Simulation of the acoustic radiation emitted by vibrating structures

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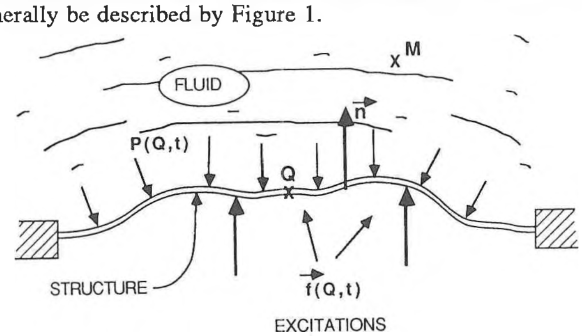
1. Introduction

Challenge of noise control for the nineties will be to leave out trial and error methods which have proven to be efficient only in specific and well known situations in order to tackle the difficult and complex, but inevitable, problem of structural acoustic and vibrations.

In order to control noise at source during the designing phase, a radiation model of a semi-complex structure has been developed. Based on analytical approach using a variational method, this model allows to predict the effects of the boundary conditions [J. Acoust. Soc. Am., 88(6), 1990] as well as stiffeners and added masses with force or moment type of excitation. To meet industrial applications, calculations have been extended to mechanical sources of vibrations (electric motor, engine) which are attached to large thin structure and act as noise radiator. A theoretical analysis of the problem is presented. The actual force input into the structure is the boundary result of both the output impedance of the source and the input impedance of the structure. A quadripole approach for the source assembly enables to calculate the force input, the kinetic energy, the radiation efficiency and the overall sound power. The key novelty of this method lies in its capacity to predict, a priori, the radiated noise in various configurations allowing the designer to rationally choose the best configuration in terms of noise control. Some applications will be presented. In general, guidelines for optimal acoustic conception, at designing stage, will be suggested.

2. Basic theory

Structural acoustic and vibration behaviors of a structure can generally be described by Figure 1.



W(Q,t) = normal displacement of the structure
P(M,t) = acoustic pressure

Figure 1. Main parameters in structural acoustic and vibration problems

Usually this fluid structure interaction problem is described by the following equations:

For the structure
- Dynamic response

$$\rho_s \frac{\partial^2 w}{\partial t^2}(Q,t) + L_e(w(Q,t)) = \vec{f}(Q,t) \bullet \vec{n} - p(Q,t) \quad (1)$$

L_e : Space differential operator $\vec{f}$: Excitations p: Fluid loading

- Boundary conditions (2)

For the fluid
- Wave equation

$$\nabla p^2(M,t) - \frac{1}{C_o^2}\frac{\partial^2 p}{\partial t^2}(M,t) = 0 \quad (3)$$

- Sommerfeld condition (4)

$$-\rho_o \frac{\partial w}{\partial t}(Q,t) = \vec{\nabla} P(Q,t)\,\vec{n}(Q,t) \quad (5)$$

with a $e^{-j\omega t}$ frequency dependance the (3), (4), (5) lead to (6)

$$p(M) = -\rho_0 \omega^2 \int_{(s)} G(Q',M)\, w(Q')\, dQ' \quad (6)$$

which introducing (6) into (1) bring to:

$$-\rho_s \omega^2 w(Q) + L_e(w(Q)) = \vec{f}(Q)\vec{n} + \rho_o \omega^2 \int_{(s)} G(Q',Q)\, w(Q')\, dQ'$$

The displacement of a given structure, in the presence of the fluid, is the solution of integro-differential system.

Under this form one may easily understand why so many approaches tend to simplify the problem. It is not an easy one. However, if one wants to solve it as rigorously as possible, there will be essentially two choices:

3. Generalized analytical approach

The method used to obtain the structural acoustic and vibration behaviors of the plate is based on a variational approach with a discretisation of the solution which is using polynomial function such as

$$\{\phi_m(x) \bullet \phi_n(y)\} = \left\{\left(\frac{2x}{a}\right)^m \left(\frac{2y}{b}\right)^n,\ \begin{matrix} n = 0 \ldots N; \\ m = 0 \ldots N \end{matrix}\right\}$$

The system of equations to be solved are the following
- Equation of motion of the structure
- Boundary conditions along structure contour
- Equation of motion of the surrounding acoustic media
- Sommerfeld conditions

This leads to a linear system which is

$$\{-\Omega[M_{nmpq}] + [K_{nmpq}]\}\{a_{nm}\} = \{F_{nm}\} + j\,\Omega[Z_{nmpq}]\{a_{nm}\}$$

where Ω is the pulsation, M_{nmpq} the generalized mass, K_{nmpq} the generalized stiffness, F_{nm} the mechanical forces, and the last term the acoustic pressure $\{a_{nm}\}$ are the unknown modal coefficient of displacement. This acoustic pressure is calculated with the Rayleigh integral. More details are given in Berry's paper [1]. It is worth noticing that the variationnal approach permits to add several degrees of complexity on the structure. The most important features are described in figure 2.

