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WHITE NOISE ANALYSIS OF NON LINEAR SYSTEMS WITH APPLICATION TO THE AUDITORY SYSTEM

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ABSTRACT

Linear systems analysis with tones, clicks or white noise results essentially in the same information: the impulse response or frequency response of the system. In non-linear systems, such as the auditory system, these three methods provide different results. Furthermore the outcome will be level dependent. Higher order cross correlation with gaussian wide band noise as input signal provides, in principle, a method to analyze non-linear systems. For that purpose one needs noise with a zero valued second order auto correlation function. Commercially available pseudo-random noise generators do not produce noise with satisfactory properties. Software generated noise with gaussian amplitude distribution can easily be generated on basis of the uniform distributed random number generator. Using noise with these improved characteristics we studied neurons in the auditory midbrain of the grassfrog. Two examples are shown and a comparison is given between the results obtained with the white noise method and the more traditional pure tone analysis.

SOMMAIRE

L'analyse des sytèmes linéaires au moyen de sons purs, de clics ou d'un bruit blanc donne essentiellement la même information: la réponse impulsionnelle ou la réponse en fréquence du système. Dans les systèmes non-linéaires tel que le système auditif, ces trois méthodes donnent des résultats différents. De plus, la réponse dépendra du niveau d'excitation. L'inter-corrélation d'ordre supérieur avec une bande de bruit gaussien comme signal d'entrée fournit, en principe, une méthode d'analyse des systèmes non-linéaires. A cette fin, on a besoin d'un bruit dont la fonction d'auto-corrélation de second degré est nulle. Les générateurs de bruit pseudo-aléatoire disponibles sur le marché ne produisent pas un bruit qui rencontre ces exigences. Un bruit généré par programmation avec une distribution gaussienne d'amplitudes peut facilement être produit à partir d'un générateur de nombres uniformément distribués au hasard. En recourant à un bruit comportant de telles caractéristiques, nous avons étudié des neurones du cerveau moyen de la grenouille. Deux exemples sont présentées ainsi qu'une comparaison entre les résultats obtenus avec le bruit blanc et la méthode plus conventionnelle de l'analyse aux sons purs.

INTRODUCTION

Traditionally the auditory system has been studied using rather simple and mostly deterministic stimuli, such as continuous sinusoids, tone-pips or tone-bursts, repetitive click trains, and sinusoidally amplitude- or frequency-modulated tones.These stimuli have the advantage that they are characterized by a only a few independently variable parameters. For instance, continuous tones are completely determined by frequency and sound pressure level, and the response of neurons in the auditory system is usually presented in its dependence on these parameters, e.g., as tuning curves or iso-intensity contours. The tuning curve is a plot of sound pressure level-frequency combinations that produce a prespecified change in the spontaneous firing rate of neurons. The iso-intensity contour represents the firing rate of a neuron as a function of frequency for a constant sound pressure level. Tuning curves therefore represent an equal-output criterion, iso-intensity contours an equal-input one. If the system under study were linear, then both measurements could easily be converted into each other, however, the auditory system is highly nonlinear and tuning curves and iso-intensity contours generally highlight different aspects of its functioning. Constant response criteria can only be used if the non linear element is a threshold detector and the last stage in the chain of transformations. The problem with the tuning curve approach is that it is only based on a constant change in the magnitude of a response, for example in the firing rate of the neuron, the vibration amplitude of the basilar membrane or the receptor potential amplitude. A true constant response criterion should also consider phase (latency) and harmonic distortion.

A further consequence of the nonlinear nature of the auditory system is that the frequency response measured with one frequency at a time - the harmonic analysis method - cannot describe the behavior of the system for complex multi-frequency stimuli. A very simple multi-frequency stimulus is Gaussian wide band noise (GWN), that has wide spread use in linear systems analysis. Cross correlation between the system output and the GWN results in an estimate of the impulse response, and through Fourier transformation in an estimate of the frequency response of the system. This technique can also be applied when the output signal is discrete such as for a neural spike train. Depending on the precise technique used, the form of the nonlinearity that results in the spike production, and the form of the impulse response itself, the cross correlation technique either results in an unbiased estimate, a scaled version or a linear combination of the impulse response and its first derivative. For auditory nerve fibers with characteristic frequency below 4 kHz it has been established that an unbiased estimate is obtained (De Boer and De Jongh, 1978; Eggermont et al.,1983a). One manifestation of the nonlinear nature of the auditory system is that the estimate of the impulse response depends on the spectrum level of the noise that is used: in general the impulse response shows greater damping for higher noise levels, and consequently a broader frequency response (Moller, 1986). Therefore it is expected that nonlinear systems analysis offers some more insight into the specific nature of the auditory nonlinearity, which by virtue of the audibility of cubic difference tones is known to be at least of third order. The reader has to keep in mind that in non linear systems the result of an input-output characterization has by definition not much predictive value. Because the superposition principle only holds for linear systems, knowledge of the input-output relation of a non linear system is only valid for the actual test stimulus itself. Using the Wiener-kernel formalism a complete description of the non linear system will require determination of all Wiener kernels (Johnson,1980) which is usually not feasible because of computational constraints.

In this paper the main emphasis will be on the use of GWN stimuli, how to study their first and second order statistical properties and to find out how these affect the stimulus response relationships that are revealed by first- and second-order cross correlation. Methods to reduce computation time will be briefly discussed. Examples will be given from a study into the functioning of the auditory midbrain of the grass frog.

