BAGPIPES A PROGRAM FOR ANALYZING ACOUSTIC TRANSMISSION IN DUCTWORK

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Introduction

The finite element method provides an effective means for modelling acoustic propagation in HVAC systems and pipe networks. However, due to extensive lengths of ducting and the degree of mesh refinement required to model high frequency sound transmission, resulting systems of equations can be very large. It is essential to reduce their size so that computer memory limits are not exceeded and computation time is reduced.

In BAGPIPES, system equations are generated automatically in the form of duct superelements. During generation, the superelements are formed by successively enclosing layers of elements, so that the complete superelement is described by only the number of nodes needed to describe a single layer. Consequently the interior nodes, which are not required for superelement conductivity, do not add to the size of the system. Limits on the model size arise from the number of superelement nodes required for connectivity.

Once the superelements have been assembled, nodes which are not required for the application of boundary conditions and sound sources, or for the solution output are condensed from the system. The result of these reductions in savings in run time and the ability to model far larger systems than would otherwise be possible within the confines of a personal computer.

The program has been used to model several fittings commonly found in HVAC systems. In addition, a network including a plenum chamber was modelled and results compared with a complementary experimental study of the acoustic transmission through the network.

Theory

This section contains an outline of the finite element acoustic theory utilized in the program. The matrix methods for substructuring and application of boundary conditions are described.

The finite element equations for modelling an acoustic volume have been derived previously and are given by Craggs [1]. At frequency λ the nodal system of equations has the form

$$([K] - \lambda^2[P])\{p\} = \{Q\}$$

where

[K] and [P] are square symmetric matrices derived from the kinetic and potential energy, respectively

{p} and {Q} are vectors containing the nodal pressures and volume source terms, respectively.

In the following the frequency dependent coefficient matrix ({K} - λ^2[P]) will be represented by the single stiffness matrix [S].

Substructuring

When the nodal pressure and sources at interior nodes in a length of duct may be eliminated from the system, the duct length can be modelled as a single "superelement". The advantage of this approach is that the superelement has only as many nodal variables as the number of nodes lying on the end planes of the duct.

In addition, for prismatic ducts where the geometric changes are one dimensional, the duct superelement can be generated by assembling an element representing a single layer of the duct and cascading this layer sequentially n times to generate 2^n layers. This applies to duct segments consisting of straight sections or arc sections. When the layers differ, a new layer must be formed and cascaded with the previous assemblage each time. This applies to sections containing bends or where the dimensions of the layers parallel to the duct axis is non-uniform.

The procedure is summarized as follows. For two duct segments A and B the system of nodal equations are:

$$\begin{bmatrix} S_{11}^A & S_{12}^A \\ S_{21}^A & S_{22}^A \end{bmatrix} \begin{Bmatrix} p_1^A \\ p_2^A \end{Bmatrix} = \begin{Bmatrix} Q_1^A \\ Q_2^A \end{Bmatrix}$$

$$\begin{bmatrix} S_{11}^B & S_{12}^B \\ S_{21}^B & S_{22}^B \end{bmatrix} \begin{Bmatrix} p_1^B \\ p_2^B \end{Bmatrix} = \begin{Bmatrix} Q_1^B \\ Q_2^B \end{Bmatrix}$$

Where the subscripts 1 and 2 denote nodes on the endplanes and p_2^A is synonymous with p_1^B and Q_2^A with Q_1^B.

The system of equations representing the coupled model is

$$\begin{bmatrix} S_{11}^A & S_{11}^A & 0 \\ S_{21}^A & D & S_{12}^B \\ 0 & S_{21}^B & S_{22}^B \end{bmatrix} = \begin{Bmatrix} p_1^A \\ p_2^A \\ p_1^B \end{Bmatrix} \begin{Bmatrix} Q_1^A \\ Q_2^A \\ Q_1^B \end{Bmatrix}$$

where $D = S_{22}^A + S_{11}^B$

Solving for p_2^A from the second row

$$p_2 = -D^{-1}S_{21}^A p_1^A - D^{-1}S_{21}^B p_2^B$$

and eliminating p_2 from the third row results in the condensed system

$$\begin{bmatrix} S_{11}^A - S_{12}^A D^{-1} S_{21}^A & -S_{12}^A D^{-1} S_{12}^B \\ -S_{21}^B D^{-1} S_{21}^A & S_{21}^B - S_{21}^B D^{-1} S_{12} \end{bmatrix} \begin{Bmatrix} p_1^A \\ P_2^B \end{Bmatrix} = \begin{Bmatrix} A_1^A \\ Q_2^B \end{Bmatrix}$$

or

$$[S^*]\{p\} = \{Q\}$$

If the layers are all equal the procedure is

1. Form $[S^A] = [S^B]$
2. Form $[S^*]$ n times, each time setting $[S^A] = [S^*]$ to form a superelement consisting of 2^n layers.

