Predicting Acoustic Radiation from Coupled Fluid/Structure Systems : A Comparison of Two Computer Codes

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1 Introduction

Predicting the acoustic radiation arising from fluid/structure interaction can be a difficult problem particularly when the acoustic medium is a dense fluid. This area has been studied extensively at both Defence Research Establishment Atlantic (DREA) and Martec Limited (under contract to DREA) resulting in the development of two suites of computer codes for the prediction of radiated noise from vibrating submerged structures. Both use finite element methods to model the structure and either finite element or boundary element methods to model the surrounding fluid.

The program COUPLE [1, 2, 3] has been developed at DREA and is used in conjunction with the finite element analysis program, VAST (Vibration And STrength) [4], and the boundary element code, BEMAP (Boundary Element Method for Acoustic Prediction) [5]. The program AVAST (Acoustic VAST) has been developed at Martec Ltd. under contract to DREA and is also used with VAST.

2 Theory

COUPLE

In the COUPLE suite of programs, VAST is used to create the matrices for a finite element model of the structure, resulting in the structural dynamics equation

$$[\mathbf{K}-\omega^2\mathbf{M}]\,\{\delta\} = \mathbf{L}\,\mathbf{p} + \mathbf{f}_s \tag{1}$$

where $\mathbf{M}$ and $\mathbf{K}$ are the discretized mass and stiffness matrices, $\mathbf{L}$ is an interface matrix relating surface pressures to structural forces and $\mathbf{f}_s$ represents the externally applied structural loads. A finite element model of the fluid surrounding or contained in the structure is also generated using equations based on the classical wave equation in pressure. COUPLE assembles this fluid model into the fluid dynamics equation

$$[\mathbf{H}-\omega^2\mathbf{Q}]\,\mathbf{p} = \rho\omega^2\mathbf{L}^T\{\delta\} \tag{2}$$

where $\mathbf{H}$ and $\mathbf{Q}$ are the fluid 'stiffness' and 'mass' matrices, $\mathbf{p}$ is the fluid pressure vector, and $\{\delta\}$ is the structural displacement vector. COUPLE then provides the means to combine these two equations. When combined (with $\mathbf{f}_s$=0), (1) and (2) form the unsymmetric system:

$$\left[\begin{bmatrix} \mathbf{K} & -\mathbf{L} \\ 0 & \mathbf{H} \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{M} & 0 \\ \rho\mathbf{L}^T & \mathbf{Q} \end{bmatrix}\right] \begin{Bmatrix} \delta \\ \mathbf{p} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{3}$$

which must be symmetrized to maintain compatibility with the VAST eigensolver used. One symmetrical system available in COUPLE, which can easily be derived from the above equation, is:

$$\left[\begin{bmatrix} \mathbf{K} & 0 \\ - & \frac{1}{\rho}\mathbf{Q} \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{M}+\rho\mathbf{L}\mathbf{H}^{-1}\mathbf{L}^T & \mathbf{L}\mathbf{H}^{-1}\mathbf{Q} \\ - & \frac{1}{\rho}\mathbf{Q}\mathbf{H}^{-1}\mathbf{Q} \end{bmatrix}\right] \begin{Bmatrix} \delta \\ \mathbf{p} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{4}$$

This combined dynamics eigenproblem is output from COUPLE and solved using the VAST solvers. The resulting eigenvectors of the fluid/structure system, together with an applied dynamic load vector, are used by the VAST frequency response module to generate the surface velocities on the fluid/structure interface.

The COUPLE suite uses these surface velocities as input boundary conditions to the program BEMAP. BEMAP uses the Helmholtz integral relation,

$$\alpha p(q) = \int_S \left\{ p(\zeta)\frac{\partial G(\zeta,q)}{\partial n_\zeta} + i\omega\rho v(\zeta) G(\zeta,q) \right\} dS_\zeta \tag{5}$$

where G is the Green's function:

$$G(\zeta,q) = \frac{e^{ik(q-\zeta)}}{(q-\zeta)} \tag{6}$$

and

$$\alpha = \begin{Bmatrix} 4\pi & (q \text{ exterior}) \\ 2\pi & (q \text{ surface}) \\ 0 & (q \text{ interior}) \end{Bmatrix} \tag{7}$$

to predict the acoustic radiation from a vibrating surface, S_ζ, at any point, q, in the acoustic field. In order to avoid numerical difficulties at characteristic wave numbers, BEMAP overdetermines the surface Helmholtz equations with a series of interior equations (CHIEF method [6]).

AVAST

The AVAST suite of programs also uses VAST as a basis to establish a finite element model of the structure under consideration. AVAST uses the modal characteristics of the dry structure to establish a mobility matrix, $\mathbf{M}(\omega)$, relating surface nodal velocities ($\mathbf{v}$) to applied structural loads ($\mathbf{f}$):

$$\mathbf{v} = \mathbf{M}\mathbf{f} \tag{8}$$

Using this expression, an equation relating the normal velocity at the surface ($\mathbf{v}_n$) with the surface pressures ($\mathbf{p}$) and the externally applied loads is constructed:

$$\mathbf{v}_n = \mathbf{T}\,[\mathbf{M}\mathbf{f}_s + \mathbf{M}\mathbf{T}\mathbf{Q}_T\mathbf{p}] \tag{9}$$

where the matrix $\mathbf{Q}_T$ provides a transformation from surface pressures to nodal forces and $\mathbf{T}$ is a coordinate transformation matrix.

