DIGITAL GENERATION OF THE HIGH QUALITY PERIODIC AUDIO SIGNALS WITH THE AID OF A D/A CONVERTER AND COMPUTER

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Very often in many areas of acoustics, sound engineering, psychology, psychoacoustics, audiology etc. there is a need for high quality sinusoidal, square wave, series of clicks and other types of periodic signals.

A computer - based digital function generator can generate any arbitrary type of signal in the frequency range from 0 Hz to 20,000 Hz, S/N = 95 dB and with no harmonic or intermodulation distortion (when based on 16 bit, 44.1 kHz D/A converter) . Frequency stability is determined by a quartz clock in the D/A converter which has an accuracy in the order of $1/10^6$.

Methods for sinusoidal signal synthesis

Generation of sinusoidal waves is of primary importance, since due to the Fourier theorem any periodic wave may be synthesized via additive synthesis of pure tones with appropriate amplitudes and phases. There are many alternative methods to digitally generate pure tones [1,2,3].

Very often real-time synthesis is accomplished on DSP (Digital Signal Processing) chips by using a sine function look-up table stored in the internal ROM (Read Only Memory) [2]. A limitation to this approach is the short length of the sine table (N).
In order to synthesize any arbitrary frequency using the look-up table method, one has to synthesize the values of the sine function using an interpolation process [2]. With linear interpolation between sine samples, this leads to S/N=64 dB for N=64, S/N=76 dB for N=128 and S/N=88 dB for N=256 due to the generation of THD by this algorithm [2]. Also this noise has an undesirable nature, since it is coherent with the generated signal (higher harmonics are generated) [5]. The only way to improve this situation is to use a longer look-up table, or by the filtering of generated signal in the analog or digital domain (which could jeopardize the real-time performance).

Synthesis of more complex signal in real time (combinations of several sinusoids with certain amplitudes and phases) could put too much performance demand on the computer's DSP chip or microprocessor.

RAM-based method for sinusoidal signal synthesis

Another way to generate sinusoidal signal is to generate audio file, which contains appropriate samples in RAM (Random Access Memory) or on the Hard or Optical Disk. Then reading of this file in real time to the D/A converter is performed to generate the desired audio signal [6]. However there is a limit on the duration of this signal due to memory consumption = 88,200 Bytes/sec (for 16 bit, 44.1 kHz D/A converter).

In many situations long durations of signal are required. In this case the most appropriate way to generate sine wave is to construct the audio file in RAM in such a way as to be able to read this file to the D/A converter over and over again (looping). To accomplish seamless (or click-less) looping through the file, the fundamental period of the sine wave must be:

$$T_o = N \cdot T_s \quad \text{where: } T_s = 1/f_s \text{ - sampling interval}, \; N = 3,4,5,\ldots\text{etc- length of RAM buffer} \tag{1}$$

Fundamental frequency (first harmonic) generated in this case is :

$$f_o = 1/T_o = 1/(N \cdot T_s) \tag{2}$$

Higher harmonics of fundamental frequency can be also generated:

$$f_m = m \cdot f_o = (m/N) \cdot f_s \quad \text{where: } m = 1,2,3,\ldots m_{MAX}, \; m_{MAX} = \mathrm{INT}(N/2), \; m\text{- harmonic number (for fundamental m=1)} \tag{3}$$

Formula for the n-th sample in the RAM buffer, to generate the fundamental or it's higher harmonic is given by:

$$A[n] = A_o \sin[(2 \cdot \pi \cdot m \cdot / N) \cdot n + \varphi_m] + \text{OFFSET} \tag{4}$$

where: A_o - signal amplitude (represented by real number)
$\pi = 3.14....$
$n = 0,1,2...(N-1)$
φ_m - arbitrary phase (any real number)
OFFSET - number representing offset required by D/A converter (OFFSET=32,767.5 for 16 bit D/A converter)

Sequence of samples going to D/A converter during generation of the sine wave in real-time is as follows:

(5)

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A[0], A[1],.....A[N-1], A[0], A[1],......A[N-1], A[0], A[1],.....etc
|-----------------------| |----------------------| |------------------
first loop                second loop              third loop  --- →
                                                               time
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Eq. (4) and (5) is the basic formula for generation of continuous sine wave.

Note 1: As can be seen from eq. (3) max frequency which could be generated is $f_m \le f_s/2$ (where $f_s/2$ is a Nyquist frequency determined by the sampling frequency f_s)

Note 2: Quantities in eq. (4) should be represented by numbers with adequate precision in order to obtain amplitudes A[n] which exceed the precision required by the D/A converter (for example 24 bit precision for 16 bit D/A converter). Rounding of the A[n] should be the last operation performed before storing them in RAM. Otherwise additional harmonic and nonharmonic (intermodulation) distortion will be generated due to the intermediate round-off errors.

Example: generation of frequencies 1Hz, 2Hz, 3Hz......20,000Hz

Assume that we have a computer with a 16 bit, 44.1 kHz D/A converter (CD-quality). If we take a RAM buffer size of N=44,100 (88,200 Bytes); then from formula (3) we can see that the available frequencies are:

$$f_m = m \ [\text{Hz}] \quad \Rightarrow \quad f_1 = 1\text{ Hz}, \; f_2 = 2\text{ Hz}, \; f_3 = 3\text{ Hz}, \; \ldots, \; f_{20,000} = 20,000\text{ Hz} \tag{6}$$

Note 1: Precision of these frequencies is determined only by the precision of the D/A converter's clock which is in the order of $1/10^6$, and in the case of a 1000 Hz tone this leads to an expected error in the order of ± 0.001 Hz.

