EXPERIMENTS ON ACTIVE POWER MINIMISATION

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1. INTRODUCTION

Active control of sound has been proven as a viable method of controlling low frequency, tonal noise in enclosures[1,2]. This method of sound control has thus far relied on a strategy of direct minimisation of the acoustic potential energy present in the enclosure. This paper suggests an alternative approach to active control. Here we consider the power outputs of sources and attempt to manipulate them to produce a desired result.

Because of their nature, these two strategies require different types of information to achieve control. The direct minimisation of energy requires information about the field as a whole using an array of microphones, typically sixteen to thirty two in number. The minimisation of total power output, however, is concerned with information purely local to the sources themselves (i.e., pressure and source strength) and does not concern itself with information general to the field. Although these approaches appear different in nature, they produce very similar results.[3]

2. THEORY

2.1 Power Outputs of Sources

The power output of a harmonic point monopole source is given by $W = \frac{1}{2}\mathrm{Re}\{q^*(\omega)p(\omega)\}$ where $q(\omega)$ is the complex strength of the source, $p(\omega)$ is the complex acoustic pressure at that source and Re denotes the real part of the bracketed quantity. If we consider a single channel active control system, i.e., one primary source and one secondary source, we can define the pressures at these sources in terms of their source strengths and transfer impedances, as

$$p_p = Z_{ps}q_s + Z_{pp}q_p \tag{1}$$

$$p_s = Z_{sp}q_p + Z_{ss}q_s \tag{2}$$

where q_p and q_s are the source strengths of the primary and secondary, Z_{pp} and Z_{ss} are the radiation impedances of the primary and secondary, respectively, and Z_{sp} and Z_{ps} are the transfer impedances between the sources ($Z_{sp} = Z_{ps}$ due to reciprocity).

We can combine these equations with that for the power to give expressions for the power outputs of the two sources in terms of their source strengths and their radiation and transfer impedances. The impedances are split into their real and imaginary components such that Z = R and jX:

$$W_p = \frac{1}{2}[|q_p|^2 R_{pp} + \frac{1}{4}q_s^* Z_{sp}q_p + \frac{1}{4}q_s Z_{sp}q_p^*] \tag{3}$$

$$W_s = \frac{1}{2}[|q_s|^2 R_{ss} + \frac{1}{4}q_s^* Z_{sp}q_p + \frac{1}{4}q_s Z_{sp}q_p^*] \tag{4}$$

The total power of the two sources can now be written as output:

$$W_T = \frac{1}{2}[|q_p|^2 R_{pp} + |q_s|^2 R_{ss} + q_s^* R_{sp}q_p + q_s R_{sp}q_p^*] \tag{5}$$

2.2 Optimal Secondary Source Strength

To actively control the sound field, we vary the strength of the secondary source to minimise the total power output. The optimal values of secondary source strength to produce this result is given by setting the differentials of W_T with respect to the real and imaginary parts of q_s to zero, which gives[3]

$$q_s = q_{s0} = \frac{-q_p R_{sp}}{R_{ss}} \tag{6}$$

If we substitute this optimal value of q_s (equation (6)) into the expression for secondary power (equation (4)), we arrive at a very interesting result. That is, under these conditions the secondary power output becomes exactly zero.[3]

$$\text{If } q_s = q_{s0}, \quad W_s = 0. \tag{7}$$

There are an infinite number of other combinations of the real and imaginary parts of the secondary source strength ($q_s = q_{sr} + jq_{si}$) which also produce zero power output of the secondary source in the presence of the primary source. If q_p is assumed real, the values of q_{sr} and q_{si} which satisfy the condition $W_s = 0$ can be written, from (4), as

$$q_{sr}^2 + q_{si}^2 + q_{sr}\left(\frac{q_p R_{sp}}{R_{ss}}\right) + q_{si}\left(\frac{q_p X_{sp}}{R_{ss}}\right) = 0 \tag{8}$$

which is plotted in Figure 1. The centre and radius of this circular contour are the values of q_s which maximise the power *absorption* of the secondary source:[3] $q_{sa} = -q_p Z_{sp}/2R_{ss}$.

It is important to note here that the secondary source strength which minimises total power output q_{s0} is entirely out of phase with the (real) primary source strength q_p, equation (6). This value of q_s must thus be given by the non-trivial intersection of equation (8) with the q_{sr} axis, as illustrated in Figure 1. If q_s is thus arranged to be out of phase with q_p and its source strength gradually increased, it initially absorbs sound power. If the magnitude of q_s is further increased, the amount of absorbed power is decreased until its power output is exactly zero when $q_s = q_{s0}$, in which case the total power output of the two sources is optimally reduced.