AVAST also uses the Helmholtz integral relation as described by equation (5); however, AVAST uses a set of exterior equations to overdetermine the system [7] [8]. From the Helmholtz equation, AVAST constructs the surface relation

$$\mathbf{A}\mathbf{p} + \mathbf{B}\mathbf{v}_n = \mathbf{0} \tag{10}$$

and the exterior pressure field relation

$$\mathbf{D}\mathbf{p} + \mathbf{E}\mathbf{v}_n = \mathbf{p}_e \tag{11}$$

where

$$\mathbf{A} = \int_{S'} \frac{\partial G}{\partial n} dS - \mathbf{C} \qquad \mathbf{B} = i\omega\rho \int_{S'} G dS \tag{12}$$

$$\mathbf{D} = \frac{1}{4\pi} \int_S \frac{\partial G}{\partial n} dS \qquad \mathbf{E} = \frac{i\omega\rho}{4\pi} \int_S G dS \tag{13}$$

and where

$$\mathbf{C} = 4\pi + \int_S \frac{\partial}{\partial n}\left(\frac{1}{r}\right) dS \tag{14}$$

Since the surface pressures are initially unknown, some form of approximation is required and the one chosen for this method is the plane wave impedance relationship:

$$\mathbf{p} = \rho c \mathbf{v}_n \tag{15}$$

Substituting (15) into equation (9) and solving for $\mathbf{v}_n$ yields:

$$\mathbf{v}_n = [\mathbf{I} - \rho c \mathbf{T}\mathbf{M}\mathbf{Q}]^{-1}\, \mathbf{T}\mathbf{M}\mathbf{f}, \tag{16}$$

The resulting normal velocities may be substituted into equations (10) and (11) and the interface pressures may be solved for in a least squares sense by solving the following overdetermined system of equations:

$$\begin{bmatrix} \mathbf{A} \\ \mathbf{D} \end{bmatrix} \mathbf{p} = \left\{ \begin{array}{c} -\mathbf{B}\mathbf{v}_n \\ \mathbf{p}_e - \mathbf{E}\mathbf{v}_n \end{array} \right\} \tag{17}$$

The surface pressures calculated in (17) may now be substituted into (9) and the updated normal velocities compared to the original. AVAST then iterates, assuming the velocities are different, through equations (10), (11), (17), and (9) until convergence.

3 Numerical Modelling Results

A model of a cantilevered flat plate vibrating in air was chosen as one of the test cases for comparing the two codes. The model consisted of an aluminum plate (305mm × 254mm × 13mm) clamped in a rigid base as shown in Figure 1. A sinusoidal point load was applied as shown and the resulting directivity patterns at various frequencies determined. One such pattern, measured at 3092 Hz with a load applied at the same frequency, is shown in Figure 2. As can be seen from the figure, the results from the two different codes match extremely well. They also compare well with experimentally measured values.

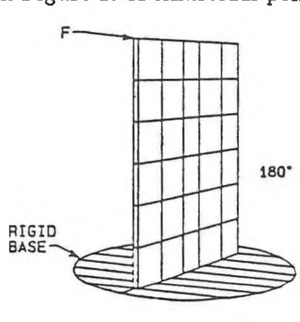

Figure 1: Flat Plate Model

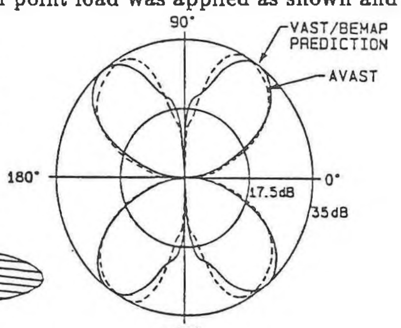

Figure 2: Directivity Pattern

Another test case to be used for comparison is a submerged cylinder with an internal dynamic load. This model incorporates the more difficult case of a dense fluid. One such model is shown below in Figure 3.

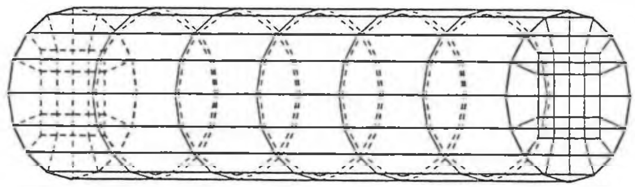

Figure 3: Ring-Stiffened Cylinder Model

4 Conclusions

Two suites of computer codes have been developed to predict acoustic radiation from vibrating structures submerged in a fluid. Results from test cases in air have agreed well with each other and with experimental results. A sufficient number of test cases has not yet been run to determine which suite uses the least computer time. Problems involving structures submerged in a dense fluid are currently being examined, but there appears to be a lack of experimental data for verification.

References

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