# ACOUSTICS OF PIPING AND DUCTS

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## ABSTRACT

This paper describes a computerized procedure for the analysis of the acoustics of piping and duct work systems. The procedure can be used for the analysis of complicated network systems with multiple inputs and outputs. It combines transfer matrix and finite element methods, using finite element methods to model more complex elements and using exact solutions for the simpler interconnecting parts. The method has applications in modelling pulsations in gas pipelines, the design and analysis of muffler systems and calculating sound transmission in ventilation ducts.

## SOMMAIRE

Cet article fait état d'une procédure d'analyse acoustique des systèmes de conduites et de canalisation. La procédure permet l'analyse de systèmes en réseaux complexes avec entrées et sorties multiples. Elle allie les méthodes de matrices de transfert et des éléments finis, en ayant recours aux méthodes des éléments finis pour modéliser les éléments les plus complexes et aux solutions exactes pour les composantes d'inter-connection les plus simples. La méthode peut s'appliquer à la modélisation des pulsations dans les gazéoducs, à la conception et à l'analyse des systèmes de silencieux et au calcul de la transmission sonore dans des conduites d'aération.

## 1. INTRODUCTION

This paper is concerned with the development of a procedure to be used on digital computers for studying the acoustics of piping and duct systems.

Various methods have been used for the analysis of noise propagation in pipes and ducts. The ASHRAE Handbook [1] gives an approximate semi-empirical method which tracks the sound power through elements along a duct path starting from the noise source and ending at the duct outlet into a room. This method does not consider the reactive effects of sound waves reflected back down the duct.

Transfer matrix techniques have been used by Munjal [7], To [8] and others for prediction of the acoustics of piping systems and acoustic mufflers. In this method the piping system is represented by a combination of discrete elements. The acoustic pressure and volume velocity at one station in the system is then related to those at another station using a two-by-two parameter matrix. The method works well for a long string of connected elements, but becomes more difficult to use when several interconnected paths, branches and multiple inputs and outputs are to be modeled. Where several pipes connect at a junction point, the continuity of pressure and conservation of mass are used to relate these quantities. The method does not consider the geometry of a junction. One advantage is that only two-by-two complex matrices are needed; thus computers with relatively small memories can be used.

Finite element methods have been used by Craggs [4], [5] and others for examining duct elements such as elbows and acoustic mufflers and lined ducts. This method is advantageous for analyzing systems with a complex geometry, and where three dimensional wave propagation effects become important. However the method would probably lead to prohibitively large matrix equations if used to analyze entire systems.

The hybrid method proposed in this paper combines the advantages of the above methods and still uses relatively small matrices. Here the input and output points and branch or junction points are considered as nodes. A global matrix equation is assembled relating the nodal acoustic pressures to the nodal volume velocity inputs and outputs. The equation is then solved to determine the nodal pressures, by first calculating a transfer matrix for each lineal string of pipe or duct elements along the paths between nodes. The transfer matrix is then converted into a network element matrix which relates the element nodal acoustic pressures and nodal volume velocities. Each network element matrix is assembled into the global matrix in "finite element" fashion. In elements such as expansion chambers and elbows where two or three dimensional wave propagation is important, true finite element methods are used. The finite element model is then reduced to a two-by-two transfer matrix relating the element input and output and is then treated as other transfer matrix elements. Junctions of several lineal strings can also be modeled using the finite element method. Again the finite element model of the junction is reduced to a network element matrix only involving its connecting nodes. This matrix is then assembled into the overall global matrix of the system.

Once the overall global matrix is assembled, it is modified to satisfy the nodal boundary conditions. For example, the acoustic pressure at the node, the volume velocity input, the incident pressure, or the terminating impedance can be specified. The modified matrix is then inverted and the pressures at each node calculated. Acoustic pressure, volume velocity, and transmitted sound power or ratios of these quantities including magnitude and phase between different points in the system can then be calculated.

