R_{n+1} + j X_{n+1} \tag{29}$$

where R_{n+1} is a resistive component

and X_{n+1} is a reactive component

For example, consider the case of an absorbent material lining an acoustically hard backing wall. The input impedance at the surface of the absorbent material may be written in a manner similar to equation (28), except that the characteristic impedance and wave number of the propogation medium will now be complex, Beranek [3, Chapter 10.4.4], that is,

$$Z_{L_{n+1}} = - jZ_o \cdot\cot(k_o\cdot t') \tag{30}$$

where

$$Z_o = R_o + jX_o \quad \text{(A complex characteristic impedance)} \tag{31}$$

$$k_o = \beta - j\alpha \quad \text{(A complex wave number)}$$

and t' is the thickness of the absorbent material (meters)
Equation (30) may now be rewritten in hyperbolic form as:-

$$Z_{L_{n+1}} = (R_o + jX_o) \cdot \coth ([\alpha + j\beta]t') \tag{32}$$

or by expanding the hyperbolic function with complex argument into its real and imaginary parts [4], that is

$$Z_{L_{n+1}} = (R_o + jX_o) [\sinh(2\alpha t')-j\sin(2\beta t')]/[\cosh(2\alpha t')-\cos(2\beta t')] \tag{33}$$

Equation (33) may now be expressed in its real and imaginary parts as:-

$$Z_{L_{n+1}} = R_{n+1} + jX_{n+1} \quad (34)$$

where $R_{n+1} = [R_0 \cdot \sinh(2\alpha t') + X_0 \cdot \sin(2\beta t')]/[\cosh(2\alpha t') - \cos(2\beta t')]$

and $X_{n+1} = [X_0 \cdot \sinh(2\alpha t') - R_0 \cdot \sin(2\beta t')]/[\cosh(2\alpha t') - \cos(2\beta t')]$

$Z_{L_{n+1}}$ may now be substituted directly into equation (26b) to determine Z_{R_n}.

Most cases of practical interest with respect to absorbents may be developed in a similar fashion.

3.3 PANEL IMPEDANCE

The panel's mechanical impedance 'Zp' may be presented in a number of ways depending upon the characteristic behaviour being demonstrated:-

i) Massive Wall

In the event that the wall mass control's the transmission of sound over the frequency range of interest:-

$$Z_p = j\omega M \quad (35)$$

where M is the panel mass per unit area

The impedance term of equation (35) will lead to an appreciation of the transmission loss characteristic refered to as the "Mass Law".

ii) Mass Control plus Coincidence Effect

The coincidence effect, first described by Cremer [5] and based upon a matching of airborn and panel travelling waves, leads to an impedance expression which may be written after Beranek [6, Chapter 13.7], as:-

$$Z_p = j\omega M.\cos(\theta) \left[1 - \frac{\omega^2 B'}{c_0 M} \sin(\theta)\right] \quad (36)$$

where: θ is the angle of incidence
B' is the complex plate bending stiffness B(1+jn)
B is the plate bending stiffness ($Eh^3/12$)
is the internal damping factor of the panel material as given from a complex Youngs modulus
E is Youngs Modulus of the panel material
and, h is the panel thickness

iii) Modified Coincidence Concept

It has been shown by Bhattacharya et al [7], that coincidence in a 'real' panel in the presence of a backing room is caused by a complicated matching of panel and room standing waves in the presence of strong coupling factor's, in consequence it is not angle of incidence dependent and it can occur at normal incidence ($\theta = 0$) excitation: it does however occur at the fundamental frequency predicted by equation (36), thus, for representative purposes only, one may re write the mass impedance term of equation (36) as:

$$Z = j\,\omega m[1 - (\omega/\omega_c)^2\,(1+j\eta)] \qquad (37)$$

where $\omega_c = c_o^2\,\sqrt{M/B}$ and is the fundamental coincidence frequency

iv) Sprung Wall

In an attempt to better represent the characteristics of a vibrating panel Foxwell and Franklin [8] employed a mechanical mass, spring, and damper model, for which:-

$$Z_p = C + j\,(\omega M - K/\omega) \quad \text{or} \quad Z_p = C + j\,\omega M\,[1-(\omega_p/\omega)^2] \qquad (38)$$

where C is the panel damping constant
K is the panel's spring stiffness constant

and $\omega_p = \sqrt{K/M}$, the panel's in vacuo mass spring resonance.

