Numerical Methods for Solving Acoustic Radiation Problems

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1. Introduction:

In solving acoustic radiation problems, different numerical methods can be used. The conventional finite element (CE) method can model the near field if proper boundary conditions are applied, simulating a region extending to infinity. The so-called Sommerfeld radiation condition, simulating an outgoing travelling plane wave, can be appropriately imposed at a large distance from the radiating body.

Often though, one is interested in the acoustics of the far field. For this, boundary element (BE), infinite element [1] and infinite wave envelope (WE) element formulations may be used. The BE method requires only a discretisation of the sound radiating boundary and allows one to calculate the acoustic variables at an arbitrary field point. A major disadvantage however, is that the formulation yields a full complex system of non-symmetric matrices as opposed to the banded symmetric FE matrices, resulting in higher computing times and data storage problems.

Both the infinite and infinite wave envelope element are special elements that are matched onto a conventional finite element mesh modelling the near field. The infinite element is a conventional finite element where the shape functions are modified by adding a wavelike variation $\exp(-ikr)$ and where the computational domain in the radial direction is mapped to infinity. However because of the exponential factor, special integration procedures are needed, other than the Gauss Legendre quadrature formula, in calculating the system matrices.

The infinite wave envelope element differs from the infinite element in the use of the complex conjugate of the shape function as a weighting function in a modified Galerkin procedure [2]. Therefore the exponential factor vanishes, allowing the use of the Gauss Legendre quadrature formula in the evaluation of the appropriate system terms. A disadvantage is that the symmetry of the banded system matrices is destroyed.

In the following work, an n^{th} order infinite wave envelope element will be presented. With this element, an arbitrary number of acoustic degrees of freedom can be specified. Results from both 2D and axisymmetric models are presented.

2. Formulation:

In Figure 1, the geometry mapping of the infinite wave envelope element is shown for 2D and axisymmetric problems. Mapping functions $M_i(s,t)$ are defined [3] which map the unbounded real element onto a unit parent element, such that the inverse mapping along each infinite radial edge yields:

$$t = 1 - \frac{2a_i}{r} \qquad i = 1,2$$

Where a_i indicates the distance from the element to the source of the outgoing travelling wave.

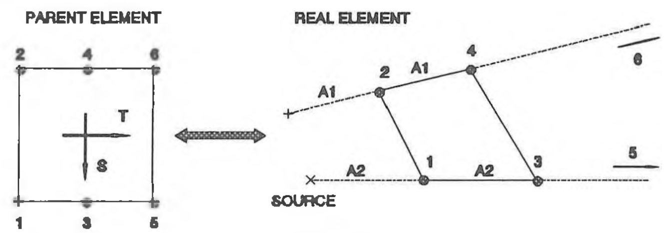

Figure 1 Geometry mapping of the WE element.

The shape functions are chosen to be linear in the angular s-direction $S_i(s)$, while in the radial r-direction a shape function of order n can be specified. These n^{th} order functions are generated in the parent element by adding acoustic nodes between the geometry nodes, 1-3 & 2-4, and forming the Lagrangian polynomials $T_i(t)$ for the respective nodes. Due to the geometry mapping a n^{th} order polynomial in the parent element will render a function of the form:

$$\frac{a_1}{r} + \frac{a_2}{r^2} + \ldots + \frac{a_n}{r^n}$$

in the radial direction of the real element, which appropriately models the amplitude decay of an outgoing travelling wave in 3D. Since the amplitude decay in 2D is approximately $1/\sqrt{r}$, the radial portion of the shape function is corrected by a factor of $\sqrt{r}$ [1].

The total n^{th} order shape function then becomes:

$$N_i(s,t) \quad = \quad S_i(s)\ T_i(t)\ \exp\left[-ik\,a(s,t)\frac{1+t}{1-t}\right]$$

Finally by using the complex conjugate of these shape functions as weighting functions, the acoustic mass and stiffness matrices can be evaluated following the conventional finite element procedures.

3. Discussion and Results:

As a first example, an axisymmetric model of scattering of a plane acoustic wave from a rigid sphere is studied. The left side of Figure 2 shows a mesh using only 20 wave envelope elements, while on the right a fine mesh is shown using 6 additional layers of conventional finite elements. For comparison, an equivalent BE mesh is indicated by the thick nodes on the left hand mesh.

