HARMONOGRAPHICS: PHYSICS OF AN ARTISTIC NOVELTY OF THE VICTORIAN ERA

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Project Summary

1. Introduction to the Problem

When two pendulums swing at right angles to each other, the combination of their oscillations plotted on paper form intricate patterns called Lissajous figures. A device which makes these patterns is called a harmonograph because it records harmonic motion. I became interested in harmonographs after seeing samples of Lissajous figures and marvelling at their complexity and beauty. I wanted to know how such a simple machine could make such a wide variety of complex designs. I set out to learn about the physics and mathematics of simple harmonic motion which are reflected in the Lissajous figures.

2. Outline of the Experiment

I researched historical designs for harmonographs and built a harmonograph which clearly displayed the relationship between the two interacting pendulums. I modified the design of my harmonograph to minimize friction and wobbling. Using a metre stick to adjust the length of each pendulum and a protractor to measure angular amplitude and phase displacement, I experimented to determine what factors affect the shape of the resulting figures.

Based upon the results of my experiments and research regarding harmonic motion, I derived mathematical formulae to describe the path followed by the pen whose motion is driven by the interaction of two pendulums swinging at right angles to each other.

I developed a computer simulation to plot the x and y coordinates calculated by my mathematical formulae. This allowed me to do three things which could not be done using the physical model: 1) to verify the mathematical formulae by comparing the shapes of the figures generated by the computer program with those generated by the physical model; 2) to test more precisely the effect of varying each factor while holding all others constant and while observing the effect of specific ratios of pendulum length and angular velocity; and 3) to observe the patterns that could be generated by ideal theoretical pendulums in a frictionless system.

3. Summary of Results

a) Two pendulums with the same angular amplitude and same lengths generate a straight line.

b) The lengths of the pendulums affect the outer dimensions of the envelope.

c) The ratio of the angular amplitudes affects the cycling of the curves; e.g. a ratio of 1:4 produces 4 loops.

d) Angular displacement is changing fastest at the bottom of the pendulum's swing and slowest at each end.

e) If the ratio of the angular amplitudes of the pendulum representing the x axis to the pendulum representing the y axis is a rational number, the curve is closed and the motion repeats itself at regular intervals of time.

f) If the ratio of the angular amplitudes of the pendulum representing the x axis to the pendulum representing the y axis is an irrational number, the curve is open and the tracing will gradually fill the area enclosed by the envelope.

g) The motion of the physical pendulum is damped by air resistance and the friction between the moving parts. Thus, the angular displacement of the swing of the physical pendulum gradually diminishes to zero. The motion of the theoretical pendulum is unaffected by friction.

4. Conclusions Reached

The interference pattern generated by two orthogonal oscillations can be represented by a Lissajous figure (x,y), where

$$x = R_x \cos(\omega_x t) \qquad y = R_y \cos(\omega_y t + \delta)$$

$$\omega_x = \frac{4\theta_x}{2\pi\sqrt{\frac{l_x}{g}}} \qquad \omega_y = \frac{4\theta_y}{2\pi\sqrt{\frac{l_y}{g}}}$$

The harmonograph is far more than an artistic novelty. It is an excellent tool for studying the behaviour of oscillating systems. Because mechanical and electromagnetic oscillations are described by the same basic mathematical equations, the behaviour exhibited by the harmonograph is related to the behaviour of radio waves, microwaves, visible light, and alternating current circuits.

Beyond its popularity in the Victorian parlour, the harmonograph has been shown to be a useful device for visualizing mathematical relationships. The intricate beauty of the patterns created by the harmonograph generates enthusiasm for comprehension of the underlying physics and mathematics.

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MUSICAL PHYSICS:
THE STUDY OF VIBRATING COLUMNS OF AIR IN MUSICAL INSTRUMENTS

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Purpose

I was trying to figure out how music is produced in a pipe organ, flute, and other brasses and woodwinds. I was also wondering if musical instrument making depends totally on the skill and experience of the draftsman, or if there is some kind of mathematical connection.

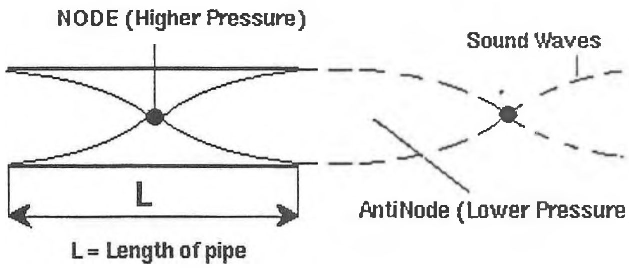
Hypothesis

I suspect that it depends on both. There must be a mathematical explanation - but how accurate is it? Well, I will study this problem and make experiments. I will try to make a 15th century pipe organ using 20th century test equipment.