CHARACTERIZATION OF STIMULI

Noise that is used to measure first order properties of the system under study should have a flat spectrum and have a bandwidth greater than that of the system, and thus a relatively short and non-oscillating impulse response. When the noise is used to measure second order properties such as energy-time densities (Mecklenbrauker, 1987), ambiguity functions, bi-spectra (Spekreyse and Reits, 1982), or second order correlation functions (Marmarelis and Marmarelis,1978), the noise must have adequate second order properties itself. Thus the second order auto correlation should be equal to zero everywhere, or equivalently its bi-spectrum should be flat. A flat second order auto correlation requires that the amplitude distribution of the noise is symmetric, i.e., the skewness should be zero. When the (0,0) element of the second order auto correlation is zero then the skewness of the amplitude distribution is also zero. We have therefore investigated the amplitude distribution, and the first and second order auto correlation functions for pseudo-noise generated by a Wavetek 132 function generator, noise characterized as random and generated by the Bruel and Kjaer dual FFT analyzer (model 2032), and noise derived from uniformly distributed, software generated, random numbers after appropriately changing

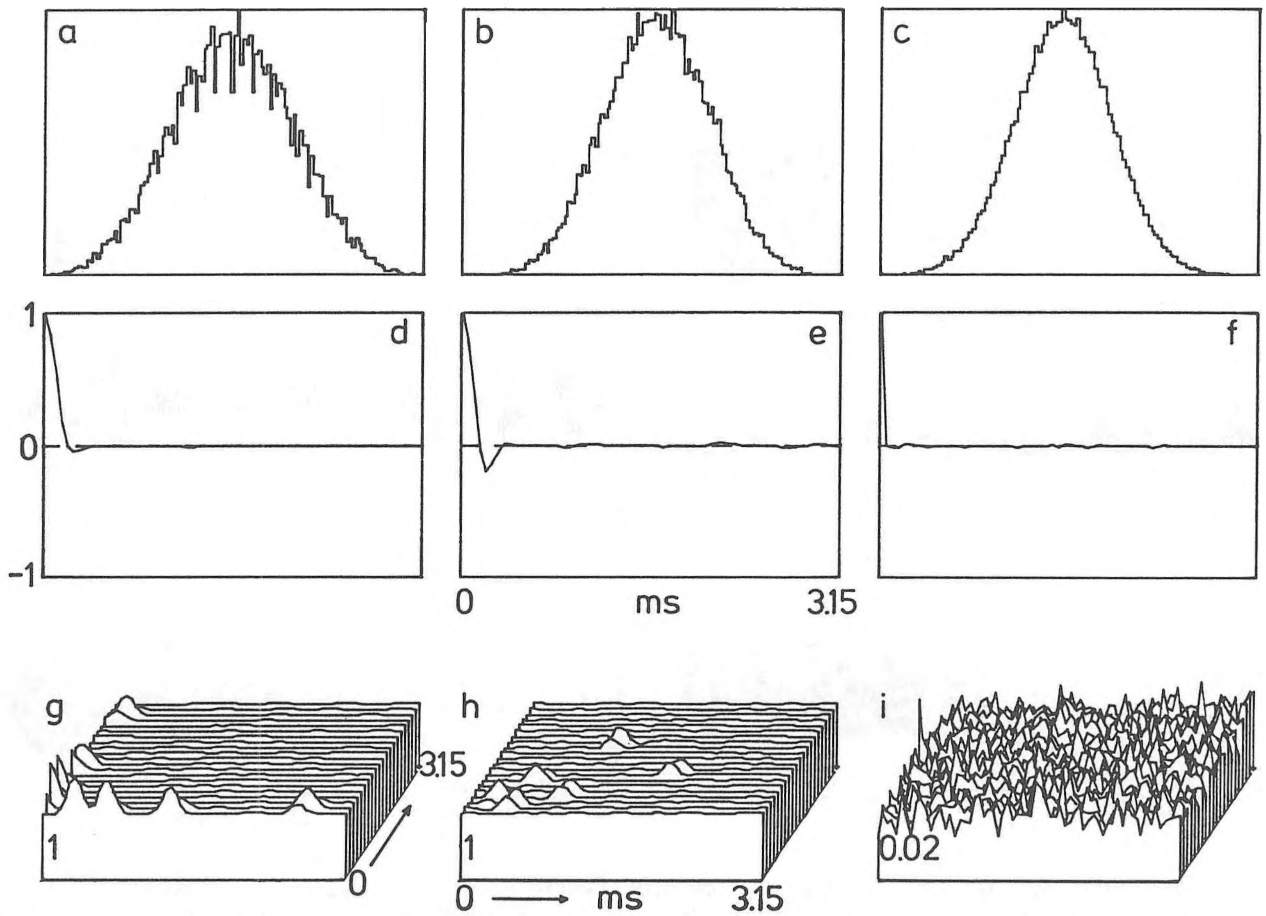

Figure 1. Characterization of pseudo-random noise.

the amplitude distribution to a standard normal distribution(as e.g. in Eckhorn and Popel,1979). The Wavetek and B&K noises were generated for a -3dB point of 2.5 and 3.2 kHz respectively and sampled at 20 kHz for a total of 32k samples.