3. EXPERIMENTS

The experiments were conducted with a pure tone excitation frequency of about 90 Hz in an enclosure of approximate dimensions 6 m × 2.2 m × 2.2 m. The volume velocity of two loudspeakers acting as the primary and secondary sources were monitored using the method described in reference 4. The primary source was driven at a constant level and the phase of the secondary source adjusted to be 180° with respect to that of the primary. The power output of the two sources was measured by monitoring the average product of their volume velocity with their near field pressures.[4] The measured values of W_s and W_T as the secondary source amplitude was gradually increased are shown in Figure 2, together with J_p, the level of the sum of the squared outputs of 32 monitor microphones also deployed in the enclosure. It can be seen that W_s initially is negative but then rises to a positive value, as expected. The value of q_s for which $W_s = 0$ corresponds closely to the minimum in the measured values of W_T, confirming the prediction above. This minimum in total power output also coincides with the minimum in the mean square pressure in the enclosure J_p indicating that minimising W_T does achieve global control, even though only measurements local to the secondary source are used in achieving this minimum.

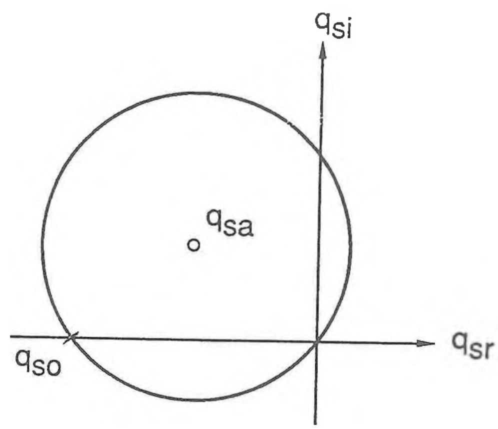

Figure 1. Locus of real and imaginary components of the secondary source strength (q_{sr} and q_{si}) which cause its power output to be zero in the presence of a primary source.

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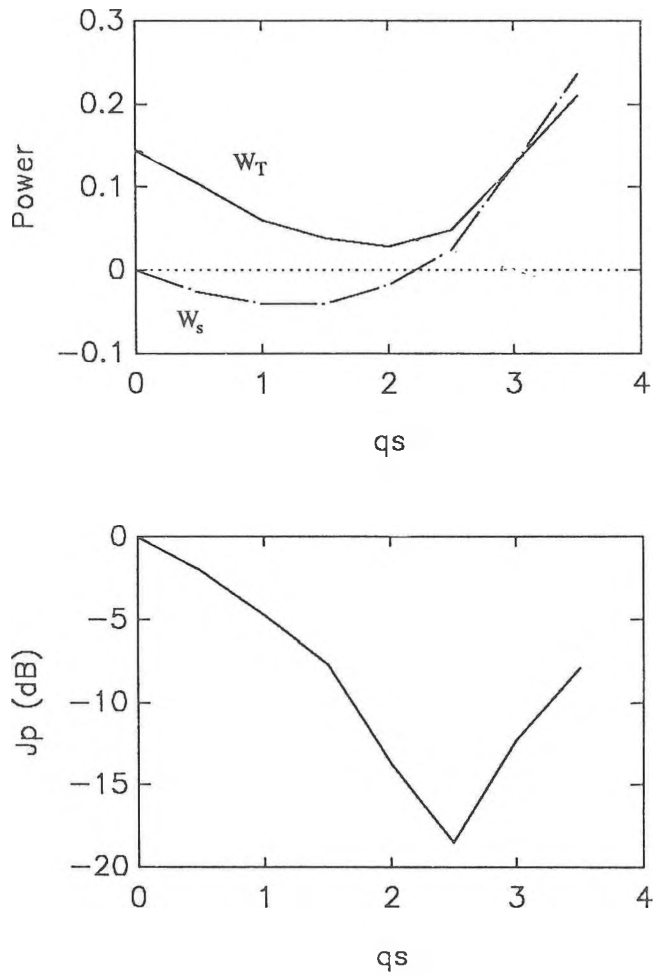

Figure 2. Power output of secondary source W_s (-·-·) total power output of primary and secondary sources W_T (___), and mean square pressure J_p in the experimental enclosure as the secondary source strength was increased.