VERTICAL DYNAMIC FORCES FROM FOOTSTEPS

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ABSTRACT

Vertical dynamic forces from walking and running were measured by load cells placed between temporary supports and the centre of a 17-m-long floor strip. The measurements showed that the dynamic forces are composed of wave trains of harmonics of the walking or running rate. The significant low frequency contributions are generally contained within the first three to four harmonics.

To represent the contribution of each harmonic to the total dynamic forces, a 'dynamic load factor, α' was defined as the ratio of dynamic force amplitude of the harmonic to the weight of the person. Footstep forces were produced by three male subjects; for walking, the maximum dynamic load factors measured and corresponding frequencies are: α_1 = 0.56 at 2.4 Hz for the first harmonic, α_2 = 0.28 at 5.4 Hz for the second, α_3 = 0.12 at around 7.8 Hz for the third, and α_4 = 0.08 for the fourth harmonic. For running, maximum values of α_1 were 1.45 at 3.0 to 3.5 Hz for the first harmonic, α_2 = 0.4 for the second, α_3 = 0.2 for the third, and α_4 = 0.1 or less for the fourth harmonic.

RÉSUMÉ

Les forces dynamiques verticales du pas de marche et du pas de course ont été mesurées au moyen de cellules extensométriques placées entre des supports temporaires et le centre d'une piste de 17 m. Les mesures ont révélé que les forces dynamiques se composent de trains d'harmoniques de la fréquence du pas. Les contributions importantes à basse fréquence sont généralement associées aux trois ou quatre premières harmoniques.

Pour représenter la contribution de chacune des harmoniques aux forces dynamiques totales, on a défini un "facteur de charge dynamique, α" comme étant le rapport de l'amplitude de la force dynamique de l'harmonique et du poids de la personne. Les forces de pas ont été produites par trois hommes. Au pas de marche, les facteurs de charge dynamique maximaux mesurés et les fréquences correspondantes étaient : α_1 = 0,56 à 2,4 Hz pour la première harmonique, α_2 = 0,28 à 5,4 Hz pour la deuxième, α_3 = 0,12 à 7,8 Hz environ pour la troisième, et α_4 = 0,08 pour la quatrième. Au pas de course, les facteurs maximaux étaient : α_1 = 1,45 entre 3,0 Hz et 3,5 Hz pour la première harmonique, α_2 = 0,4 pour la deuxième, α_3 = 0,2 pour la troisième, et α_4 = 0,1 ou moins pour la quatrième.

INTRODUCTION

Walking and running are two of the most common forms of dynamic excitation produced by occupants in buildings, yet relatively little quantitative data is available about them. To provide some data on this topic, a study of the forces produced by walkers and runners was undertaken at the Division of Building Research of the National Research Council of Canada.

A number of previous studies of forces induced by human activity can be found in Refs. 1 to 4. These studies simulated the forces due to a continuous series of steps by combining duplicates of the force produced by a single step separated by time delays corresponding to the walking rate (3,4). The present investigation measures the dynamic forces produced by a continuous series of steps; possible inaccuracies arising from the creation of sequences consisting of single step pulses are thus avoided.

A knowledge of the forces produced by footsteps is of practical significance: a) in understanding the nature of undesirable floor vibrations produced by occupants; and b) in affording the possibility of predicting during the design stage the probable behaviour of a floor subjected to walking or running. This is of ever-increasing importance since lighter structural systems and longer spans tend to make floors more prone to occupancy-induced vibrations.

TEST PROCEDURE AND ANALYSIS

The forces from walking and running were measured at mid-span of a simply supported floor strip consisting of two open-web joists 914 mm deep, spaced at 1.76 m and spanning 17.04 m. The joists are covered by pre-cast concrete panels of 100 mm thickness, each measuring 2.13 m across and 1.19 m along the floor span. A plan view and elevation are shown in Figure 1.