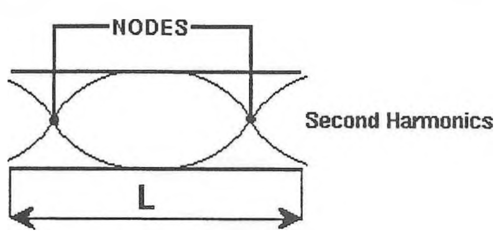
Theory

I did some studies and found out that all pipe instruments produce sound from vibration of the standing sound waves in a column of air in the pipe. Frequency of the tone played depends on the length of the pipe. I also found out that vibrations of the air column is set up by the vibrating lip of the player of brass instruments and by the airstream directed against one edge of an opening for woodwind instruments. The air within the tube vibrates with a variety of frequencies, but only certain frequencies persist. They are called resonant.

Theory of the Open Tube

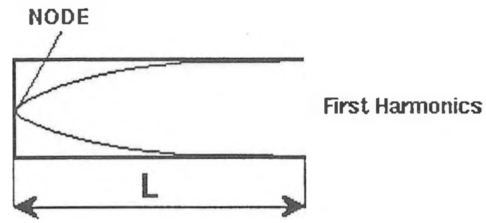


The graph within the tube represents standing waves (motion of air molecules) for air to vibrate, there must be at least one NODE within the tube. If the frequency doubles - there will be two nodes - and the tone will sound one octave higher, say C to C_1 it is called SECOND HARMONICS. I can easily demonstrate it in my experiment.

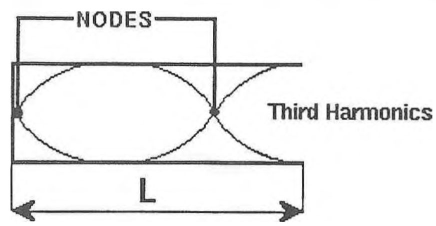


As you see it, the pipe will vibrate in resonance every time the tone is increased 1 octave - C to C_1 to C_2, etc.

Theory of the Closed Pipe



For the closed pipe, there is always a NODE at the closed end and an ANTINODE at the open end. The only difference is that there are only ODD HARMONICS presented so the pipe will be in resonance only every second octave, like C_1 to C_3 to C_5, etc.

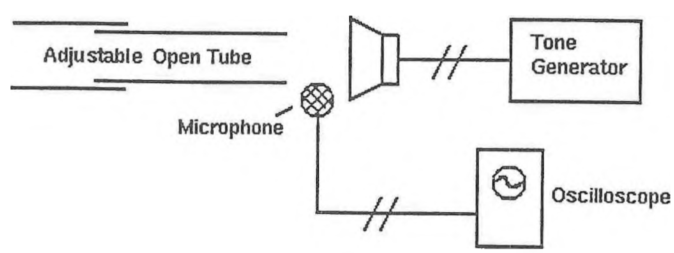


I also discovered that for the same frequency - say 440 Hz = note A, the length of the open pipe must be 2x longer than for the closed pipe (open pipe - 162 mm; closed pipe - 81 mm).

Test and Experiment Results

Basically, I made four experiments with interesting results.

Open Pipe Experiment No. 1.



1) I calculated the length of the pipes from the formula and recorded it in the table.

2) I placed an adjustable open pipe on the table with a speaker as the source of vibrations (tone) - say 440 Hz = note A, which was supplied by the tone generator.

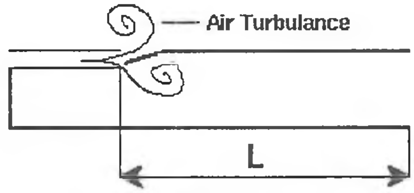
3) Then I slowly adjusted the length of the pipe and at the same time I observed the wave form on the oscilloscope. At the certain point I noticed increase in the volume and wave pattern grew taller due to the increase of volume of the sound because the pipe was in resonance with the frequency of 440 Hz.

4) I measured the length of the pipe and recorded it on the table. The difference between the calculated and tested length was 34 mm.

I them repeated the procedure 8 times for all notes of the musical scale between C to C_1. Then I cut all 8 pipes from plastic water pipes and made a small electronic pipe organ. The pipes are removable so they can be used in other demonstrations like the seashell theory. When you hold a seashell next to your ear, you are just hearing the "tube" that resonates to certain frequencies in the spectrum of background sounds and one pitch is dominant. By listening to all pipes in a certain order you can produce a tune.

Open Pipe Music Experiment No. 2

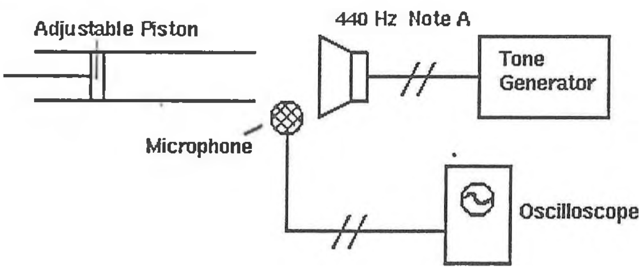
I calculated all the lengths of the pipes for the musical scale of C_1 to C_2. Then I made eight whistles according to the length I calculated.



After this was done, I played each note and tuned the whistle by trimming them (by ear) with a piano. I found this extremely difficult to do, and they are still somewhat out of tune by a few H. There is also a big difference between "calculated" and "tuned" lengths. Perhaps because of air turbulence.