## 2. THEORY

The detailed theory of transfer matrix methods and finite element methods used in this paper are presented by Craggs [4] and To [8]. The procedure used to link these methods is outlined below. The basic parameters used are the acoustic pressure given in the form $P \exp(j\omega t)$

and the volume velocity V exp(jωt) where P and V are complex quantities.

## 2.1 Two-node elements

Given an acoustic element with one input and one output, referred to subsequently as a two-node element, the four-pole transfer function matrix can be used to relate the pressures and volume velocities at the input (P1,V1) and output (P2,V2) by

$$\begin{Bmatrix} P1 \\ V1 \end{Bmatrix} = [a1] \begin{Bmatrix} P2 \\ V2 \end{Bmatrix} \tag{1}$$

where [a1] is the four-pole parameter matrix. In the above form [a1] is a two-by-two matrix with complex elements. The pressures and velocities can be split into real and imaginary parts for computational purposes. The matrix [a1] then becomes a four-by-four matrix with real elements. If a string of several two-node elements are connected in series then, for example, given a string of five elements a single transfer matrix [a] can be obtained relating the input of element 1 (station 1) to the output of element 5 (station 6) by

$$\begin{Bmatrix} P1 \\ V1 \end{Bmatrix} = [a] \begin{Bmatrix} P6 \\ V6 \end{Bmatrix} \tag{2}$$

where

$$[a] = [a1]\ [a2]\ [a3]\ [a4]\ [a5]. \tag{3}$$

If stations 1 and 6 are global nodes then equation (2) can be rearranged to give

$$[b] \begin{Bmatrix} P1 \\ P6 \end{Bmatrix} = \begin{Bmatrix} V1 \\ V6 \end{Bmatrix} \tag{4}$$

where [b] is the element network matrix relating the element pressures and volume velocities. Each network element matrix can be assembled into the global matrix [B] to give the following matrix equation

$$[B]\ \{P\} = \{V\} \tag{5}$$

where {P} is the vector of global node acoustic pressures and {V} is the vector of global node external input and output volume velocities.

The most basic two-node element is a rigid straight pipe for which an exact solution exists for one dimensional fluid motion along the pipe (refer to To [8]). This is the basic element used to connect other more complex elements where the one dimensional analysis is not applicable.

## 2.2 "Finite Element" Two-node elements

The finite element method can be used to model subcomponents of a system in which three dimensional waves are important. However, if the input and outputs to the subcomponent are simple pipes then it is possible to construct a much reduced connective matrix using the procedure outlined below.

If the input and output pressures are contained in the vector Pb and all other internal nodal pressures within the finite element model in vector Pi then the mathematical model of the component is in the form of the partitioned matrix equation:

$$\begin{bmatrix} a & . & b \\ . & . & . \\ c & . & d \end{bmatrix} \begin{Bmatrix} Pb \\ \cdots \\ Pi \end{Bmatrix} = \begin{Bmatrix} Vb \\ \cdot\cdot \\ \theta \end{Bmatrix} \tag{6}$$

where Vb is a vector of input and output node volume velocities and $\theta$ represent a vector of zeros. This equation can be solved to eliminate the internal pressures Pi to give:

$$[e]\,\{Pb\} = \{Vb\} \tag{7}$$

This equation is now of the form given in equation (4) so that [e] is now the element network matrix and could be assembled directly into the global matrix [B] given in equation (5). Alternatively matrix [e] in equation (7) can be converted to transfer matrix form if this element is within a string of lineally connected elements.

## 2.3 Elements with three or more nodes

If several strings of two node elements meet at a common node then the network element matrices for each string can be assembled into a global matrix equation in "finite element fashion". This is based on equating the pressures at the node and using the equation for continuity of volume velocity at the node. This however does not consider the geometry of the junction and is only valid when the pipe or duct cross section dimensions are small compared to the wavelength of sound. The junction can be modeled using a finite element model as described above for two-node elements. On each input or output boundary connecting to the rest of the system, the pressures are constrained to a single value and an equation similar to equation (7) is obtained where [e] is now a network element matrix for the junction which can be assembled into the global matrix [B]. Note in this case since [e] now relates the pressures and volume velocities at more than two nodes it cannot be converted into a transfer matrix as was done with two-node elements.