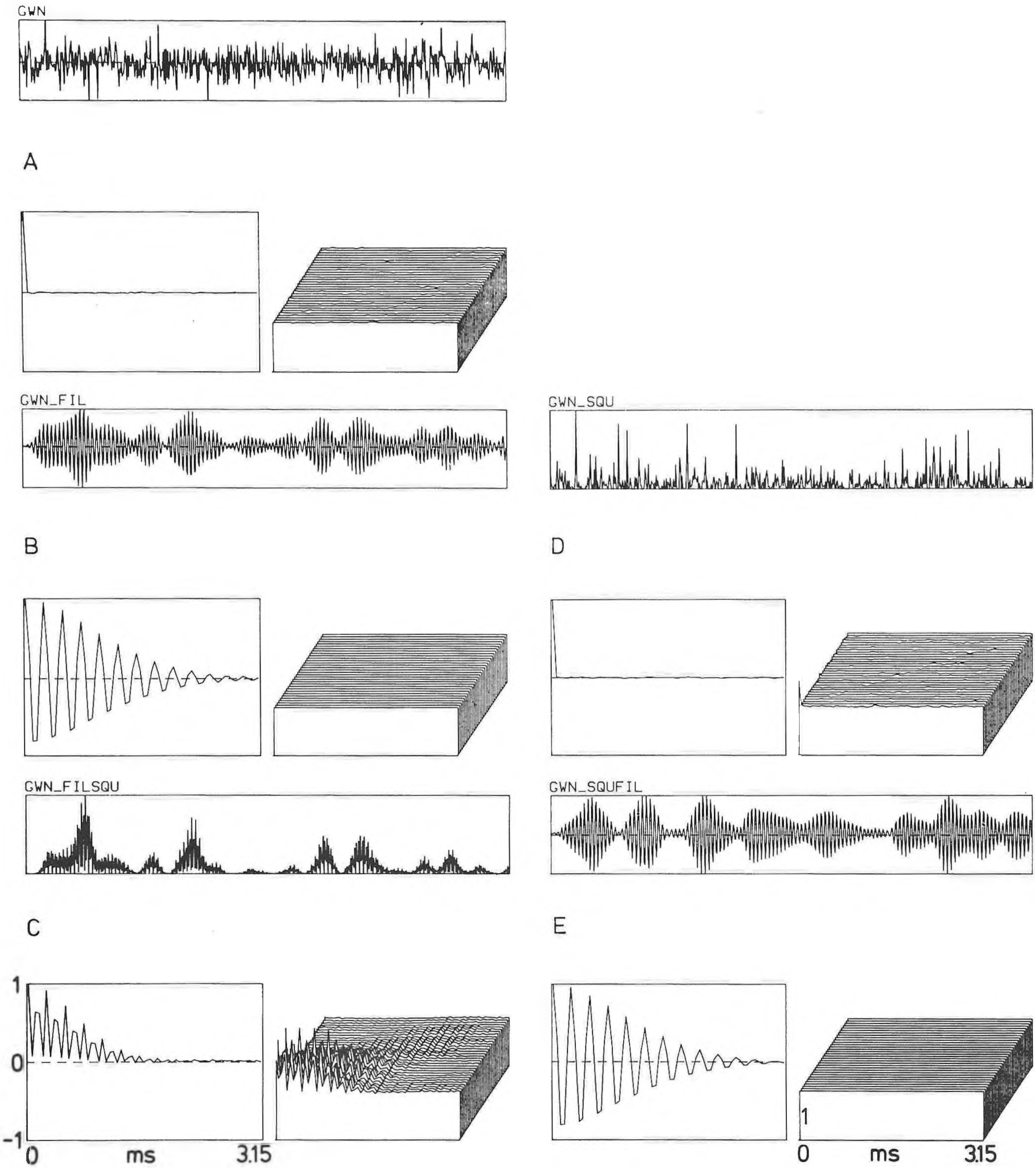

Figure 2. First and second order auto correlation functions for A) Gaussian wide band noise (GWN), B) 2 kHz bandpass filtered noise, C) filtered and squared noise, D) squared GWN, and E) squared and filtered noise.

All amplitude probability densities appear to be symmetrical; for the Wavetek noise with sequence length (2^{15}-1) the distribution is very close to Gaussian , the B&K noise and the computer generated noise do have a Gaussian probability density function (Figure 1 a,b,c). The first order auto-correlation functions for the three types of noise were computed for 64 lags (3.2 ms) and all of them are acceptably close to the ideal one as obtained for the computer generated noise (Figure 1 d,e,f).

The second order auto-correlation properties for the three types of noise differ greatly, however. Defined as:

$$R_{xxx}(\tau_1,\tau_2) = \frac{1}{T}\int_0^T x(t)\, x(t-\tau_1)\, x(t-\tau_2)\, dt \tag{1}$$

for T sufficiently large, the second order auto correlation function can be represented in pseudo 3-dimensional view as in Figure 1 g,h,i. Part g shows the result for the Wavetek, part h the so-called random noise from the B&K, and part i the computer generated noise. One can observe that only the computer generated noise has satisfactory second order properties, and that the "random" B&K noise clearly has periodic components in it and therefore should better be labeled pseudo-random. Basically this means that the two commercial noise sources studied do not produce noise with properties that are acceptable for second order analysis of non-linear systems.