The impedance term of equation (38) will lead to an appreciation of transmission loss characteristics refered to as 'Stiffness Control', Damping Control', and 'Mass Control'.

v) Multiple Panel Resonance

Multiple 'Mass-Spring' resonance will be encountered in 'real' panels and their frequency will depend upon the panel characteristics such as size and boundary conditions. In general each resonance or 'eigen' frequency will have an associated impedance term and the sum of such terms will be similar to the addition of resistance in parallel; the impedance will also depend upon the spatial nature of the forcing function and upon the location considered on the panel surface which is generally evidenced by the panel velocity being coordinate dependent, thus for illustrative purposes it is necessary to consider an average impedance based upon surface averaged panel velocities and forces. The surface average impedance for the case of a simply supported panel subjected to normal incidence excitation may be written from reference [9, Appendix I], as:-

$$1/Z_p = 4.(2/\Pi)^4 \sum_{q=1}^{\infty} \sum_{r=1}^{\infty} 1/(qr)^2\,Z_{qr} \qquad (39)$$

where $Z_{qr} = j\,\omega M\,[1-(\omega_{qr}/\omega)^2]$ and is the qr^{th} 'modal' impedance

$\omega_{qr} = \Pi^2\,(B'/M)^{\frac{1}{2}}\,[(q/b)^2 + (r/c)^2]$ and is the qr^{th} panel 'eigen' frequency
b is the panel length
c is the panel breadth
q is a modal integer, q = 1,3,5 etc.
r is a modal integer, r = 1,3,5 etc.
Π 3.142 .... .

It may be noted that panel damping is now incorporated within the expression for 'eigen' frequency, by way of the internal damping factor occuring within the bending stiffness B'.

The series summation of equation (39) converges quite rapidly for terms involving eigen frequencies ω_{qr} greater than the excitation frequency ω, in addition one may note that the series summation $4 \times (2/\Pi)^4 \ \Sigma \ 1/(qr)^2 = 1$.
These features may be employed to yield computational economies.

3.4 PRESSURE AND PANEL VELOCITY

The velocity of any panel may be written in terms of pressure and impedance via the relationships shown in equations (7) and (11), that is:-

$$V_{P_i} = P_{L_i}/Z_{L_i} = P_{R_i}/Z_{R_i} = (P_{L_i} - P_{R_i})/Z_{P_i} \tag{40}$$

Also, from equation (23)

$$V_{P_{i+1}} = V_{P_i} \cdot Z_{B_i}/(Z_{A_i} + Z_{L_{i+1}}) \quad , \ i = 1 \text{ to } n-1 \tag{41}$$

Thus, equations (40) and (41) will allow any pressure ratio or ratio of pressure to panel velocity, to be determined.

Equation (19) may be employed to determine the pressure at any point between vibrating panels or between the last panel and the systems terminating impedance.

3.5 SOUND PRESSURE RATIO

As a measure of a panel system's performance, a sound pressure ratio (SPR) may be employed, that is :-

$$\text{SPR} = 20 \ \text{Log}_{10} \left(|Pi'| / |P_{R_n}| \right) \tag{42}$$

where $|Pi'|$ is the pressure amplitude of the incident pressure wave on the left hand side of the first panel.

and $|P_{R_n}|$ is the pressure amplitude on the right hand side of the n^{th} panel

The sound pressure ratio as defined in equation (42) is similar to the term 'Transmission Loss' based upon a ratio of incident intensity to transmitted intensity; this similarity becomes exact when the pressure wave on the transmitted side of the n^{th} panel is freely propogated; however the ratio of pressures employed here avoids complexities associated with the definition of intensity in the complicated sound fields caused by the presence of backing walls or rooms, whilst still providing a strong indicator of performance as might subjectively be judged.

For the present analysis, it is necessary to deduce a relationship between the incident pressure amplitude 'Pi' and the total pressure on the surface of incidence P_{L1}.

Proceeding in a manner shown by Richards and Mead [10], the total pressure on the incident surface may be written as:- $P_{L1} = Pi' + Pr' + Prr$

where Pi' = incident pressure wave
Pr' = reflected pressure wave
Prr = re-radiated pressure wave caused by panel vibration

Via equation (9), one may write the re radiated wave as:-

$Prr = -\rho c_0 \cdot V_{P1}$ and, given that $0 \leqslant Pr' \leqslant Pi'$,

as a first approximation $Pr' = Pi'$ thus $P_{L1} = 2p_i' - \rho c_0 \cdot V_{P1}$

or replacing P_{L1} by the product of panel velocity and input impedance, one may write:-

$$Pi' = V_{P1} \cdot (Z_{L1} + \rho c_0)/2 \qquad (43)$$

4. APPLICATIONS

In order to illustrate the use of the forgoing analysis, three case studies will be considered.

1. Single leaf panel backed by a cavity
2. Double leaf panel
3. Multiple leaf panel

4.1 VIBRATING PANEL BACKED BY A CAVITY

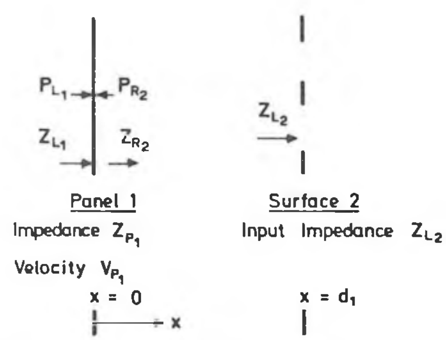

Figure 2 - Model describing parameters for a vibrating panel backed by a cavity.