At centre span under each joist a piezoelectric force transducer was inserted between a temporary support and the bottom chord, and pre-loaded to approximately

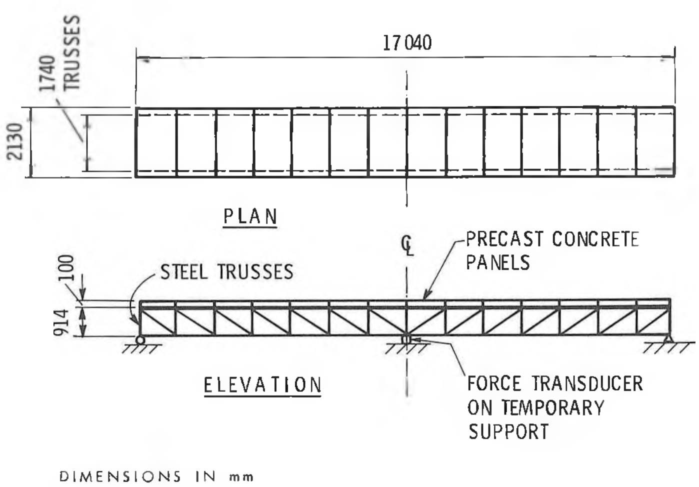

Figure 1. Test floor strip.

4000 N. The signals from the two transducers were added, amplified and recorded. Analysis commenced with high-pass filtering the signals at 0.2 Hz to eliminate the low frequency drift as the subject moved across the floor strip. The signals were then digitized and analyzed on a Fourier analyzer and by Fast Fourier Transform (FFT) routines on a minicomputer. Digital filtering was carried out by 'windowing' in the frequency domain. The peak amplitudes near the middle of the signal train were used as a measure of signal magnitudes (see Figs. 2 and 3).

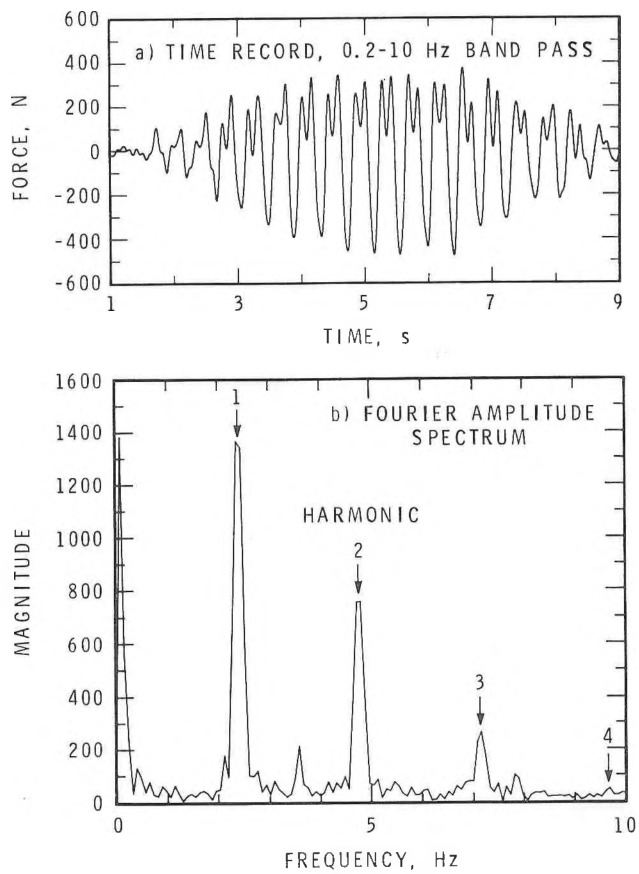

Figure 2. Walking forces at 2.4 steps/second, subject A.