If the layers differ the procedure is, for n-1 layers

1. Form $[S^A]$
2. Form $[S^B]$

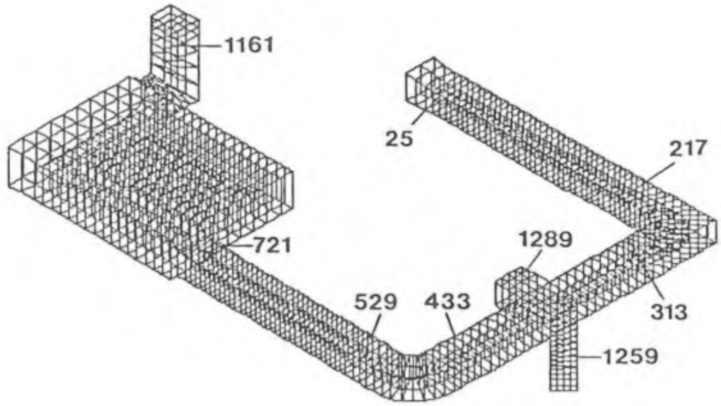
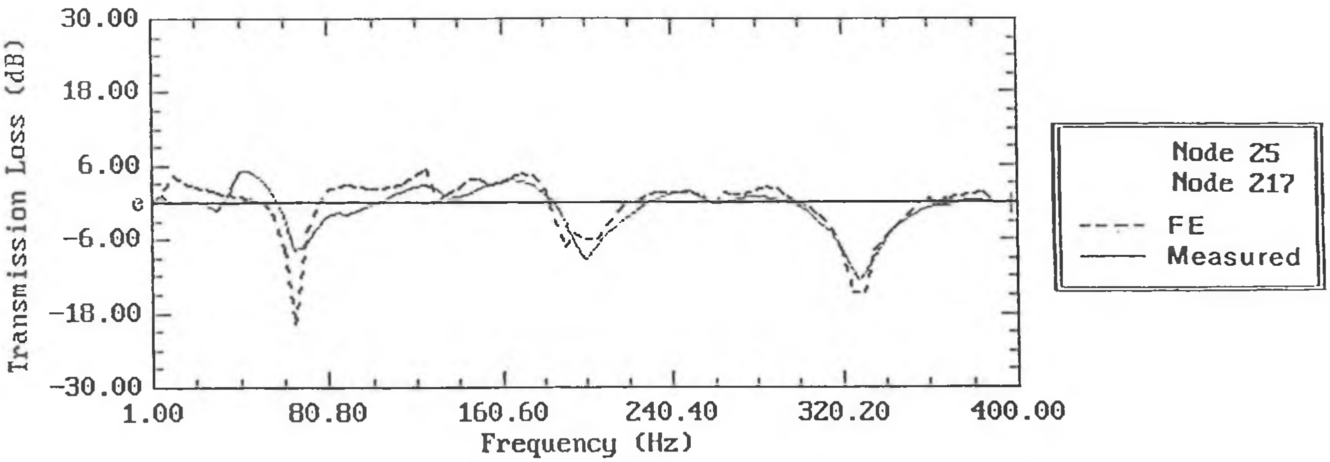
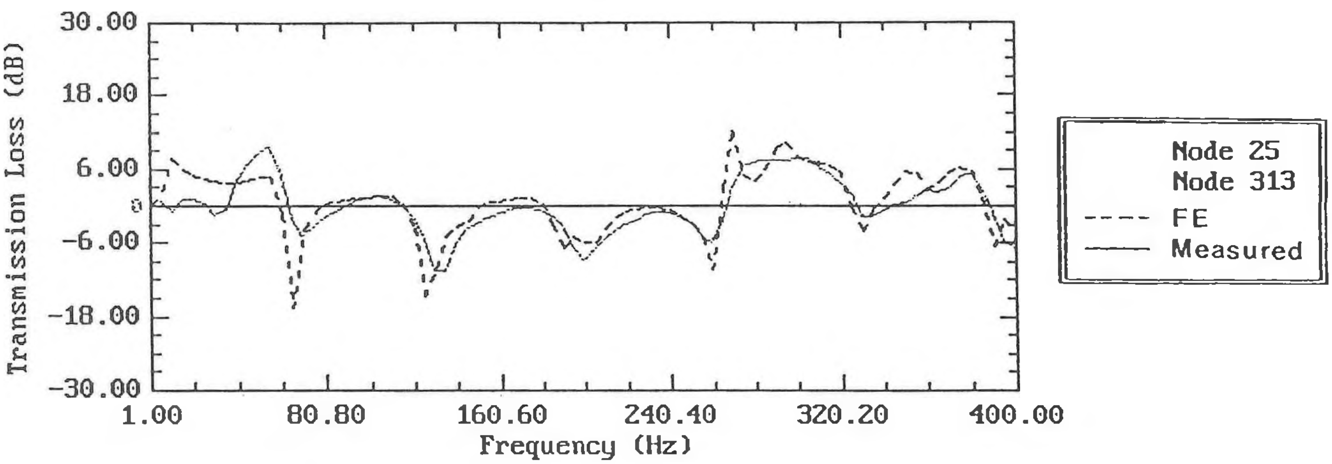


Figure 1





3. Form $[S^*]$ and set $[S^A] = [S^*]$

4. Form $[S_B]$

These procedures allow very large systems to be represented without having to generate the huge matrices which would be associated with the standard finite element methods.

Experimental Validation of Program

A three dimensional duct rig was specially built in order to provide test data to compare with the computer model results. The right was modular in construction so that bends, junctions and plenum chambers could be tested separately. A complete network finite element model is shown in Figure 1. Results for the transmission loss at various points in the network are shown. There is very good agreement.

References

Craggs, A. (1972) "The Use of Simple Three-Dimensional Acoustic Finite Elements for Determining the Natural Modes and Frequencies of Complex Shaped Enclosures", Journal of Sound and Vibration, 23(3), p. 331-339.

Craggs, A. and Fuchshuber, P.J. (1990) BAGPIPES, A Finite Element Program for Pipe Acoustics", Dept. of Mechanical Engineering Report #75, University of Alberta.