The model of Figure (2) consists of a panel capable of vibration, backed by a cavity of depth d_1. The backing cavity is terminated by a surface having an as yet arbitary input impedance Z_{L2}.

Equations (15) and (26b) may be applied to determine the system's input impedance Z_{L1}, that is

$$Z_{L_1} = Z_{P_1} + Z_{R_1} \qquad (44)$$

where $$Z_{R1} = \frac{Z_{L2}\cos(kd_1) + j\,\rho c_0 \sin(kd_1)}{\rho c_0 \cos(kd_1) + j\,Z_{L2}\sin(kd_1)} \qquad (45)$$

The sound pressure ratio may be written from equation (42) as:-

$$SPR = 20.\log_{10}\left(|Pi'|/|P_{R1}|\right) \qquad (46)$$

substituting for $P_{R1} = V_{P1} \cdot Z_{R1}$ from equation (40), and P_i from equation (43) into equation (46), and expressing impedance terms in their 'normalised' form, the sound pressure ratio may now be written as:

$$SPR = 20\log_{10}\left|(Z'_{p1} + Z'_{R1} + 1)/2\,Z'_{R1}\right| \qquad (47)$$

Z'_{R1} will be derived from equation (45) upon consideration of the three choices for the normalised termination impedance Z'_{L2} deduced from equations (27), (28), and (29). For example consider the normalised form of equation (29), $Z'_{L2} = R_2 + j\,X_2$, (partially reflecting and absorbing) which upon subsituation into equation (45) yields

$$Z'_{R1} = \frac{R_2 \cdot \cos(kd_1) + j\,[\sin(kd_1) + X_2 \cdot \cos(kd_1)]}{[\cos(kd_1) - X_2 \cdot \sin(kd_1)] + j\,R_2 \cdot \sin(kd_1)} \qquad (48)$$

where d_1 is the distance from the vibrating panel to the begining of the absorbinq and reflecting layer.

If the normalised termination impedance Z'_{L2} is due to an absorbent material lining an acoustically hard backing wall, R_2 and X_2 above will be expressed by the normalised form of R and X defined after equation (34).

4.2 DOUBLE LEAF PANEL

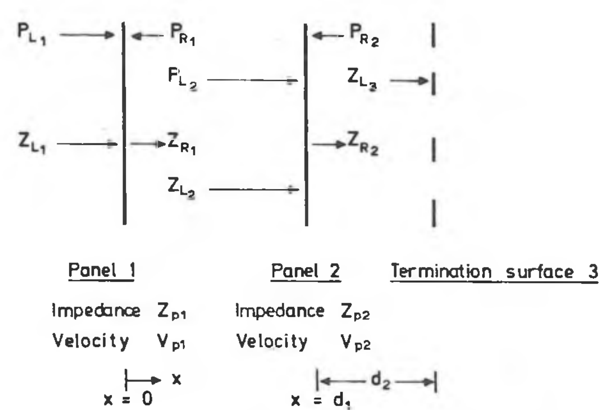

Figure 3 - Model describing parameters for a double leaf panel system.

The model of Figure (3) consists of two vibrating panels separated by an air gap of width d_1. To the right and at a distance of d_2 from the second panel, exists a termination surface having an arbitary input impedance Z_{L3}:

The sound pressure ratio for the model of Figure (3) may be written from equation (42) as:-

$$SPR = 20 \log_{10} (|Pi'|/|P_{R2}|) \tag{49}$$

where P_i is the incident pressure

and P_{R2} is the pressure to the right of the second panel

The incident pressure P_i' may be written directly from equation (43) that

is $Pi' = V_{P1} (Z_{L1} + \rho c_0)/2$

whilst the pressure existing the right of the second panel may be written from equation (40) as:-

$$P_{R2} = V_{P2} \cdot Z_{R2} \tag{50}$$

The second panel velocity V_{P2} may be written in terms of the first panel velocity V_{P1} via equation (23), and the impedance to the right of the second panel Z_{R2} may be expressed in terms of the termination impedance Z_{L3}, via equation (26a); thus:-

$$P_{R2} = V_{P1} [Z_{B1}/(Z_{A1}+Z_{L2})][Z_{A2}-Z_{B2}^2/(Z_{A2}+Z_{L3})]$$

or, rearanging terms

$$P_{R2} = V_{P1} \{Z_{A2}[Z_{B1}/(Z_{A1}+Z_{L2})]-Z_{B2}[Z_{B1}/(Z_{A1}+Z_{L2})][Z_{B2}/(Z_{A2}+Z_{L3})]\} \tag{51}$$

Equation (43) for P_i' and equation (51) for P_{R2} may now be substituted into equation (49) to yield:-

$$SPR = 20 \log_{10} | (Z_{L1} + \rho c_0)/2\ Z_2 | \tag{52}$$

where

$$Z_{L1} = Z_{P1} + Z_{A1} - Z_{B1}^2/(Z_{A1}+ Z_{P2}+ Z_{A2} - Z_{B2}^2/[Z_{A2}+ Z_{L3}]) \tag{53}$$

and

$$Z_2 = Z_{A2} [Z_{B1}/(Z_{A1}+ Z_{L2})] - Z_{B2}[Z_{B1}/(Z_{A1}+ Z_{L2})][Z_{B2}/(Z_{A2} + Z_{L3})]$$

The panel impedance terms Z_{P1} and Z_{P2} may be chosen from equations (35) to (38), whilst the termination impedance Z_{L3} may be chosen from equations (27) to (29).