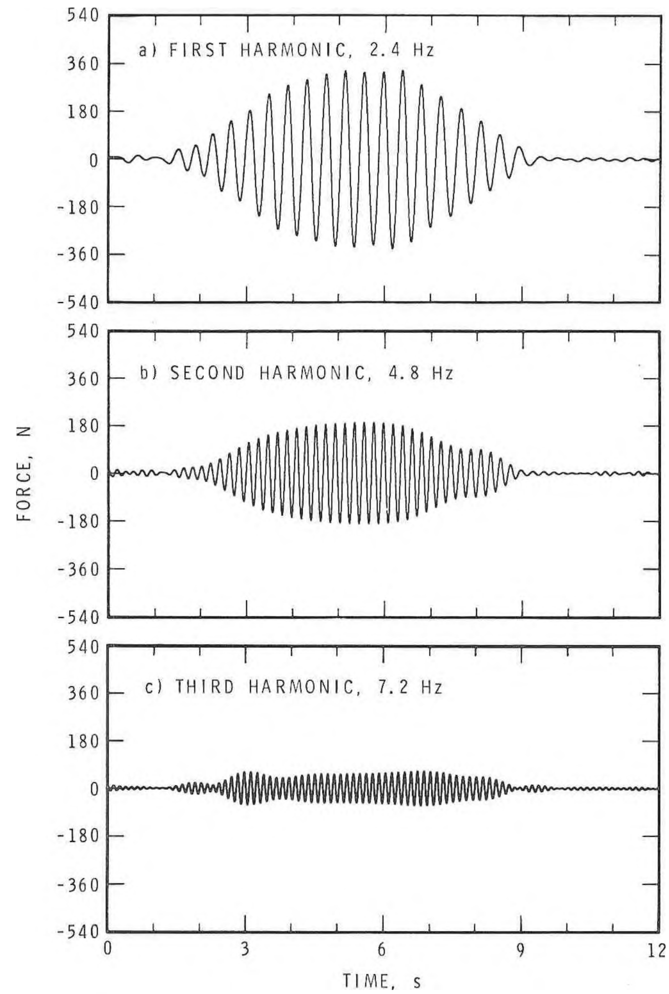

Figure 3. Harmonic components of force from walking at 2.4 steps/second, subject A.

The tests were conducted by playing pre-recorded pulses at the desired walking or running rate through loudspeakers, and requesting the subject to walk or run along the floor strip at the specified rate using a stride of his own choosing. For walk tests in which stride was varied, the subject walked first at his natural stride, and then at a shorter and a longer one.

To compensate for the dynamic amplification effects near the lowest resonance frequency of the supported floor strip, correction factors were determined from 2 to 10 Hz by comparing the load cell output with the known shaker force that was applied at centre span.

Since the lowest resonance frequency of the instrumented floor strip was 12 Hz, the useful results were thus limited to frequencies below about 10 Hz. Above that frequency, the dynamic amplification becomes large and rather sensitive to damping. A frequency range from 0.2 to 10 Hz was considered adequate for present purposes since the majority of problems associated with footsteps have occurred in floors with fundamental frequencies below about 8 Hz (5).

CHARACTERISTICS OF FOOTSTEP FORCES

The distinction between walking and running is that during walking the person always has one foot in contact with the floor or ground, whereas during running, contact is lost. The walking or running frequency in Hz is defined as the rate at which the feet make contact with the floor. Walking and running were performed by three male subjects, whose physical characteristics are given in Table 1.

TABLE 1

Physical Characteristics of Male Test Subjects

Subject	Weight (N)	Height (m)	Age (y)
A	734	1.82	48
B	800	1.78	41
C	690	1.88	23

Walking

A typical time record of walking forces, low-pass filtered at 10 Hz, is shown in Figure 2a. The record contains only the dynamic portion of the induced forces, as the static components have been eliminated by the 0.2 Hz high pass filter. These forces are bounded by a parabolically shaped envelope which is the static influence line for the mid-span support of the floor strip (6). The Fourier amplitude spectrum of this force record is shown in Figure 2b. The spectrum shows that the forces produced by walking consist of distinct frequency components at integer multiples of the walking rate, with spectral amplitudes that decrease with increasing frequency. The first three or four harmonics comprise the main dynamic components of walking forces in the frequency range of interest. As the time record in Figure 2a shows, the forces are periodic, with a repetition rate equal to the walking rate.

Running

The time signatures of forces from running are similar to those for walking, except that the pulses associated with each step are zero during the time the runner is airborne. Nevertheless, the Fourier spectrum of the force record again contains discrete frequency components at integer multiples of the running rate, as shown in Figure 4.