The coupling between the source excitation and the structure is made through a similar approach of the one used by Snowdon. This allows to take into account the interaction between the impedance of the input system and the local mechanical impedance of the plate.

4. Results

The modal permits a very interesting and meaningful parametric study. The key informations are the force transmissibility, the quadratic velocity, the radiation factor and the overall acoustic power. The type of excitation can vary, the plate can have any type of boundary conditions with added mass and added stiffeners. To show the importance of the coupling interaction between the excitation source and the structure, a basic example is given in figure [3]. One could see that classical force transmissibility described in textbook with the hypothesis of an infinite mechanical impedance of the supporting structure (fig. 3.a) is far more different than the actual value transmissibility (fig. 3.b). More complex phenomena will be exposed in the presentation.

5. Conclusions

A general analytical approach describing the vibro-acoustic behavior of a semi-complex system has been developed. It allows to understand the physical phenomema involved, gives quantitative results and reveals a good tool for engineers who want to make acoustic design a priori.

6. Bibliography

[1] A. Berry, J.-L. Guyader and J. Nicolas, "A general formulation for the sound radiation from plates with arbitrary boundary conditions", J.A.S.A., 88(6), 1990.

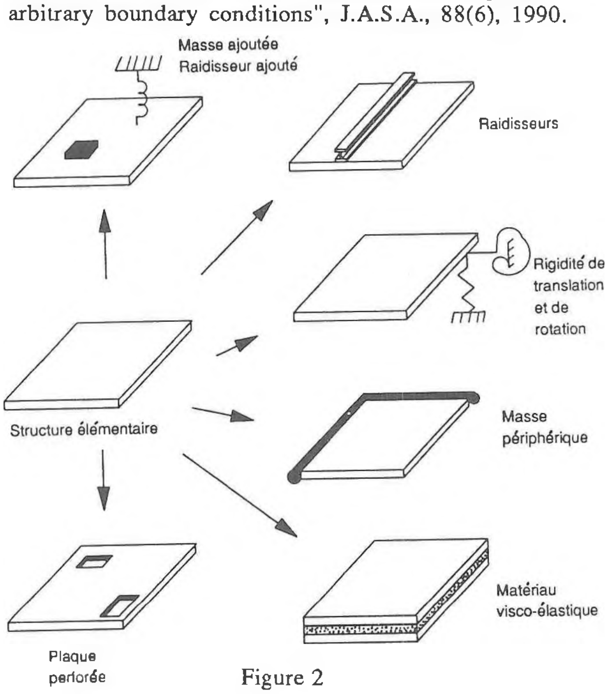


Figure 2

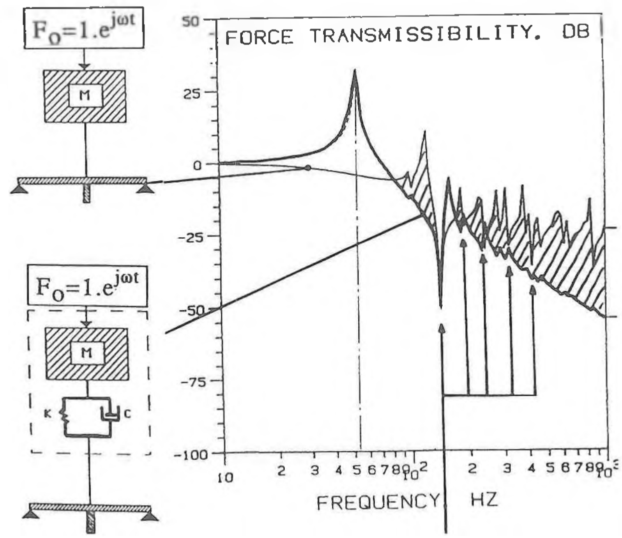


Figure 3.a: Force transmissibility with a rigid structure (infinite impedance)

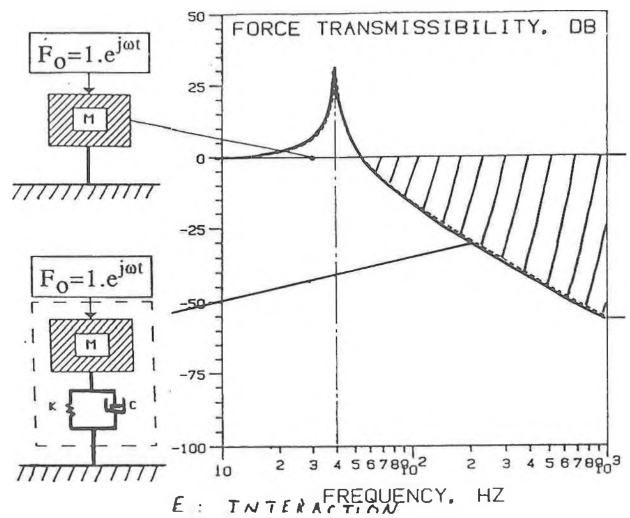


Figure 3.b: Force transmissibility with structure interaction (finite impedance)