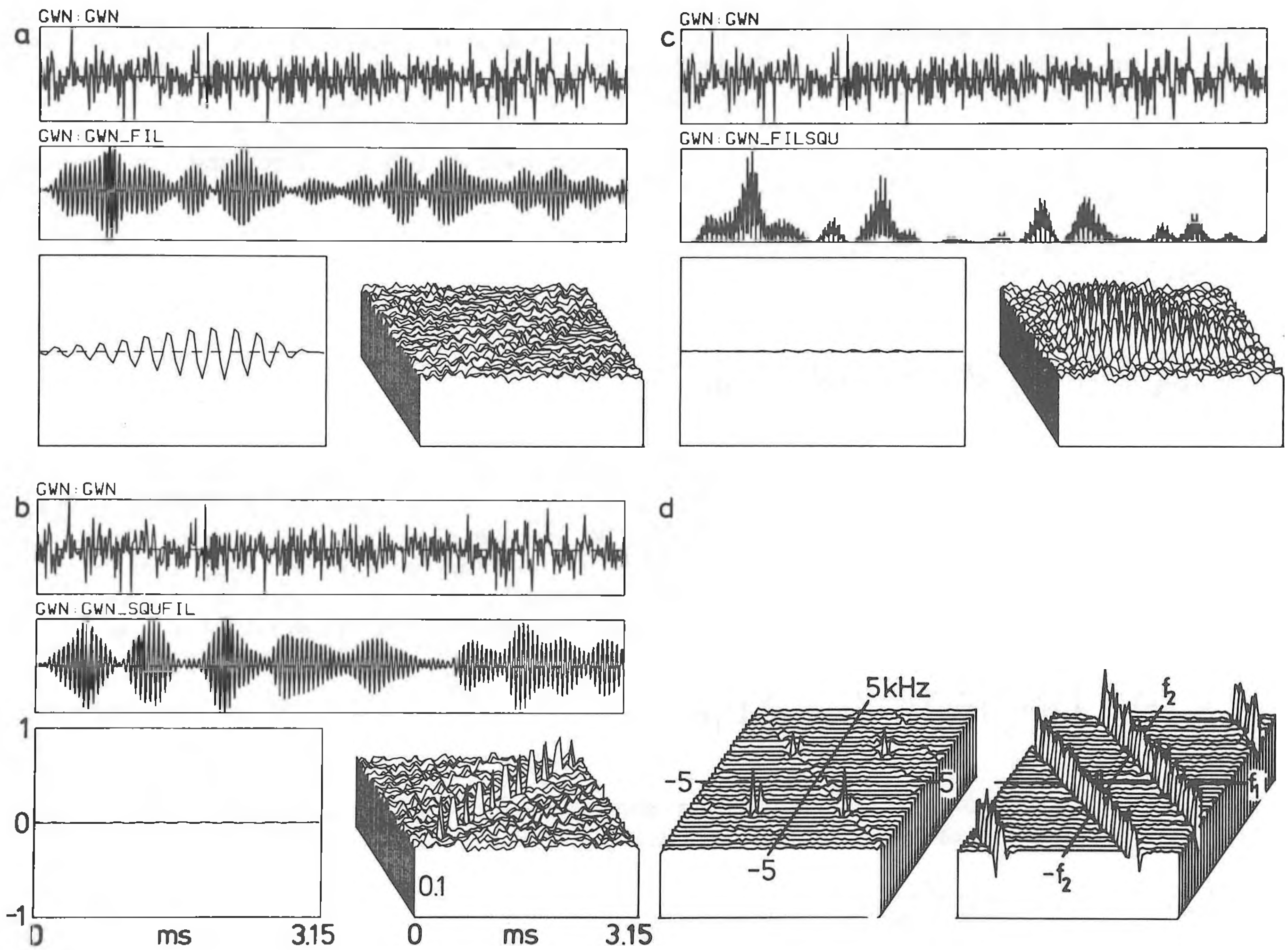

Figure 3. First and second order cross correlation functions for the three systems under consideration (a, b, c) and the bispectra for the systems in b and c (d).

STIMULUS RESPONSE RELATIONSHIPS

The power of non-linear analysis is best demonstrated for a model system. For that purpose we introduce two non-linear systems consisting of a linear band-pass filter (center frequency 2 kHz, slopes 135 dB/oct, -6 dB bandwidth 125 Hz) and a squarer in cascade: one system has the filter at the input, the other system has the squarer at the input. The input to the systems will be gaussian wide-band noise (GWN) and we will show the properties of signals and system through their first and second order correlation properties. Figure 2 presents the format for the auto-correlation analysis of the signals at various points in the systems. First of all the properties of the GWN are repeated in part a, part b gives the same for the filtered GWN showing the effects of the filter in the first order auto-correlation. Because the filter is linear the second order auto-correlation is not affected. Part c shows the signal properties after the signal is filtered and then squared: the first order auto-correlation is now the squared version of the filtered one and the second order auto-correlation reflects both the skew and the filter properties. In part d we study the first transformation in the squarer-followed-by-filter system and observe that squaring the signal does not affect the first order auto-correlation but the induced skewness clearly shows in the (0,0) element of the second order correlation which now has a value of 0.5. Passing this squared signal through the filter produces a first order auto correlation which is identical to that in the absence of the non-linearity, the second order auto correlation is zero since the filtering operation removes all skew from the signal.