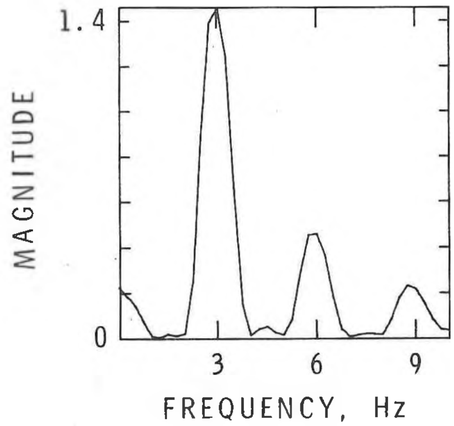

Figure 4. Fourier amplitude spectrum for running at 3.0 steps/second.

MATHEMATICAL REPRESENTATION OF FOOTSTEP FORCES

Walking

The analysis of force records indicates that the walking forces (F(t)) can be represented by the following expression:

$$F(t) = P\left(1 + \sum_{n=1}^{N} \alpha_n \sin\left(n2\pi ft + \phi_n\right)\right) . \qquad (1)$$

Here, P = static weight of subject;
α = Fourier amplitude or Fourier coefficient;
n = order of harmonic of the walking rate (n = 1, 2, 3...);
f = rate of walking in Hz;
t = time variable;
ϕ = relative phase angle;
N = total number of harmonics.

The dynamic component of the walking force in Eq. (1) is represented by the summation term, which is a Fourier series with Fourier coefficients (α_n) at the discrete frequencies (nf). From the analysis of time records, the relative phase angle (ϕ_n) was determined to be about 0°, 90°, and 0° for n = 1, 2, and 3, respectively, for walking rates between 2.0 and 2.4 Hz. The time signatures of the three lowest and most significant harmonics that comprise the walking forces in Figure 2 are presented in Figures 3a, b and c. These were obtained from the time record shown in Figure 2a by digital bandpass filtering in the frequency domain using Fourier transform techniques (7). The centre amplitudes of the harmonics, normalized by the weight (P), then represent the Fourier coefficients (α_n).

Running

Periodic dynamic forces produced by running can also be represented by Eq. (1). However, the relative phase angles of the harmonics are not as clearly defined as those for walking.

VARIATION OF DYNAMIC FORCES WITH STEP FREQUENCY

Walking

The key parameters in Eq. (1) that describe the dynamic force from walking are the Fourier coefficients (α_n) and the walking rate or step frequency (f). In a manner similar to that used in the description of rhythmic forces (8), the Fourier coefficients (α_n) are called 'dynamic load factors,' defined as the ratio of the dynamic force amplitude of each harmonic to the weight of the subject. In this paper, the variation of α with step frequency was studied for walking rates from 1.0 to 3.0 Hz using three different male subjects (A, B and C, Table 1). The results are shown in Figures 5a, b, and c.

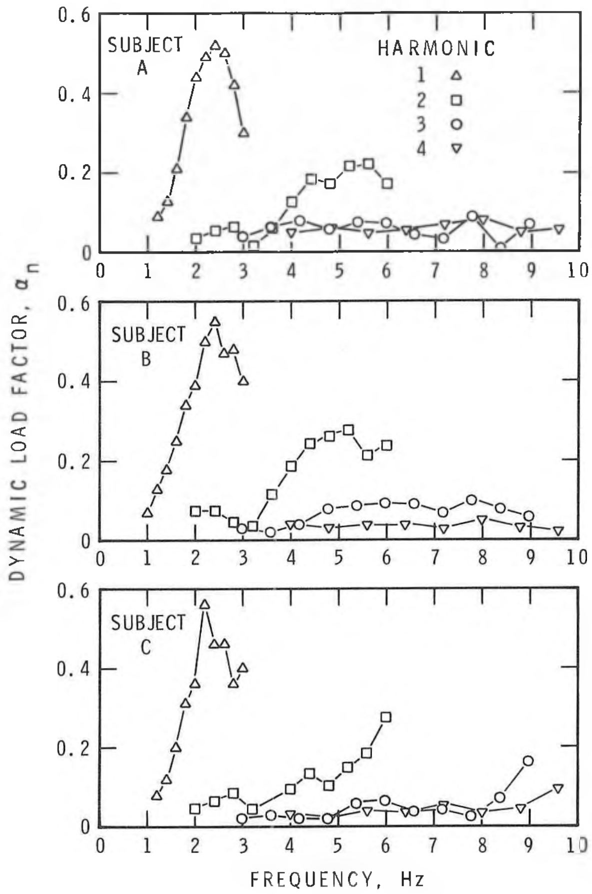

Figure 5. Dynamic load factor for walking.

