SYMPHONIC BELLS OF 'FANTASTIC' PROPORTION

Daryl Caswell
University of Calgary
Mechanical Engineering Dept.
2500 University Dr. N. W.
Calgary, Alberta

1. THE PROBLEM:

Producing the sound of large church bells in an orchestral setting is often referred to as the last unsolved problem of the orchestral percussionist. The problem is as old as the symphonic orchestra and has resulted in many amusing and often musically disastrous attempts at finding a solution. Among the more successful attempts (ranging from an eight foot long tubular chime played from a step ladder, to throwing open the back door of the concert hall and playing the local church carillon at the prescribed moment in the score) is the use of flat metal plates. The disadvantages of using metal plates to imitate the sound of church bells centre on the difficulties encountered in accurately predicting the natural frequency of the plate, achieving sufficient amplitude and avoiding unharmonious overtones.

2. THE USUAL SOLUTION:

The free vibration of plates under every conceivable method of support has been the subject of countless publications and research projects in the last 100 years. [ref. 1-8] Mathematical analysis requires the solution of the differential equation of motion:

$$D\nabla^4\omega + \rho\frac{\partial^2\omega}{\partial t^2} = 0$$

where ω is the transverse deflection and ∇ is the biharmonic differential operator.

$$D = \frac{Eh^3}{12(1-\upsilon^2)}$$

is the flexural rigidity where E is Young's modulus; h is the plate thickness; υ is Poisson's ratio. Mass density per unit area of plate is represented by ρ and t is time.

At first glance, the problem merely requires the researcher to plug the proper numbers into one of the many methods stemming from the work of Rayleigh and Ritz near the turn of the century. However, the results are unsatisfactory because of the following difficulties:

a. The theory assumes that the plate is thin. However, the fundamental frequency of a thin plate will not dominate acoustically when a freely suspended plate is struck. One or more of the harmonics will be louder than the natural or fundamental frequency.

b. There is no theoretical method to predict the strength of the sound and match it to that of a real bell.

c. While plate theory permits the determination of the frequency content of a given plate, the reverse is not true. Given the frequency content of a bell there is no means to determine the plate dimensions which will yield an approximation to the desired response.

d. Until recently, the freely suspended plate was subject to large errors in frequency analysis (>6%) because of the difficulty in correctly modelling the boundary conditions.

3. THE 'MUSICAL' SOLUTION:

Given the serious difficulties encountered in attempting a purely mathematical solution, I decided to develop an artistically driven method by allowing competent musicians with an extensive knowledge of the problem to guide the necessary experimentation. Foregoing the use of acoustical analyzers which are at best averaging instruments, the musicians verbally described the sound they were looking for and, with the help of our musically inclined shop technicians, isolated the most appropriate plate material, the optimum thickness for proper amplitude, and the approximate length to width ratio which seemed to give a bell-like frequency response.

Now it was possible to go back to the analyzers, the mathematics and the computer to determine what it was about the 'subjective' input of the musicians that resulted in the final choice of material, thickness and shape. Analysis showed that the solution existed in a very narrow window of dimension which lies between a thin plate and a thick plate as well as between a plate and a beam. The mathematical disadvantage is that the assumption of thinness is lost but this was compensated for by the fact that the plate could be treated as a beam as far as determining the fundamental frequency. This resulted in improved accuracy. Arising from this artistically driven analysis was the interesting observation that there is a critical length to thickness ratio below which the fundamental frequency of a plate will not dominate in the frequency response.

4. RESULTS:

Employing the information gained from quantifying the musical appropriateness of the chosen plate parameters, a set of bell plates has been produced for performances of Berlioz's 'Symphonie Fantastique' by the Calgary Philharmonic and the Vancouver Symphony. The final movement of the work requires the tolling of distant church bells calling the witches back from their revel. Further experimentation produced a gong-like sound for the Calgary Opera's production of Puccini's 'Madame Butterfly' as well as the church bell signalling the lover's meeting time in Verdi's Falstaff.

5. REFERENCES

1. **Gorman, Daniel J.** 1982 'Free Vibration of Rectangular Plates' (Elsevier, New York).
2. **Leissa, A. W.** 1969 NASA SP-160 'Vibration of Plates'.
3. **Leissa, A. W.** 1973 Journal Of Sound And Vibration 31(3), 257-293. The Free Vibration Of Rectangular Plates.
4. **Rayleigh, Lord** 1894 'Theory of Sound', vol. 1, second edition (Macmillan and co., London).
5. **Ritz, W.** 1909 Annalen der Physik, fourth series, vol.28, p. 737, 'Theorie der Transversalschwingungen einer quadratischen Platte mit freien Randern'.
6. **Timoshenko, Stephen, D. H. Young, and W. Weaver,** 1990 'Vibration Problems in Engineering', fifth edition (Wiley, New York).
7. **Waller, Mary D.** 1961 'Chladni Figures, A Study in Symmetry' (G. Bell and Sons Ltd., London).
8. **Warburton, G. B.** 1954 Proceedings of the Institute of Mechanical Engineers, ser. A,168,371-384. The Vibration Of Rectangular Plates.