## 3. COMPUTER PROGRAM DESCRIPTION

The computer program was developed on a Hewlett Packard 9816 desktop computer in the Basic 4.0 programming language. It is divided into several subprograms which are loaded and run separately through a small main program to reduce memory requirements. These include a data input program, a calculation program, and a program for displaying and plotting results. A separate program is used to generate finite element model data files for components such as elbows, branches, expansion chambers and transitions etc.

### 3.1 Data input program

The data for a given duct or piping system is entered in this section of the program. Once the data is entered the user can either go to the calculation section of the program or store the data in a disk data file. The file can later be recalled and the data modified, for example, if some components of the system are changed.

The first section of this program is used to enter data for the system nodes which are defined as any points where external input or outputs connect to the system and junction points between subsystems. The data entered for each node includes its geometric coordinates and the boundary condition at the node. Boundary conditions which are presently included are the total pressure, incident pressure, volume velocity input, or the terminating acoustic impedance. Real and imaginary parts of the above quantities can be specified.

The second section of the data input program is used to enter data for two-node subsystems. These are defined as lineally connected strings of two-node elements. The data entered includes the start and end global node numbers to which the string connects, the list of elements within the subsystem, the end coordinates, cross section dimension, type number and sub-type number of each element. As an option, the fluid properties can be specified for each element to allow,for example, variation of temperature and pressure throughout the system.

The third section of the data input program is used to enter data for three-node elements and at present is restricted to finite element models built from four or eight degree of freedom plane isoparametric elements. Data entered for each three-node element includes connecting global node numbers, a cross-section dimension, an element type and sub-type number and the fluid density and speed of sound within the element.

Elements connected to more than three global nodes have to date not been considered but could easily be incorporated into the program if required.

The fourth section of the data input program allows input of data for element sub-types. Types of elements include a straight rigid pipe, other elements which can be modeled directly using two-by-two transfer matrices and general types of finite element models. The finite element models incorporated to date include four or eight node axisymmetric isoparametric elements and four and eight node plane isoparametric elements. For each type of element sub-types can be created. For example, one element type is an expansion chamber with insertion tubes at each end. Data entered for each sub-type of this element includes the ratios of the chamber diameter and outlet diameter to the inlet diameter, and the ratio of the insertion lengths to the chamber length. For each finite element model sub-type, a data file name is entered. The data file contains the information defining a specific finite element model such as a particular elbow or branch configuration.

### 3.2 Calculation program

Once the input data has been entered directly into memory or loaded from a data file then the calculation program can be executed. First the frequency range and increment between points at which the calculations are to be performed is entered. To save memory the results are saved only for global nodes selected. The program then calculates and stores the complex pressures and input volume velocities for each global node selected at each frequency value in the selected range. The program then branches to the results presentation program.

### 3.3 Results presentation program

This program allows the calculated global node pressure and volume velocity results to be converted to the desired output which can then be displayed on the CRT display, or sent to a plotter or printer. The values of pressure and volume velocity at selected points between elements within subsystems can also be displayed if the values for the global nodes at each end of the subsystem have been saved. Pressures, volume velocities, or transmitted sound power can be graphed, stored or recalled from data files as a function of frequency. These quantities or ratios of these quantities can be graphed in terms of the real and imaginary parts or phase and magnitude where applicable, and on logarithmic or linear axes. This flexibility allows direct presentation of results, for example, in terms of transmission loss, insertion loss or attenuation. Logarithmic scales can be referenced to user chosen values so the sound pressure levels and sound power levels within the system can be presented.

## 4. EXAMPLES

The first example illustrates the use of the method to solve a simple piping network problem. The remaining examples show the use of the finite element method to model individual system components.