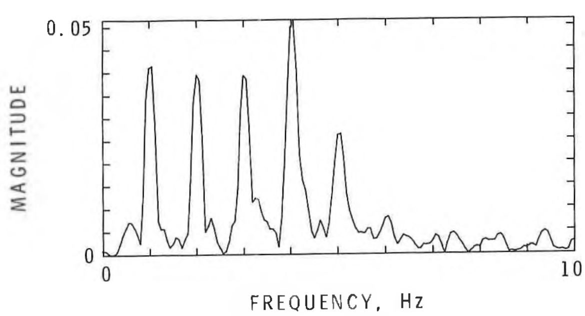

Figure 6. Fourier amplitude spectrum for walking at 1.0 steps/second, subject B.

At very low walking rates (near 1.0 Hz) the first four harmonics have comparable dynamic load factors. Additional higher harmonics are also evident, as shown in Figure 6 by the Fourier amplitude spectrum of a walking record at 1.0 Hz. At higher footstep frequencies the dynamic load factor of the first harmonic (α_1) is the largest and reaches a maximum near 2.4 Hz for all three subjects. For subjects A and B the following can be observed from Figures 5a and b. The maximum dynamic load factors for the second harmonic (α_2) are somewhat less than half the maxima of α_1. This peak occurs at about 5.4 Hz, slightly more than twice the frequency of the maximum for α_1. The dynamic load factors of the third harmonic (α_3) are also somewhat less than half of α_2, with the peak occurring near 7.8 Hz. This is slightly more than three times the frequency of the maximum dynamic load factor for the first harmonic. This shift in frequencies of the maximum dynamic load factors for the second and third harmonics from integer multiples of the frequency at which the maximum (α_1) occurs can be attributed to the fact that higher walking rates produce larger forces during the heel landing and toe push-off phases of impact. The result is a relatively larger proportion of the higher harmonics than at lower walking rates.

The results for subject C (Fig. 5c) differ somewhat from those of subjects A and B (Figs. 5a and b) in that the second and third harmonics do not exhibit the same shape as those for the other two subjects. Subject C shows a relative lack of second and third harmonic components in his walking forces at walking rates between 2 and 2.6 Hz, and the dynamic load factors reach a maximum at frequencies of 6 Hz for the second harmonic and 9 Hz for the third, corresponding to a 3 Hz walking rate. On the other hand, there is good agreement in the variation of α_1 among the three subjects, both in the general shape and the frequency at which the maxima occur. Maximum values of the dynamic load factors among the three test subjects were: $\alpha_1 = 0.56$, $\alpha_2 = 0.28$, $\alpha_3 = 0.12$, and $\alpha_4 = 0.08$. This excludes the extreme values near 9 Hz for subject C.

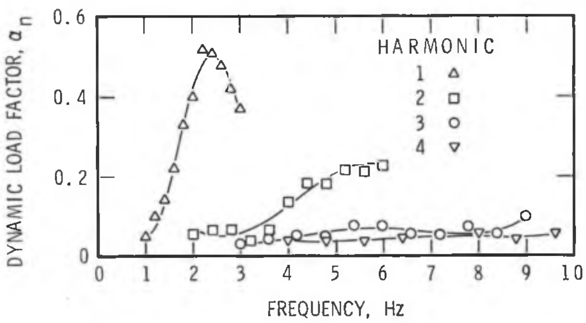

Figure 7. Averaged dynamic load factors for walking, subjects A, B and C.