As we know from linear systems theory the Fourier transform of the auto-correlation function of the output signal for a GWN input signal gives an estimate of the modulus of the frequency response but leaves us in the dark about the phase relationship between input and output signal (Bendat and Piersol, 1971). In order to identify the system completely we need to use the cross-correlation between input and output signals. The first and second order cross-correlations are defined as:

$$R_{yx}(\tau_1) = \frac{1}{T}\int_0^T y(t)\, x(t-\tau_1)\, dt \tag{2}$$

$$R_{yxx}(\tau_1,\tau_2) = \frac{1}{T}\int_0^T y(t)\, x(t-\tau_1)\, x(t-\tau_2)\, dt \tag{3}$$

In Figure 3a we show results for the analysis of the linear filter: samples of the input and output signal are shown together with both cross-correlation functions. The first order cross-correlation results in an unbiased estimate of the impulse response of the filter. For this linear system the second order cross-correlation should theoretically be zero, in this case for a limited signal length the extremes are smaller than 0.01. For the squarer-filter combination the first order cross-correlation nearly disappeared, as it should because:

$$R_{yx}(\tau) = \int h(\tau_1)\, h(\tau_2)\, R_{xxx}(\tau_1,\tau_2,\tau)\, d\tau_1\, d\tau_2 \tag{4}$$

In contrast the second order cross correlation shows on the diagonal a stretched out version of the impulse response of the linear filter because:

$$h_2(\tau_1,\tau_2) = \delta(\tau_1-\tau_2)\, h(\tau_1) \tag{5}$$

The filter-squarer combination (Figure 3c) again does not show a first order cross-correlation, however, a distinct second order cross-correlation equal to:

$$h_2(\tau_1,\tau_2)=h(\tau_1)\,h(\tau_2) \tag{6}$$

A bi-spectrum analysis of the two non linear systems shows (Figure 3d) that for the filter-squarer combination the only output is for signals at the filter frequency, however, for the squarer-filter an output is produced whenever the sum or difference of two frequencies equals the filter frequency. This illustrates the formation of sum and difference frequencies when a multifrequency signal is presented at the input of the squarer. Such phenomena are found in the auditory system, indicating that the filter stage is not likely to precede the nonlinearity.

THE AUDITORY NERVOUS SYSTEM

At this point we introduce a model representing current thinking about the auditory peripheral system with exclusion of the middle ear. The model (Figure 4) consists of a non-linear filter, located in the basilar membrane, that receives energy from an active process located in the outer hair cells (Ashmore, 1987), followed by a synapse responsible for auditory adaptation (Eggermont, 1973, 1975, 1985; Smith, 1979; Harris and Dallos, 1979), and finally a pulse generating device: the auditory neuron. Because techniques to deal with non-linear filtering are lacking we modify the model into a band-pass non-linearity (BPNL) model. In such a model the non-linear filter is split up into two linear filters with an algebraic non-linearity sandwiched in between. The first filter is a sharply tuned band-pass filter representing the tuning properties of the basilar-membrane-outer-hair-cell system, the second filter is a low-pass filter representing the action of the hair-cell-neuron-synapse (see Pfeiffer,1970; Johannesma, 1971; De Boer, 1976).

In the present model the occurrence of an action potential z(t) depends on the signal y(t), the generator potential of the auditory neuron. In case the pulse generator produces spikes proportional to y(t) it can be demonstrated that :

$$R_{zx}(\tau)=R_{yx}(\tau) \tag{7}$$

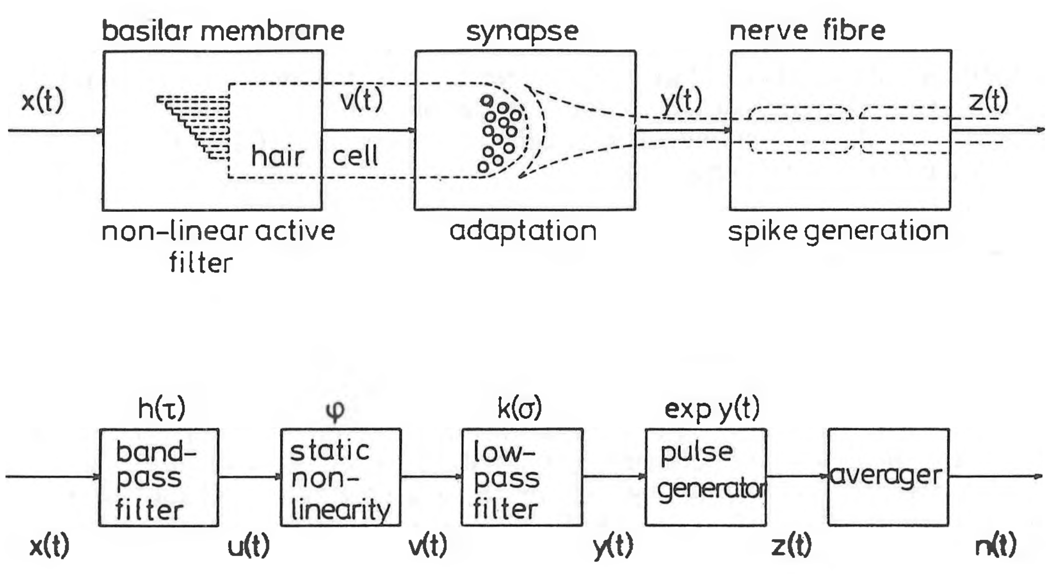

Figure 4. A model for the peripheral auditory nervous system, and its adaptation for non-linear systems analysis (from Eggermont et al., 1983c).