### 4.1 Simple piping network

This example considers a piping network shown in Figure 1. All pipes in this network have 0.05 m diameters. The system has three volume velocity inputs given by $V_1 = 1$ cu.m/s, $V_2 = -0.707 - 0.707j$ cu.m/s and $V_3 = -0.707 + 0.707j$ cu.m/s (independent of frequency) and three connecting pipes with anechoic terminations. A fluid density of 1.21

kg/m$^3$ and a speed of sound of 344 m/s has been used throughout the system. Each length of pipe between junctions and between junctions and boundary terminations has been modeled using an exact one dimensional pipe transfer matrix solution. The transfer matrix for each pipe section was converted by the computer program to a network element matrix. These were assembled to give a global matrix equation which was solved to obtain the pressures and volume velocities at each junction and termination. Figure 1 shows the ratios of the pressures at the three anechoic terminations to the volume velocity at input 1. Note that the calculations were performed at 5 Hz intervals so that some of the local maximum and minimum values may not be truly represented.

Each length of pipe in this simple system could be replaced by a string of several components. The computer method then combines the transfer matrices for the components in the string so that the final global matrix equation for this new system would be the same size as solved in the simple system.

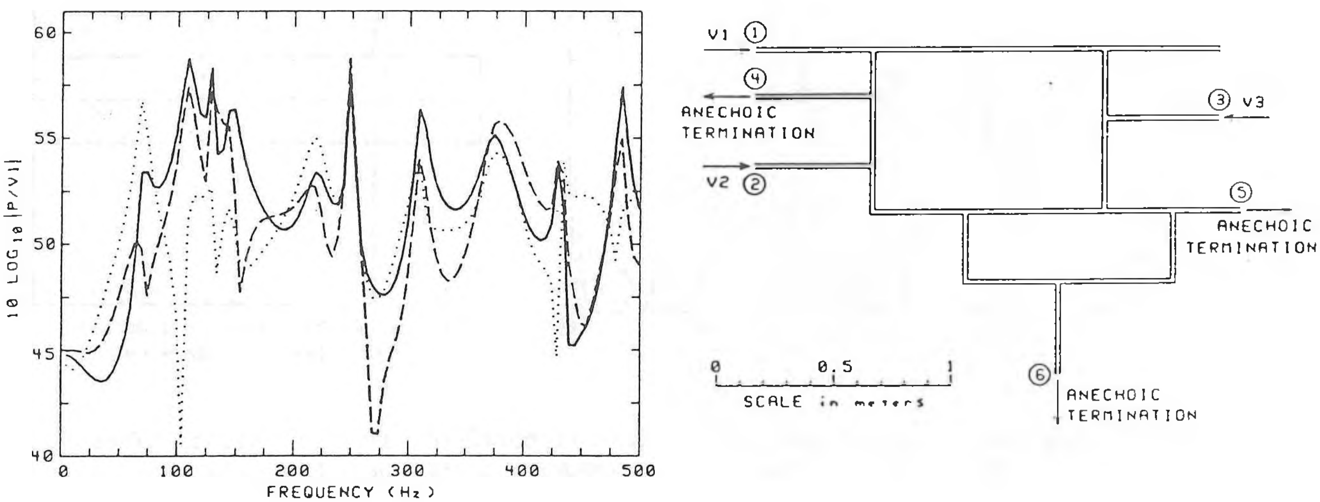


Figure 1. Piping network transfer impedance wrt Station 1
Stn. 4 - dotted, Stn. 5 - dashed, Stn. 6 - solid

In this example at each junction the pipes have been linked to a single node and the solution does not take into account the detailed geometry of the junctions or elbows and is thus only valid at low frequencies. Valid predictions could be extended to higher frequencies by using finite element methods to model the junctions and elbows. These could then be linked with one dimensional pipe elements.

### 4.2 Expansion chamber with insertion tubes

This example considers an expansion chamber with insertion tubes. The system considered starts with an arbitrary length of straight pipe which has an incident pressure boundary condition specified at the inlet. This is attached to one end of the expansion chamber. The outlet is attached to an arbitrary length of straight pipe with an anechoic termination.

The expansion chamber was modeled using the finite element method with isoparametric axisymmetric ring elements having eight degrees of freedom. The element mesh is shown in Figure 2. The pressures at nodes on the input and output surfaces were constrained to a single value and the finite element solution converted to a transfer matrix for the expansion chamber. The transfer matrices for the expansion chamber and the connecting one dimensional pipe elements were then combined and converted to a global network matrix equation which was solved to obtain the pressure at the anechoic termination.