The data for the three subjects at each frequency were averaged and are plotted in Figure 7; a fourth-degree polynomial curve was fitted to the resulting points. The maximum values for this polynomial fit are: $\alpha_1 = 0.52$ at 2.4 Hz, $\alpha_2 = 0.24$ at 5.6 Hz, $\alpha_3 = 0.06$ at around 6 Hz, and $\alpha_4 = 0.05$ at around 8 Hz. α_3 and α_4 could well be considered constant from about 4 to 9 Hz at the above values. This again ignores the 9 Hz data point, which is strongly influenced by the contribution from subject C.

Variation of α with length of step The variation of α with step length was investigated at the 2 and 2.4 Hz walking rates using subject A. The results are shown in Figure 8. The 0.9 m step length was the natural stride of the walker. For the three step lengths investigated here, the dynamic load factors increase with step length for both the 2 and the 2.4 Hz walking rates.

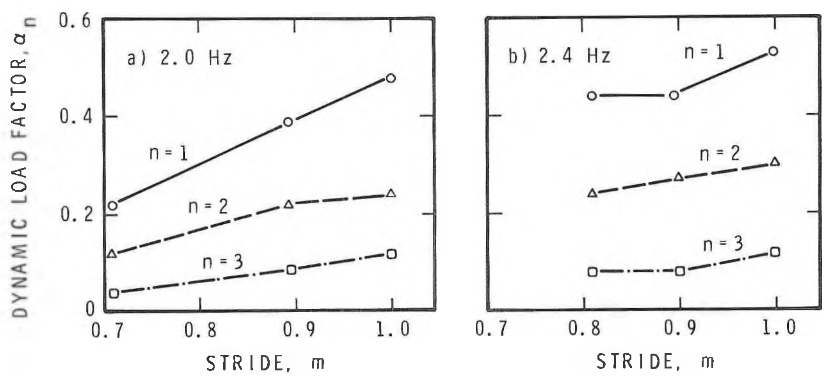

Figure 8. Variation of dynamic load factor with step length and walking rate.

Running

The dynamic load factors (α) for running are plotted against running rate for the three male subjects in Figures 9a, b and c. The results show the first harmonic to be the most significant, increasing monotonically from approximately 0.5 at 1.5 Hz to a maximum of 1.5 at a frequency of 3.6 Hz. Thereafter a gradual decrease in α_1 is indicated. The second harmonic initially decreases from a high of about 0.4 around 3.0 Hz, to a low of 0.1 around 4.0 Hz, and then gradually increases to between 0.35 and 0.47 around 7.0 Hz. The third harmonic remains relatively constant at less than

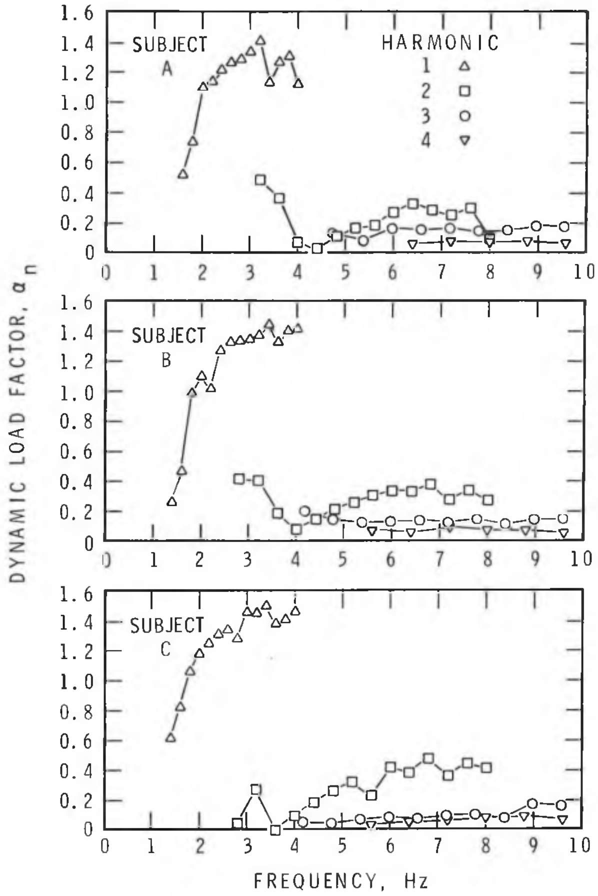

Figure 9. Dynamic load factor for running.