This appears to be the most likely situation for the auditory nerve fibers (De Boer and De Jongh, 1978). Thus instead of using, the rather inaccessible analog signal, y(t) we can perform the correlation with the spike train as the output signal (De Boer and Kuyper, 1968). The method can be extended to second order cross-correlation (Hermes et al.,1981; Eggermont et al.,1983a). In both cases the signal to noise ratio decreases by a factor of 1.8 respectively 3.4 as compared to the correlation on the basis of the analog signals.

Since the z(t) are unitary pulses occurring at times t_n, the cross correlation takes the form of averaging the pre-spike stimulus:

$$R_{zx}(\tau)=\frac{N}{T}\frac{1}{N}\sum_{n=1}^{N}x\left(t_n-\tau\right) \tag{8}$$

respectively the lagged product function:

$$R_{zxx}(\tau_1,\tau_2)=\frac{N}{T}\frac{1}{N}\sum_{n=1}^{N}x\left(t_n-\tau_1\right)x\left(t_n-\tau_2\right) \tag{9}$$

of the stimulus that precedes the action potentials. It turns out (Moller, 1986) that the first order cross correlation is stimulus dependent, at least in rodents, and becomes progressively more damped with increase in noise level. This is again a strong indication of a non-linearity of at least third order.

POLARITY AND TERNARY CORRELATION FUNCTIONS

Calculating higher order correlation functions requires fast computers and although these are now readily available a discussion of computationally less demanding procedures is still useful. An idea proposed by Veltman (1966) and later elaborated upon by Wolf (1973) and Klein and Yasui (1979) is to replace both the GWN and the analog output signal by a simple transformation of these signals. Such a transformation can be replacing the signal by its sign, and it can be shown (Figure 5 a,b) that correlation between two such transformed signals, polarity correlation, leaves the shape of the correlation function intact but reduces the amplitude by a factor 2/pi, and increases the variance by a factor 1.57 for first order correlation and 2.47 for second order correlation.

A slightly better result is obtained by using a ternary transform (Figure 5 c,d) in which the signal is transformed according to:

$$x(t)>\mu+n\sigma \quad , x(t) = 1$$

$$x(t)<\mu-n\sigma \quad , x(t) = -1$$

and otherwise x(t) = 0; usually n is taken equal to 1. In this case the variances for the first and second order correlation functions increase by 1.37 respectively 1.88, the reduction in amplitude is however larger and results in about 0.24 of the original size.

A draw back of the polarity and ternary correlation procedures is that in case of an even order non-linearity following the filter, such as in our example of the filter-squarer combination, the output signal will be extremely skewed. In such a case the transformations are not justified. This can be alleviated by adding a non-correlated random signal to the output before transforming it, this of course will further increase the variance but at least allows these computational faster methods to be used.