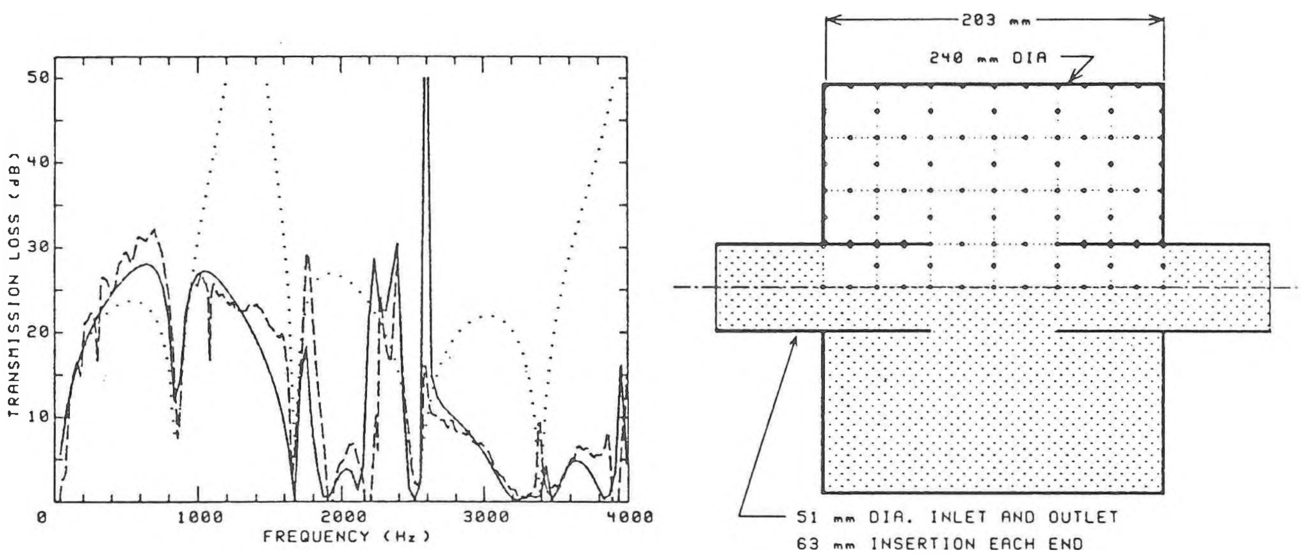


Figure 2. Axisymmetric expansion chamber with insertion tubes
FEM - solid, Experiment (MEANU) - dashed, ID - dotted

The transmission loss was then obtained by taking the ratio of the inlet incident sound power to the transmitted sound power at the anechoic termination and is shown in Figure 2 calculated at 40 Hz intervals. Also shown is an experimental result measured at the Mechanical Engineering Acoustics and Noise Unit (M.E.A.N.U.) using a two microphone technique used by Chung and Blazer [2], [3]. A third result is shown which was obtained by treating the expansion chamber as a series of one dimensional pipe elements including side branches. This demonstrates that the one dimensional analysis is inadequate for the frequency range considered.

## 4.3 Elbows

When two one-dimensional pipe elements join, the transfer matrix method and the assembly procedure of network elements does not consider the geometry of the junction. The pipes could be on a common axis or perpendicular to each other and the same result would be obtained. This is only valid for wavelengths much larger than the pipe cross section dimension. The model can be extended to higher frequencies by using a more detailed model of the junction which takes into account the junction geometry.

### 4.3.1 Square elbow

The finite element mesh used for a square elbow is shown in Figure 3. In this example an eight degree of freedom plane isoparametric element was used. As with the expansion chamber considered in section 4.2, the finite element model was linked to a one dimensional pipe element with the incident pressure specified at the inlet and to a second pipe with an anechoic termination. To compare directly with experimental results of Lippert [6] the results are shown in Figure 3 in terms of the pressure transmission coefficient (the ratio of the outlet pressure at the anechoic termination to the incident pressure at the inlet).