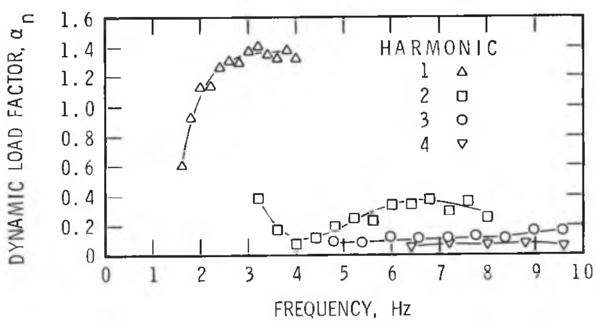

Figure 10. Averaged dynamic load factors for running, subjects A, B and C.

0.2 between 4 and 8 Hz. For the fourth harmonic, the dynamic load factor α is less than 0.09 for all frequencies investigated.

The plot of the averages of the dynamic load factors among the three test subjects is shown in Figure 10. A fourth-degree polynomial fit gives a maximum value of α_1 = 1.40 at 3.6 Hz, α_2 = 0.40 at 3 Hz and 0.35 at 6.5 Hz; α_3 and α_4 are relatively constant at 0.12 and 0.08, respectively.

A summary of the dynamic load factors for walking and running is presented in Table 2.

DISCUSSION AND SUMMARY

The technique used to measure step forces restricts the useful data to below 12 Hz, the lowest resonance frequency of the test structure. While there are high frequency components present in footsteps, it is the low frequency ones that cause most vibration problems due to walking or running in buildings.

The Fourier series representation of the dynamic components of footstep forces simplifies analytical calculations of dynamic responses of floors and other building components. This permits the treatment of a dynamic process that has so far eluded simple analysis. Not only is the numerical evaluation of the response possible, but a realistic intuitive assessment of the causes of vibration problems from footsteps becomes feasible. It is apparent from the characterization of the spectrum of the forces that a resonant condition can occur when the walking or running rate, or an integer multiple of it, coincides with a natural frequency of the floor. This coincidence can cause a large dynamic response in floors having low damping.

A comparison of dynamic load factors for walking and running shows the following.

1. The maximum dynamic load factor of the first harmonic for running is approximately three times as large as the corresponding α_1 for walking. These maxima occur around 2.4 Hz for walking and between 3.0 and 3.5 Hz for running.

TABLE 2

Maximum Dynamic Load Factors (α) for Walking and Running

Activity	Harmonic (n)	Maximum from subject A, B or C* Frequency (Hz)	Maximum from subject A, B or C* α	Maximum from polynomial fit to averages from subjects A, B, C* Frequency (Hz)	Maximum from polynomial fit to averages from subjects A, B, C* α
Walking	1	2.4	0.56	2.4	0.52
	2	5.2	0.28	5.6	0.24
	3	4.2 - 7.8	0.12	5.8	0.06
	4	8.0	0.08	7.6	0.05
Running	1	3.6	1.50	3.6	1.40
	2	6.8	0.47	3.2	0.40
				6.6	0.35
	3	4.2	0.20	6 - 8	0.12
	4	7 - 9	0.09	7 - 9	0.08

*Except for values corresponding to 9.0 Hz and above.

2. The ratio of the dynamic load factors of the first to the second harmonic (α_1/α_2) is considerably larger for running than for walking. In absolute terms, however, α_2 for running is slightly higher than for walking but is shifted to a higher frequency. The low point in α_2 occurs around 3.0 Hz for walking and around 4.0 Hz for running.

3. The dynamic load factors for the third and fourth harmonics for running are only slightly larger than those for walking.

The above comparison of dynamic load factors shows that running represents a more severe dynamic loading than walking for all four harmonics. While this is not surprising in a qualitative sense, the data provide the quantitative information to substantiate this widely-held view.

Although the measurements have been taken over a wide frequency range, regular walking occurs generally at or near 2.0 Hz, and recreational running from about 2.4 to 3.2 Hz. Thus the measured maximum force values of the lowest harmonic occur near the most common walking or running rates.

ACKNOWLEDGEMENT

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