In Figure 3, results are presented by means of a polar plot of $|P_s|/|P_i|$ at a distance of $r = 5a$. These are shown together with the analytical and BE (20 linear elements) solution. The 2nd order WE element, used in the coarse mesh (i.e. with no conventional acoustic elements), is able to model the correct general shape of the directivity pattern, but the magnitude is not very accurate in some regions. The 20 node boundary element model is slightly more accurate than the low order WE model, but it should be noted that the WE calculation time was 25 times faster

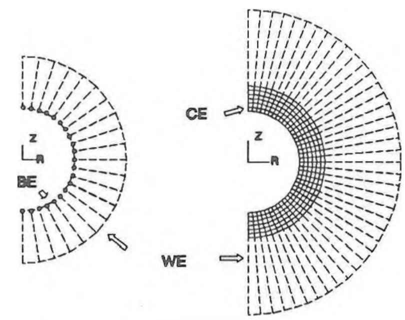

Figure 2: Axisymmetric meshes for rigid sphere.

than that for the BE solution.

Using the fine mesh, it is seen that the 5th order WE element solution matches the analytical solution very well, and was still 2.5 times faster than the BE solution.

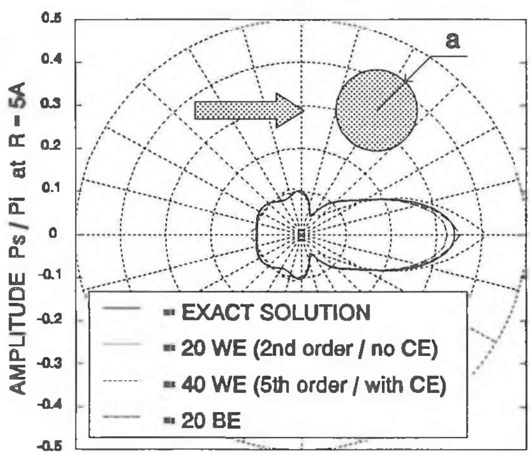

Figure 3: Scattering of an acoustic plane wave from a rigid sphere @ka = 4.

In a second example, the 2D acoustic pressure field generated by simple radiator is presented. The calculational mesh is shown in Figure 4 along with the superimposed real velocity boundary condition on the central panel. The panel is of length a, and the calculational frequency was specified at $ka = 10\pi/3$.

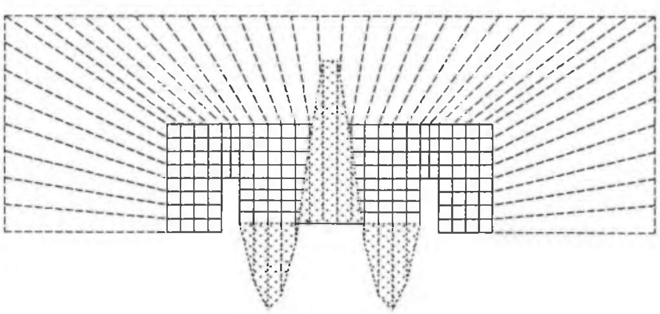

Figure 4: Simple radiator: WE mesh and velocity b.c.

The contour lines of constant pressure magnitude are shown in Figures 5 and 6 for the BE method and 7th order WE element calculation respectively. Very good comparison is obtained between the two pressure patterns, but again the WE calculation shows great speed; almost 5 times faster than the BE method.

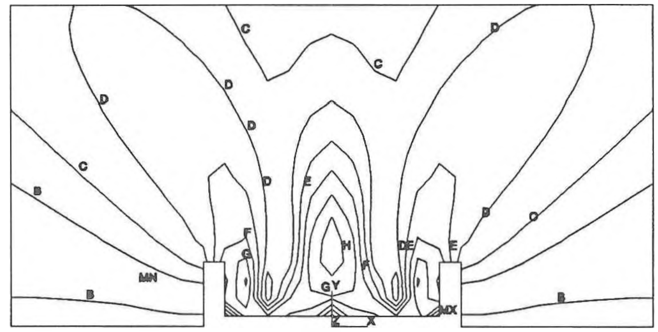

Figure 5: Simple radiator: BE solution.

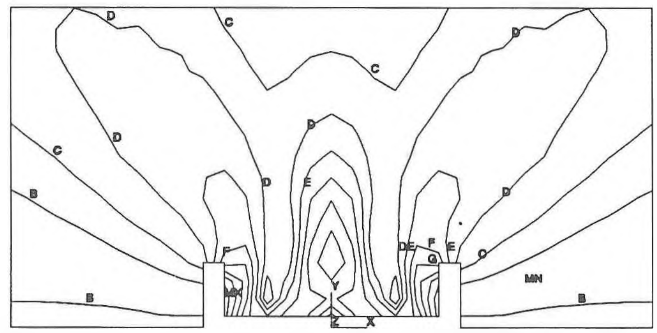

Figure 6: Simple radiator: 7th order WE solution.

4. Conclusions:

The infinite wave envelope formulation has been shown to be able to accurately model different types of acoustic radiation problems. The large calculation speed advantage over the BE method has been demonstrated. In a WE solution, normally both CE and low order WE elements are used for high accuracy solutions. Moving to higher order elements allows the element to better model the acoustic near field, and in some cases the need for CE is eliminated.

5. Acknowledgement

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6. References:

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