In general the evaluation of equation (52) and (53) must be undertaken by computational techniques, although it has been shown, Brüel [11, Chapter 5] London [12], that analytical interpretation is possible for certain simplified case studies. It may be noted that the angle of incidence dependence of the Mass-Spring-Mass Resonance as deduced by London [12] for the case of infinite panels, does not apply for the case of finite panels, Guy [13]. For finite panels, resonance occurs at about the fundamental predicted by London ($\theta=0$), but for all incident angles.

4.3 MULTIPLE PANELS

The sound pressure ratio for the general case of multiple panels may be written directly from equation (42):-

$$SPR = 20 \log_{10} (|P_i'|/|P_{R_n}|)$$

where P_i' is the incident pressure

and P_{R_n} is the pressure to the right of the n^{th} vibrating panel and its solution may be written in a manner similar to equation (52), that is:-

$$SPR = 20 \log_{10} |(Z_{L_1} + \rho c_0)/2 Z_n| \quad (54)$$

where

$$Z_{L_1} = Z_{P_1} + Z_{A_1} - Z_{B_1}^2 /[Z_{A_1} + Z_{P_2} + Z_{A_2} - Z_{B_2}^2 /[Z_{A_2} + \; .....$$
$$- Z_{B_{n-1}}^2/[Z_{A_{n-1}} + Z_{P_n} + Z_{A_n} - Z_{B_n}^2 /[Z_{A_n} + Z_{L_{n+1}}]] \; ......]] \quad (55)$$

$$\text{and } Z_n = Z_{A_n} \prod_{i=1}^{i=n-1} [Z_{B_i}/(Z_{A_i} + Z_{L_{i+1}})] - Z_{B_n} \prod_{i=1}^{i=n} [Z_{B_i}/(Z_{A_i} + Z_{L_{i+1}})] \quad (56)$$

where $n > 1$
and Π infers successive products

Analysis of equations (54), (55) and (56) may now proceed via computational techniques.

5. DISCUSSION

The present discussion is based upon the results presented in Figures (4) to (8), with the objective of illustrating typical general analysis applications; the discussion is not exhaustive, nor do the figures display all possible trends, phenomenum, or phenomena interaction.

Results arising from the analytical procedure outlined in section 4.1 with respect to single leaf panels in the presence and absence of a backing room are shown in Figures (4), (5) and (6).