Note 2: With RAM buffer size N=441,000 (882,000 Bytes), one can generate all frequencies from 0.1 Hz to 20,000 Hz with 0.1 Hz resolution.

Note 3: Other choices of N and m parameters in eq. (3) will give desired combinations in most practical applications.

Note 4: Generated files with desired frequencies could be stored on the Hard or Optical disk and recovered into RAM prior to generation of sine tone.

This algorithm was the basis for a successful implementation on the AMIGA Computer [6], and is being implemented on the NeXT Computer [7].

Improving Generator through the use of Digital Dither

The only harmonic and intermodulation distortion generated by the above described algorithm is associated with round-off error of final amplitudes A[n] before storing them in RAM (in order to generate integer numbers used by D/A converter - from 0 to 65,535 for 16 bit D/A). However as mentioned previously, due to the signal correlated nature of noise, higher harmonics and intermodulation products with the sampling frequency are generated [4,5].
Elimination of the harmonic and nonharmonic (intermodulation) distortion (which are imposed by the resolution of the D/A converter) may be accomplished by using Digital Dither, with a small white noise penalty (3 dB- for uniform-pdf dither) [5,8], resulting in S/N=95 dB for 16 bit dithered digital generator (theoretical S/N=98.2 dB for undithered 16 bit digital system).
This could be done by adding dither to each amplitude before rounding the number and sending it to the D/A converter:

$$A_D[n] = A[n] + D[n] \tag{7}$$

where: A[n] - initial amplitude of sine wave before final rounding
$A_D[n]$- final amplitude of sine wave after adding dither
D[n] - dither amplitude

An example of optimal dither in this case is a sequence of random numbers in the range [-0.5; 0.5] with uniform pdf (probability density function). For other types of dither please see ref. [5,8,9,10].

Generation of Random Digital Dither:
On some hardware, dither words D[n] can be generated in real time. In this case unrounded amplitude A[n] is retrieved from RAM, added to real-time synthesized dither D[n], then rounded and send to the D/A converter. This is the preferable way to use Digital Dither.

Generation of Pseudo-Random Digital Dither:
In the case when the hardware can't generate dither words D[n] in real-time, one can use a random sequence D[n] of the same length as A[n], add them according to eq.(9), and store the rounded value of $A_D[n]$ in RAM for use by the D/A converter. This is not as an effective method as using random digital dither, but for a long enough RAM buffer (value of N), it can decrease the level of harmonic and nonharmonic (intermodulation) distortion [7].

Synthesis of more complex periodic Waveforms

According to the Fourier theorem any bandlimited, periodic function with fundamental period $T_0 = N \cdot T_s$,can be expressed in the form:

$$A_T[n] = \sum_{m=1}^{m_{MAX}} A_m[n] = \sum_{m=1}^{m_{MAX}} A_m \sin\left[\left(2\cdot\pi\cdot m/N\right) n + \varphi_m\right] + \text{OFFSET} \tag{8}$$

where: A_m - amplitude of the m-th component
φ_m - phase for the m-th component
A_T - total amplitude of a synthesized signal

- Formula (8) can be used to synthesize many different types of bandlimited and periodic waveforms. For example: combinations of sinusoidal signals, triangular wave, square wave, series of clicks, frequency modulated signal, amplitude modulated signal etc. However, the period of modulation function is restricted to available periods $T_m = 1/f_m$, given by eq. (3). Also none of the components in eq.(8) can exceed Nyquist frequency = $f_s/2$.
- Synthesis of any arbitrary bandlimited and periodic function can also be accomplished by using any arbitrary sequence of amplitudes A[0], A[1], A[2],......A[N]. There is $N \cdot 2^n$ of such waveforms for a n-bit D/A converter and N-samples long RAM buffer. For example, pseudo-random noise can be generated in this fashion (pseudo-random due to periodicity).
- Yet another way of synthesis is to take any periodic function (with period $T_0 = N \cdot T_s$) in analytic form A= A(t) and do digital sampling (sampling in the digital domain) according to the formula:

$$A[n] = A(t_n = n \cdot T_s) \tag{9}$$

However in this case one must be certain that the function A= A(t) is bandlimited, otherwise aliasing distortion will occur [4]. If the function A= A(t) is not bandlimited, digital filtering should be performed prior to applying formula (9) in order to eliminate all frequencies above Nyquist frequency = $f_s/2$

Generating stereo signals

The procedures outlined above may be applied for synthesis of multichannel periodic and bandlimited waveforms (for hardware with multiple D/A converters). For example, for stereo (2 channel) synthesis, one can generate different or similar waveforms in each channel by using formula (4) and (8). Phase relationship between waveforms will be maintained during playback. Since phases (φ_m) in formula (4) and (8) are arbitrary real numbers, arbitrary phase shifts between waveforms can be obtained. This could be important for some audiological and psychoacoustic tests [6,11,12].

Conclusions

The most precise digital-domain method for the generation of arbitrary periodic audio signals was presented. Advantages of this method are: highest available precision with a given D/A converter, and low requirement on the speed of hardware and software, because synthesis of waveform is not performed in real-time. Hardware and software only have to be capable of doing real-time looping through sound file generated in the RAM buffer. Details of the waveform synthesis should be sufficient for easy implementation of this algorithm on different computer platforms.

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