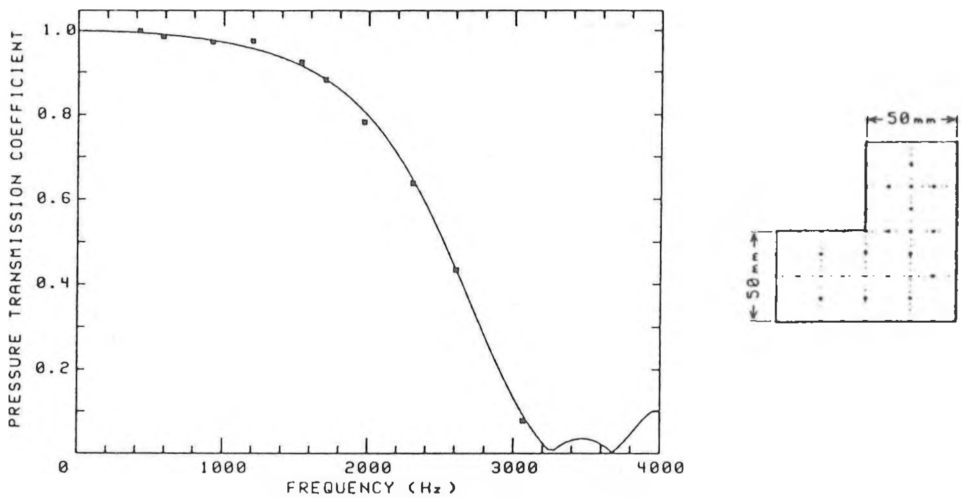


Figure 3. Square elbow with rectangular cross section
FEM - solid line, Experiment [6] - points

### 4.3.2 Round elbow with turning vane

Figure 4 shows the result obtained for a round elbow of rectangular cross section with a turning vane. The turning vane is modeled by disconnecting the nodes of adjacent elements along the line of the turning vane. Again isoparametric plane elements with eight degrees of freedom were used and the pressures at the inlet and outlet were constrained to single values and linked to one-dimensional pipes. The results are shown in terms of the transmission loss across the elbow.