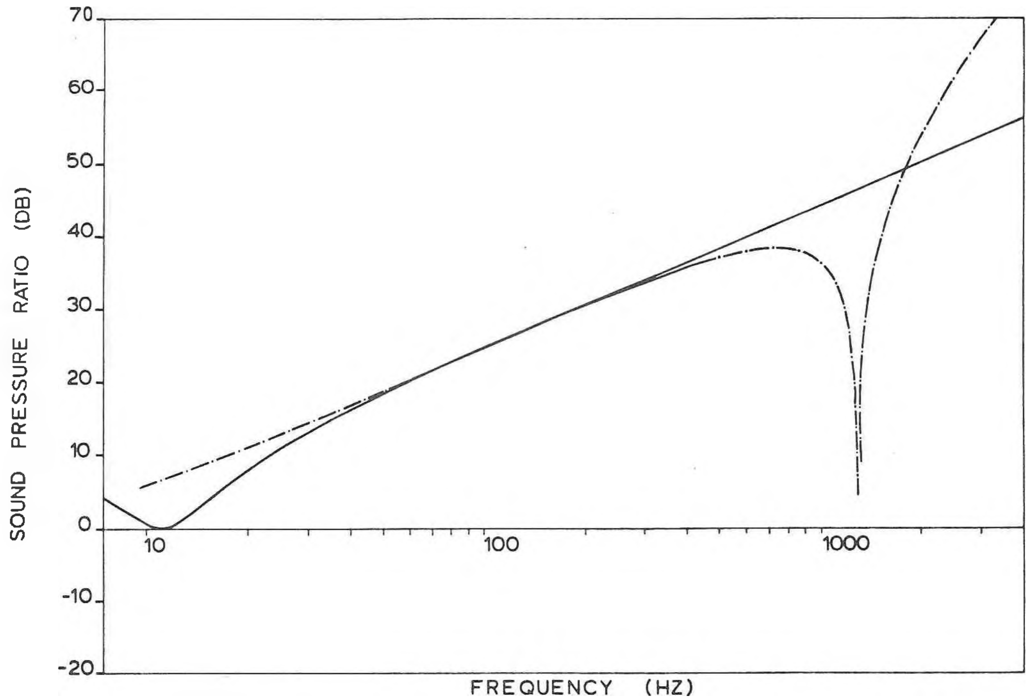

Figure 4 : Sound Pressure Ratio for a single leaf panel of glass.
Thickness 9.525mm, Density 2300 kg/m^3.
Perfect transmission to the right (Equation (27)).
—·—·— Modified Coincidence (Equation (37)).
Youngs Modulus 6.2×10^{10} N/m^2, Internal Damping $\eta = 0.002$.
——— Sprung Wall (Equation (38)).
Wall Stiffness Constant K = 108,000 N/m.

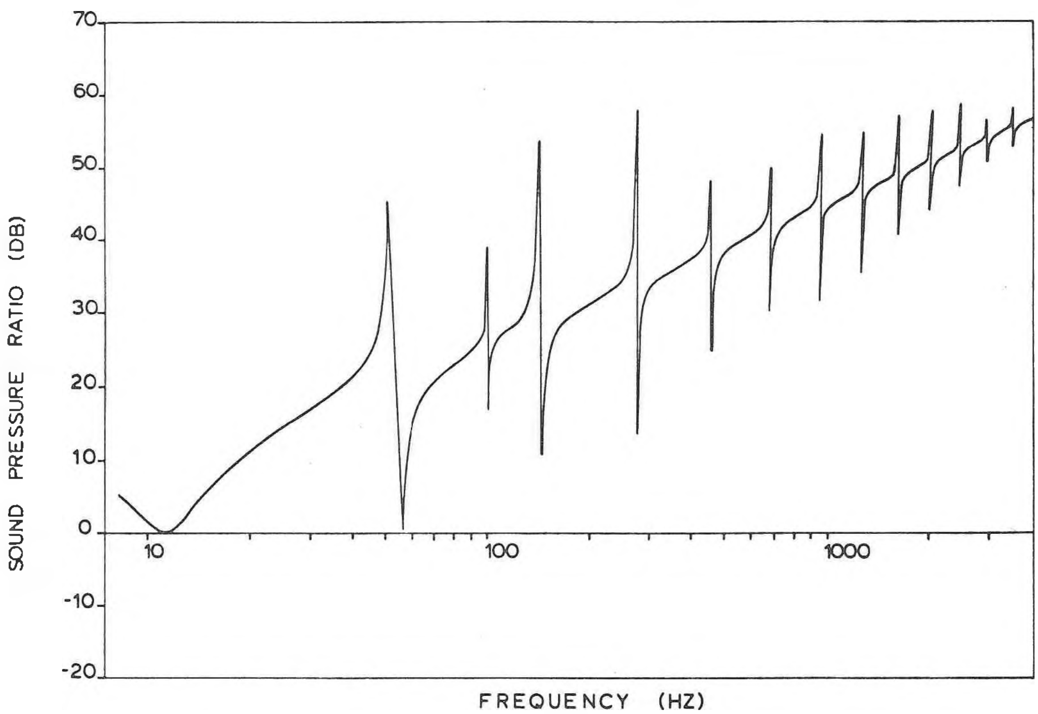

Figure 5 : Sound Pressure Ratio for a single leaf panel of glass.
Thickness 9.525mm, Density 2300 kg/m^3.
Perfect transmission to the right (Equation 27)
Multiple Resonance Panel (Equation 39).
Youngs Modulus 6.2×10^{10} N/m^2, Internal Damping $\eta = 0.002$.

Figure 4 shows the sound pressure ratio predicted for a 9.525 mm glass panel (typical of a shop window) based upon the modified coincidence equation (37), and the sprung wall equation (38). Such models are generally employed to illustrate classical control regions.

One may observe that the stiffness controled region occurs below the fundamental mass-spring resonance of 11 Hz and is therefore unlikely to be of significance for most practical purposes; the stiffness constant was chosen to yield the same fundamental resonance as predicted for a 2 x 2 metre panel (see Figure 5).

Figure (5) displays the transmission of sound when the glass panel exhibits the multiple resonance predicted for a 2 x 2 metre panel section, equation (39). The 'resonance region' can be seen over the whole frequency range thus an increase of panel damping would generally improve attenuation; it can also be seen that the sound pressure ratio tends to the 'mass law' prediction at higher frequencies although it should be noted that 'coincidence' has been ommitted from this display.