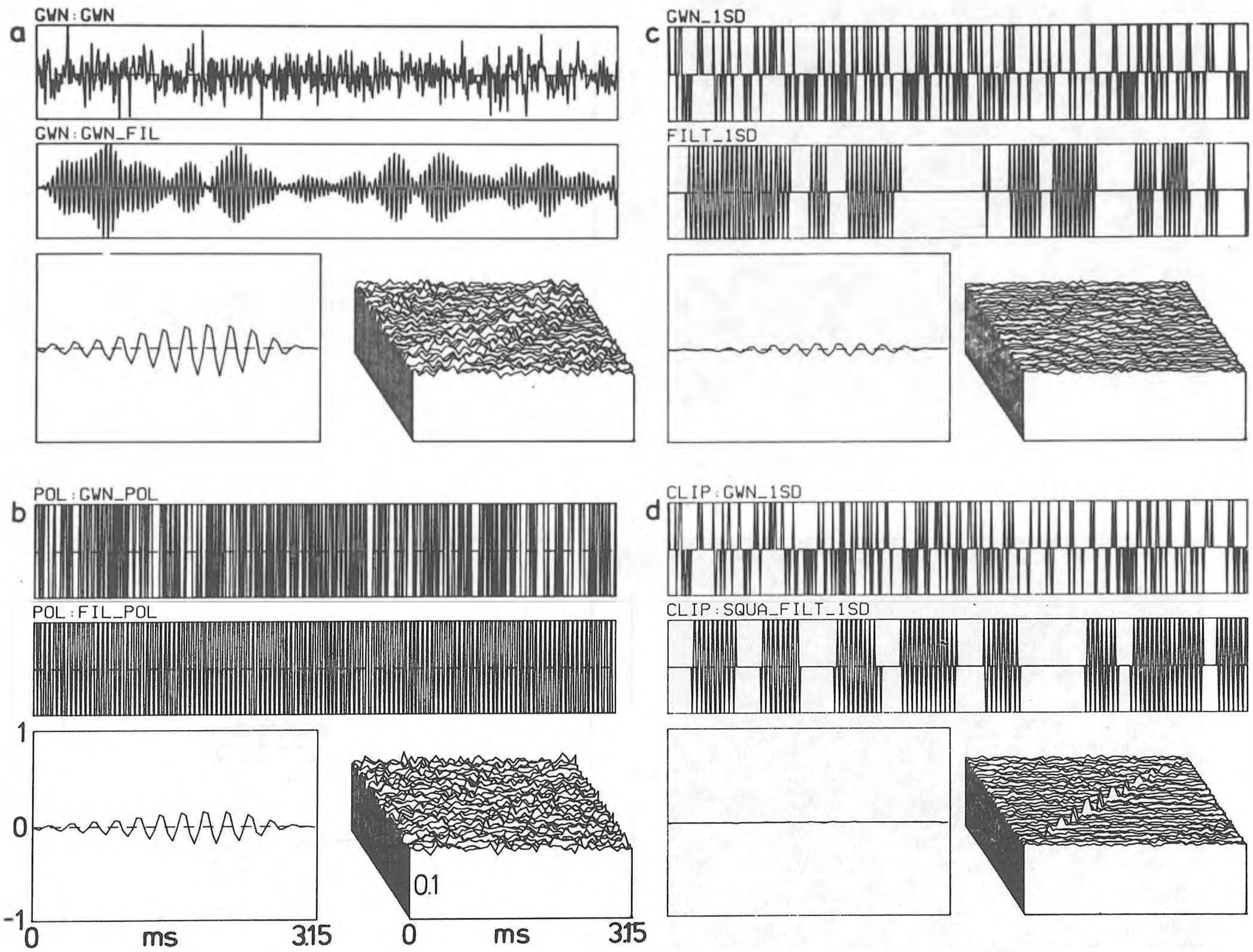

Figure 5. A comparison of cross correlation functions bases upon GWN (a), polarity representation (b), and a ternary representation (c). Part d is to be compared to Fig. 3b.

FREQUENCY-TIME REPRESENTATION OF SIGNALS AND CORRELATION FUNCTIONS

For the auditory nervous system, which is both a frequency and a time analyzer, it is intuitively more useful to represent second order correlations or bi-spectra in the form of time dependent spectra or spectrograms. The Fourier transform with respect to time of the time dependent autocorrelation function results in the spectrogram of a signal. Various forms exist such as the Wigner distribution (Yen,1987) or the Rihacek Costid (Johannesma et al.,1981). This can be extended to those parts of the stimulus signal that precede an action potential. Formally the correlation takes the form of a second order cross correlation between z(t) and x(t) in which z(t) is a series of delta functions. A Fourier transform with respect to the difference in lag times then results in a short-time-spectrogram. Averaging all these short-time-spectrograms results in the average spectrogram of signals preceding action potentials, and is considered to be causal in their generation. The latter form has been used extensively in the study of response properties of auditory neurons (Hermes et al.,1981; Eggermont et al.,1983a; Epping and Eggermont,1985). We will discuss two examples and then give an overview of results obtained in previous studies.