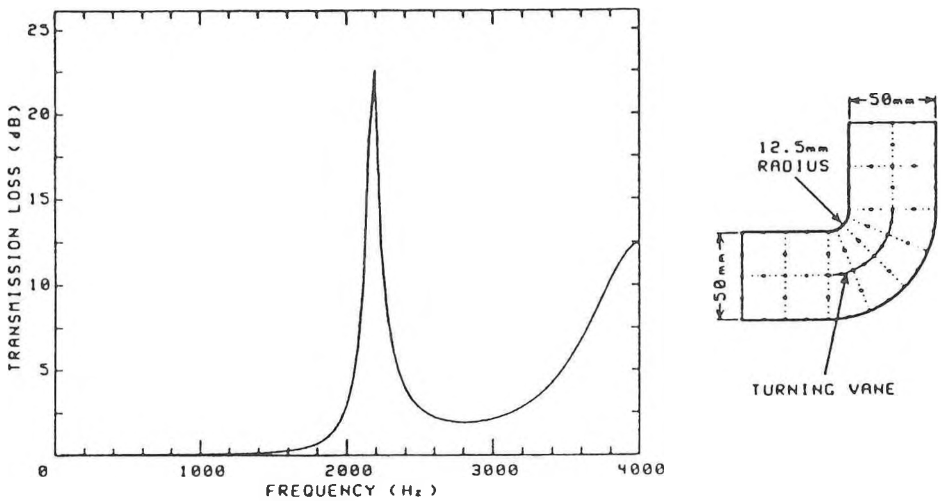


Figure 4. Round elbow of rectangular cross section with turning vane

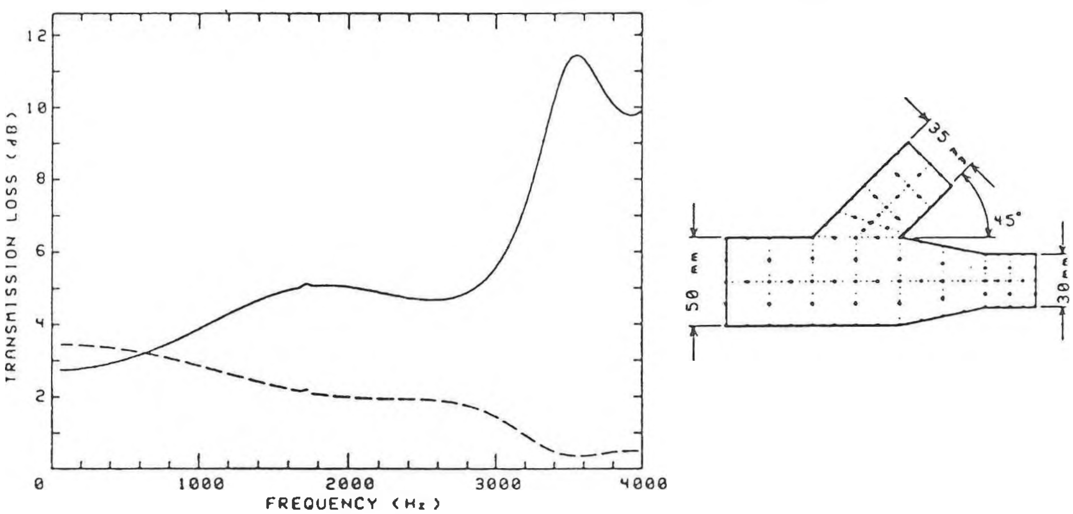


Figure 5. Rectangular duct with 45 degree branch
Branch - solid line, Continuing duct - dashed line

### 4.4 Branches or junction of more than two pipe elements

If a junction of three or more pipe elements is modeled by connecting the elements at a common node then the assembly procedure only considers the cross section areas of the joining pipes and does not consider the detailed geometry of the junction. The resulting prediction is frequency independent and is valid only when wavelengths are much longer than the cross sectional dimensions at the junction. The prediction can be extended to higher frequencies by modeling the junction using the finite element method as used above for elbows.

An example side branch in a duct of rectangular cross section is shown in Figure 5. This has been modeled using isoparametric plane elements with eight degrees of freedom. The 50 mm wide duct was considered as the inlet and transmission losses were predicted for the inlet to side branch path and the inlet to continuing duct path and are shown in Figure 5.

## 5. CONCLUSIONS AND SUMMARY

A computerized method has been developed to study the acoustics of duct and piping systems. The method combines transfer matrix procedures and finite element methods and can easily handle complicated networks with multiple inputs and outputs. It is assumed that within straight rigid piping or ducts that plane wave propagation occurs so that exact solutions can be used. In elements such as pipe junctions, branches, elbows, and in components with larger cross sectional dimensions such as plenums or expansion chambers finite element models are used to extend the validity of the method to higher frequencies. There is good agreement between the computer predictions and some experimental results obtained for individual components.

## 6. FURTHER WORK

The present work has considered components such as rigid ducts, elbows, and reactive mufflers in which there is assumed to be no internal dissipation of sound energy (with the exception of the one-dimensional rigid pipe transfer matrix element which can include viscous type fluid damping). The next step is to incorporate finite element models and possibly simpler transfer matrix models which can be used for components such as lined ducts or elbows and dissipative type silencers.

The finite elements incorporated into the program at present are limited to modeling of components having a plane or axis of symmetry. The elements used were created by applying constraints to an eight and a twenty degree of freedom hexahedral element. The hexahedral element could be easily incorporated in the program to model three dimensional components which have no symmetry. The major limitation would be the size of system that could be handled because of the limited memory of the desktop computer currently used for this work.

## ACKNOWLEDGEMENTS

The authors gratefully acknowledge the University of Alberta for providing their facilities and the National Research Council of Canada for providing funding of this work through NSERC grants A7514, A4593 and A7431.

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