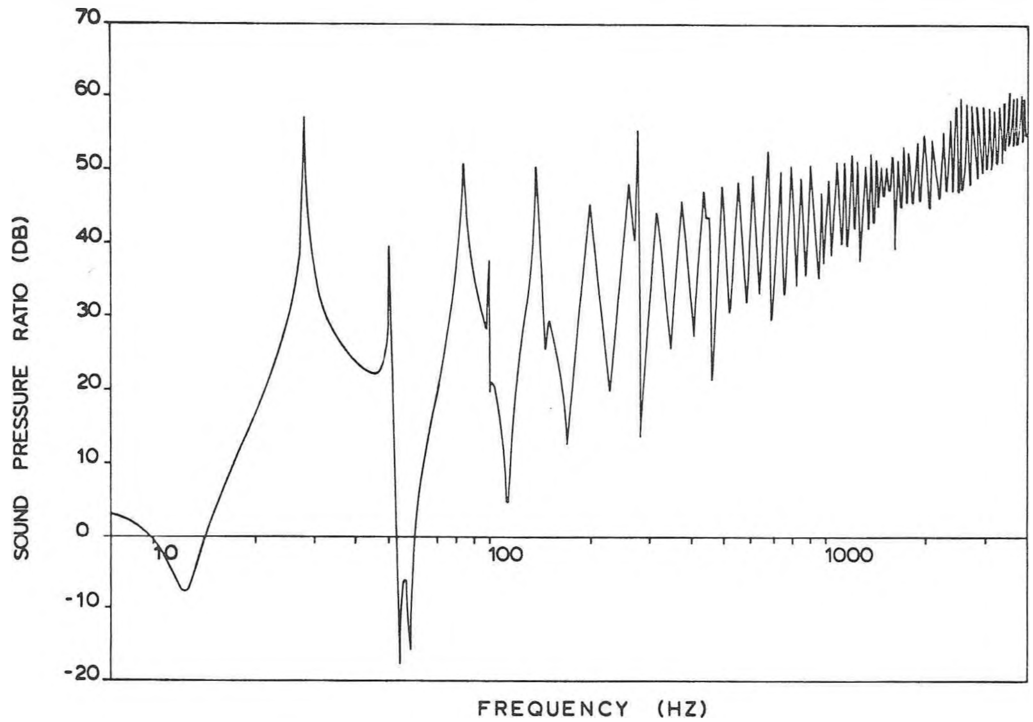

Figure 6 : Sound Pressure Ratio for a single leaf panel of glass.
Thickness 9.525mm, Density 2300 kg/m^3
Absorbent lined backing cavity (Equation 48), Distance 2900 mm.
Flow resistivity of Absorbent 20000 mks rayls/m, thickness 100 mm.
Multiple Resonance Panel (Equation 39).
Youngs Modulus 6.2×10^{10} N/m^2, Internal Damping $\eta = 0.002$.

Figure (6) displays the result of applying a 'damped' backing room to the multiple resonance panel of Figure (5). The overall cavity depth is three metres and the acoustically hard backing wall is lined with 100 mm of absorbent material having a flow resistivity of 20000 mks rayls/m. The occasions of resonance are significantly increased and the controling factors are the room depth and absorbent material; a tendency to the results of Figure (5) would be observed for increasing the room damping whilst marked increases in resonance excursions would be observed for decreasing room absorption. Resonance can be seen at lower frequencies, at which the sound pressure within the room is significantly higher than the incident pressure causing them!!