Closed Pipe Experiment No. 3

This experiment is the same in principle as experiment No. 1. The only difference is that the pipe is CLOSED at one end with an adjustable piston.



I calculated the length for 440 Hz as 195 mm. The resonance length tested as 190 mm. I didn't repeat this experiment for other notes. As I see it, the same type of testing could be used to find the resonant frequency in any hollow object of any shape which would be impossible to calculate.

Closed Pipe Music Experiment No. 4

This is somewhat like experiment No. 2, except that the pipe is closed at one end. I calculated the lengths of the pipes for the notes C_1 to C_2 in the musical scale. Then I cut all the pipe 1/2 cm longer and plugged the bottom with wooden plugs. I tuned them by adjusting the plugs in or out of the pipe. This was very successful and the difference between the calculated length and the tuned length was very small as you see in the table. You can play pipes like blowing across the top of a pop bottle. The sound made is flute-like and sounds rather nice.

Conclusions

As I see it, there is more to it in making a musical instrument than just theory. Those old craftsmen of the 15th century were very good at DOING PHYSICS without actually knowing it - THEORY came later.

I had no problem with my experiments and testing, but I had some difficulty before I used the oscilloscope in experiment 1 and 3. I was trying to use different powders in the glass tube so it would form wave patterns - as the books recommended - but it would only work when the volume was very loud. I also found it difficult to work with sound as you can't see it and it varies too much.

I could have improved the testing by using an oscilloscope to greater extent by calibrating. I also could have studied different patterns of waves of different instruments, but this was the first time I used the oscilloscope.

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Note: The actual tables of results from the four experiments as well as a diagram of the audio generator and oscilloscope patterns appeared in the complete report.

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COMPUTER VOICE RECOGNITION

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In 1950, Bell labs attempted to develop a rudimentary voice recognition, but lacked the technology for such a feat. Since them with the advent of Artificial Intelligence and more efficient information storage, voice recognition has become much more feasible. However, to date most people are unaware of its existence, and it remains quite difficult to accomplish.

Basically, this project revolves around a voice recognition system that is both user-dependent and single word; that is, one person programs in some words, and the computer will then be able to recognize the words when repeated.

When a word is spoken into the microphone, it travels to an analog-digital converter where the analog signal of the voice is turned into a digital signal understood by the computer. This digital signal is read through the printer port. The software (designed exclusively by the entrants) then produces a dot-graph plotting amplitude vs. time, and this dot-graph is then used to check the word.

Three methods were originally developed to check the word entering the system. The first method imposed a ten by ten grid on the amplitude-time graph (A-T graph) and likened the grid to a matrix; thereafter, the values were placed in the corresponding matrix locations. This matrix was then stored beside the word. The second method first imposed ten columns on the A-T graph, and the average displacement (with all values considered positive) of the points from the Time axis was stored in a 10-length array, after that ten rows were imposed on the A-T graph, and the amount of dots within each row was stored in a 10-length array. Since it was felt that the second method, by using the row aspect, had a slight advantage, a third method was developed which only used the column aspect of the second method.

In order to compare stored words with incoming words, the three methods developed their respective matrices, which were then subtracted from the stored matrices. Each location within the resultant matrix was then squared to emphasize error. And then the locations were summed, producing a number that represented deviance from the stored word.

Experiments were then performed to determine the reliability of the system for 1) Number of words in Library, 2) Number of Lessons, lesson being the amount of times each word is stored in the library, and 3) Background Noise. These tests served a dual purpose. First, they determined what factor affected the systems, and second they determined which of the three methods was best.

However, it was found that some words were had to tell apart, cases being "one" and "four" and "left" and "right". The reason for this was suspected to be the lack of the use of frequency to check the words. To this end, two methods of frequency analysis were developed. The first was a spectro-analysis, but this method hasn't yet been used to test words, though it soon will be. The second method was similar to a spectro-analysis but it only paid attention to the major frequencies by checking for dots several units above and below the T-axis rather than checking for dots on the axis. This second method was easier to implement, and thus used in lieu of the spectro-analysis because of the lime-limitation. While comprehensive tests weren't performed to check the accuracy of the frequency analysis, the system no longer mistook "one" for "four" or "left" for "right"; and it just couldn't miss with a musical note.

Further developments include more efficient amalgamation of A-T and frequency analysis. Also, applications of voice recognition are to be both simulated on the computer and demonstrated.

Our conclusions were that your system, using A-T graphs, was accurate up to ten words, at which point it became almost useless. Further, it was concluded that any real recognition system would have to have frequency analysis of some design. It was also concluded that our second method of A-T analysis (the column then the rows) was the most reliable of the three.

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Note: Accompanying appendices showing test results and sample screen output were included in the original report.

(For this project, Kevin Greer and Suresh Pereira were awarded the CAA prize in Acoustics at the 1989 Canada-Wide Science Fair. Both students have recently completed their first year with "A" standing at the University of Waterloo in Computer Science and Physics, respectively.)

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