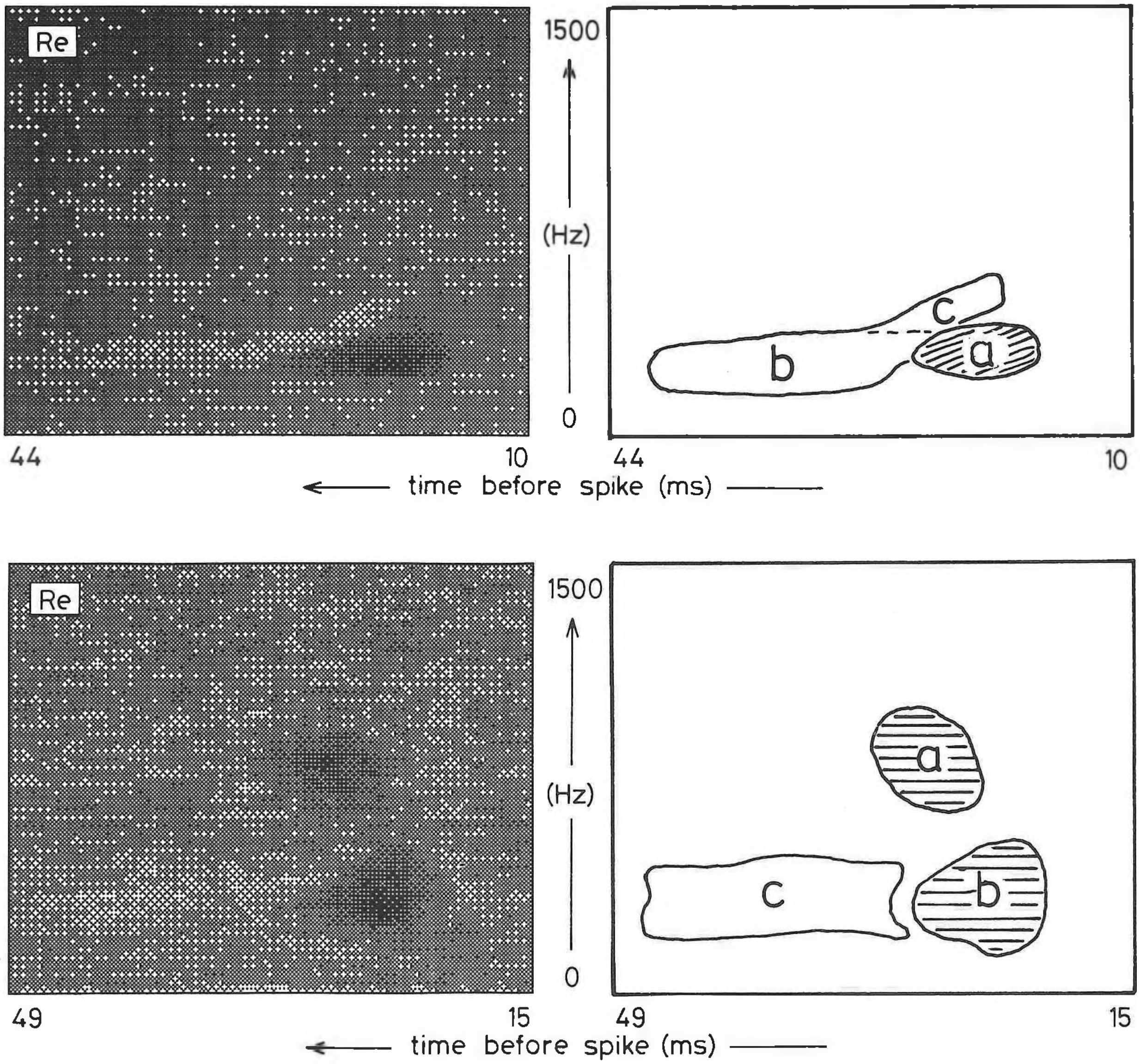

Figure 6. Average Costid's for signals preceding an action potential for two neurons in the auditory midbrain of the grassfrog.

Figure 6a shows the real part of the average Costid (short for coherent spectro-temporal intensity density) for a neuron in the auditory midbrain of the grassfrog. On the horizontal axis time before the action potential is indicated, the vertical axis represents frequency. The right hand side of the Figure schematically outlines the various areas of interest in the Costid. The coding used is a grey scale: grey is the level of the noise signal in case of random, not stimulus related, action potentials, dark represents more signal level and white less signal level than background. One can interpret part a of the Costid as those parts of the noise that increase firing probability in the neuron, i.e., those parts of the noise stimulus that are filtered out by the neuron. In this example the center frequency is about 250 Hz, the fact that this area is situated about 15 ms before the spike indicates that the latency of the neuron is about 15 ms. After this dark, activation, area one observes (part b) a lighter area indicating that in this time frame on average less noise than average was present in this frequency range. This is called post-activation suppression. Also some lateral suppression is noted (part c). The total number of spikes elicited by the noise, and thus the number of averages, was 666.

The second example (Figure 6b) shows a neuron that has a double peaked sensitivity: one region (part a) is centered around 750 Hz, the other region (b) around 250 Hz. The latter region also is accompanied by lateral suppression (c). Such double tuned neurons are quite common in the auditory midbrain of frogs and nearly absent at more peripheral levels. This indicates that neurons with widely differing best frequencies converge onto a single neuron at the level of the midbrain.

In two recent studies a total of 219 units was investigated with noise (Hermes et al.,1981; Epping and Eggermont,1985) of which 107 responded in a sustained way such that their Costids could be evaluated. Of these 107 neurons 58% showed unimodal excitatory results, 21% showed bimodal responsiveness, 19% had lateral suppression (as in Fig. 7a) and only 2% showed inhibitory Costids (i.e., certain frequencies in the noise actually decreased the spontaneous firing rate of the neuron). Comparing these results to that of an analysis with random single-frequency tone burst stimuli revealed that in 64% of the cases the same type of spectro-temporal sensitivity was obtained. The remaining 36% of the units had different spectrograms for the noise and the tonal stimuli: all of them had multimodal spectrograms for the tonal stimuli that was either changed into a unimodal one (27%) or in a different multimodal one (9%) in case of the response to noise.

DISCUSSION

The first question we obviously have to ask is what the particular benefits the GWN analysis is producing, especially in the light of its demanding computational aspects. The basic idea behind the use of gaussian white noise, is that in principle all possible stimuli are contained in it, although the probability of occurrence for each of these is very low. If we know the response of a system to noise, or approximate that knowledge by the evaluation of cross-correlation functions, it is possible to predict the response to any other stimulus, provided that our approximation was sufficiently good. This is the pervading view, however when using the cross-correlation approach the estimated Wiener kernels are only orthogonal with respect to GWN of that particular spectrum power level, unless we know all the Wiener kernels (Johnson, 1980). Furthermore when we only have an approximation of the response to GWN, because we have evaluated only a few of the possible cross correlation functions, it is impossible to obtain an accurate prediction to stimuli that are not close to GWN (e.g.,Eggermont et al.,1983 a,b,c for an experimental investigation). On the other hand it may be possible to approximate the properties of meaningful sounds such as speech by linear transformations of GWN, in that case the stimulus used for predictions is close to GWN and the procedure might work.

Stimuli used in this procedure should be very carefully evaluated before using them, because their second order properties will confound the second order cross correlations and related spectro-temporal representations. In general one cannot rely on information in technical manuals of potentially useful equipment. First of all most commercially available noise sources are destined for linear systems analysis, secondly it is not appreciated that when nonlinearities of the third order are present that this makes the estimation of the "impulse response" dependent on the noise power.

The differences in the sensitivity obtained for the noise and the tonal stimuli can be the result of the lateral inhibition that becomes manifest for the multifrequency, noise, stimulation and is absent in the tonal stimulation. Another aspect is the differing adaptation state for the neuron in case of continuous noise stimulation versus the pulsed tone burst stimulation.The differences observed between the responses of the auditory neurons to single tone stimuli and noise illustrates that in order to characterize the auditory nervous system, or parts thereof, one needs a large variety of stimuli. The theoretical background for analyzing the stimulus response relationships for GWN stimuli obviously is an important advantage. The fact that the neurons in about half of the cases do not respond in a stationary way to GWN makes it application not as widespread as wanted. The use of amplitude modulated noise stimuli can alleviate this problem.

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