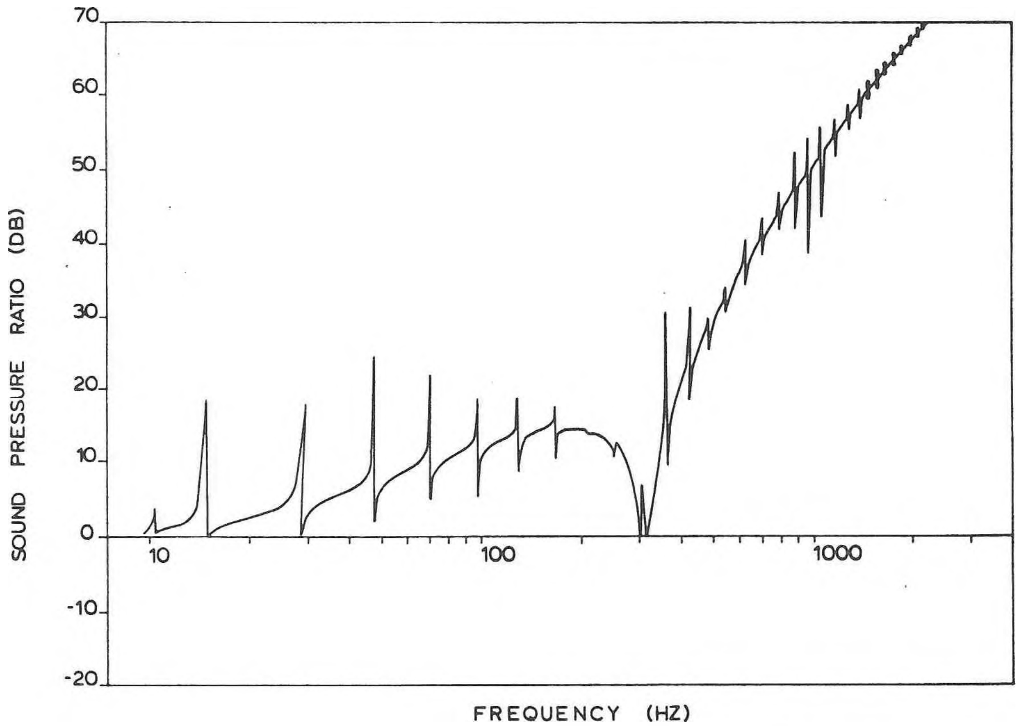

Figure 7 : Sound Pressure Ratio for a double leaf panel system of Aluminum.
Each leaf thickness 1mm, Density 2700 kg/m^3, Air Gap 25.4 mm.
Perfect Transmission to the right (Equation 27).
Multiple Resonance Panel (Equation 39).
Youngs Modulus 6.2×10^{10} N/m^2, Internal Damping $\eta = 0.0001$.

Figure (7) displays the sound pressure ratio for a double leaf panel determined in accordance with the procedure outline in section 4.2 and incorporating multiple panel resonance detailed in equation (39). The double panel is of two thin aluminum sheets, each 1 mm thick and separated by an air gap of 25.4 mm; the sheets are assumed to be square of 2x2 metres.

The mass-spring-mass resonance is clearly evident about 320 Hz, although a strong secondary resonance caused by the proximity of a panel eigen frequency can be seen about 310 Hz. The general trend is for the multiple panel resonance to be superimposed upon the curve dictated by two mass law panels separated by an air gap.

Figure (8) displays the sound pressure ratio determined for a three panel system (section 4.3) consisting of the double leaf aluminum system described for Figure 7 in association with a similar aluminum panel located one metre away, this being the typical air gap one might achieve by utilising the roof depth of an aluminum space frame. The advantage of a large air gap is that its associated mass-spring-mass resonance is located outside the frequency range of interest, such is the present case which exhibits this resonance about 40 Hz; the large air space has, however caused an air space resonance about 170 Hz thus potentially eliminating the earlier advantage. Some modification to the excursions of the 320 Hz resonance associated with the mass and smaller air gap may also be seen.

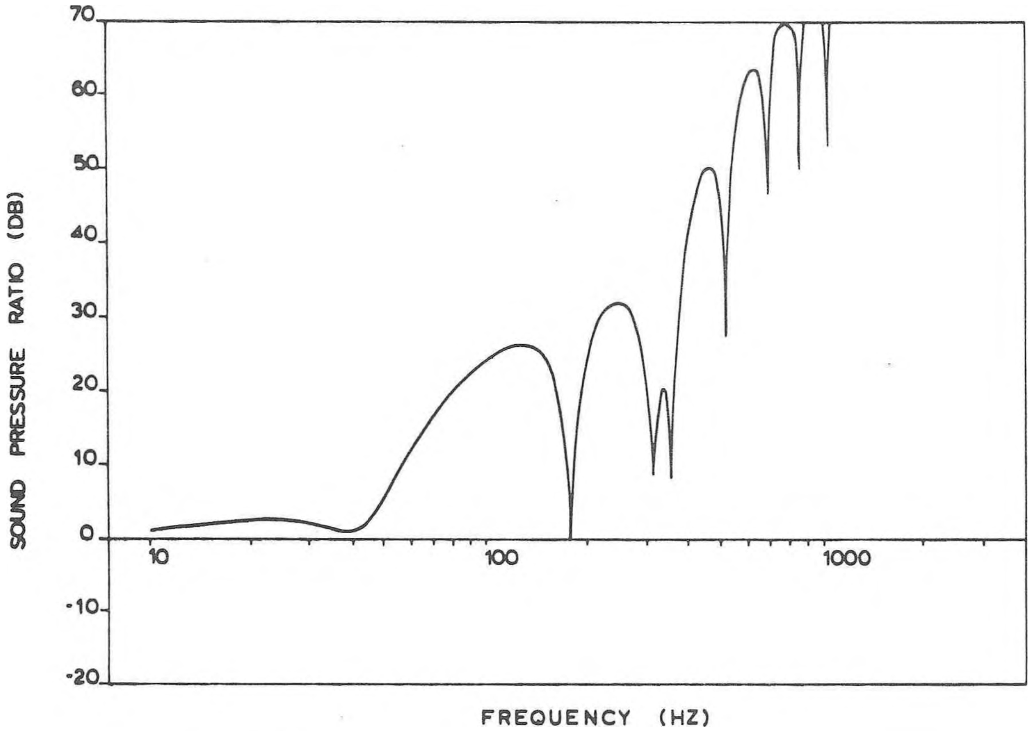

Figure 8 : Sound Pressure Ratio for a triple leaf panel system of Aluminum.
Each leaf thickness 1mm, Density 2700 kg/m^3, Air gap 25.4mm and 1000mm.
Perfect transmission to the right (Equation 27).
Mass Law Panels, (Equation 35).

The high transmission loss potential of this particular panel system may still be realised by locating absorbent material within the larger air gap; optimum absorbent material thickness, characteristic flow resistance, and location could be estimated by employing the general analysis.

All results presented in Figures 4 to 8 have been computed from the general analysis programmed in MICROSOFT BASIC on an Apple II plus microcomputer system. All computational times were 'reasonable' and as discussed are capable of visual qualative assessment.

6. CONCLUSION

A one dimensional analytical model capable of considering many features associated with vibrating panel systems has been developed and its use demonstrated by way of case studies.

It has been shown that formulations are readily generated and that they are based upon a recurring cycle involving a fixed menu of panel and termination impedance; this form of analysis is idealy suited for computation.

Graphical output from a programmed micro computer system has been shown capable of analytical interpretation, and computational times have been found 'acceptable'. Thus, the general analysis and procedure applied to a micro computer system will prove of use for instructional purposes and may also be of use for preliminary engineering design purposes.

ACKNOWLEDGEMENT

This work was financed from operating grant A0374 of the National Science and Engineering Research Council of Canada.

REFERENCES

1. Nestrov, V.S., "Sound Absorption by Multilayer Resonance Systems", Comptes Rendus (Doklady) Acad. Scie. URSS, 31 [1941], 237.

2. Kinsler, L., Frey, A., "Fundamentals of Acoustics", second edition, John Wiley & Sons, New York, London.

3. Beranek, L., "Noise and Vibration Control", McGraw-Hill Book Company, New York, London, 1971.

4. West, R.C., Selby, S.M., "Handbook of Tables for Mathematics", Revised Fourth edition, 1975, CRC Press Inc., Cleveland, Ohio.

5. Cremer, L., "Theorie der Schalldammung Dunner Wande bei Schragen Einfall", Akust. Z. 7, [1942], 81.

6. Beranek, L., "Noise Reduction", McGraw-Hill Company, New York, London, 1960.

7. Bhattacharya, M.C., Guy, R.W., Crocker, M.J., "Coincidence Effect with Sound Waves in a Finite Plate", J. Sound, Vib., (1971) 18 (2), 157.

8. Foxwell, J., Franklin, R.E., "Acoustic Effects in the Vibration of structures", Aeronautical Research Council, 19, 495, L.A. 41, 1957 (August).

9. Guy, R., Bhattacharya, M.C., "The Transmission of Sound Through A Cavity-Backed Finite Plate", J. Sound Vib., (1973) 27 (2), 207.

10. Richards, E., Mead. D., "Noise and Acoustic Fatigue in Aeronautics" (Chapter 3.4), John Wiley & Sons Ltd., London, New York, 1968.

11. Brüel, Per V., "Sound Insulation and Room Acoustics", Chapman & Hall Ltd., London, 1951.

12. London, A., "Transmission of Reverberant Sound Through Double Walls", J. Acoust. Soc. Amer. 22 [1950], 270.

13. Guy, R., "The Transmission of Airborne Sound Through a Finite Panel, Air Gap, Panel and Cavity Configuration", Acustica Vol. 49, [1982], No. 4, 323.

THE CANADIAN ACOUSTICAL ASSOCIATION — *L'ASSOCIATION CANADIENNE DE L'ACOUSTIQUE*

INVOICE / FACTURE

RENEWAL DUE / ABONNEMENT DÛ, JANUARY / JANVIER 1, 1984

PRINT COMPLETE ADDRESS INCLUDING POSTAL CODE/*INSCIRE EN CARACTERE D'IMPRIMERIE L'ADRESSE COMPLETE ET LE CODE POSTAL*

NAME/*NOM*____________________

ADDRESS/*ADRESSE*____________________

____

POSTAL CODE____________________
POSTAL

CHECK APPLICABLE ITEMS / COCHER CASES APPROPRIEES

(a)	Subscription and / or CAA membership *Abonnement et / ou adhésion à l'ACA*	$ 15.00	☐
(b)	CAA student membership *Membre étudiant de l'ACA*	$ 5.00	☐
(c)	Sustaining subscription *Abonnés de soutien*	$ 95.00	☐
(d)	Annual donation to 12 ICA *Don annuel au 12e ICA*	$ 20.00	☐
(e)	Single donation to 12 ICA *Don unique au 12e ICA*	$ 75.00	☐

Total remitted
Versement total ☐

Make cheques payable to THE CANADIAN ACOUSTICAL ASSOCIATION. Mail this form with payment to

Faire parvenir ce formulaire à l'adresse suivante en prenant soin de l'accompagner d'un chèque fait au nom de l'ASSOCIATION CANADIENNE DE L'ACOUSTIQUE.

D.A. Benwell
Environmental Health Centre
Room 233, Tunney's Pasture
Ottawa, Ontario K1A 0L2