VIBRATIONS AND SOUND RADIATION OF A CYLINDRICAL SHELL UNDER A CIRCUMFERENTIALLY MOVING FORCE

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1. INTRODUCTION

Vibrations and sound radiation by finite cylindrical shells have been extensively studied in the past few years. Usually, the authors have studied the vibrations and the sound radiation by cylinders in the case of a non-moving harmonic driving force [1]. Most papers dealing with a moving force on a cylindrical shell (axially [2] or circumferientially [3]) were only concerned about the mechanical vibrations. This is a presentation of the work under progress to develop a model including both the vibrations and the sound radiation of a simply supported cylindrical shell excited by a circumferentially moving radial point force. The motivation behind this work is the modelisation of the "pressure screens" used in the pulp and paper industry. The theoretical formulation presented in section 2 is based on a variational approach. The case of a homogeneous cylindrical shell in air is treated as a first step towards more complex structures. Numerical results in terms of quadratic velocity and radiated sound power are presented, and principal phenomena related to the moving force rotational speed are discussed in section 3.

2. THEORETICAL FORMULATION

The studied system consists of a baffled thin cylindrical shell with the simply supported boundary conditions (Fig. 1). In the case of a finite cylinder, and with a variational approach, one can find the governing equations of motion for the studied system using the Hamilton's function, which has the form:

$$H=\int_{t_o}^{t_1}\left\{T_{shell}-E_{shell}-E_{fluid}+E_{force}\right\}dt \tag{1}$$

where T_{shell} and E_{shell} are respectively the shell kinetic and deformation energy, E_{fluid} is the energy related to the exterior acoustic pressure field, and E_{force} is the energy of the rotational driving force. Using the thin shell theory and under Donnell's assumptions, the three first terms are expressed as in reference [1]. The energy term related to the radial force is

$$E_{force}=\int_V\left\{\overrightarrow{P(M)}\right\}^t\left\{\overrightarrow{U(M)}\right\}dV \tag{2}$$

where V is the volume of the cylinder, $F(M)$ is the radial force at a point M on the shell, and $U(M)$ is the displacement of point M. A radial point force located at x_o and travelling around the circumference is expressed as:

$$P(M,t)=P(x,\varphi,t)=\frac{P}{aL}\delta(x-x_o)\delta(\varphi-\Omega\cdot t) \tag{3}$$

where δ is the Dirac distribution, and Ω is the rotational speed of the force. Applying the Poisson's summation formula on (3) one can separate the space and time variables:

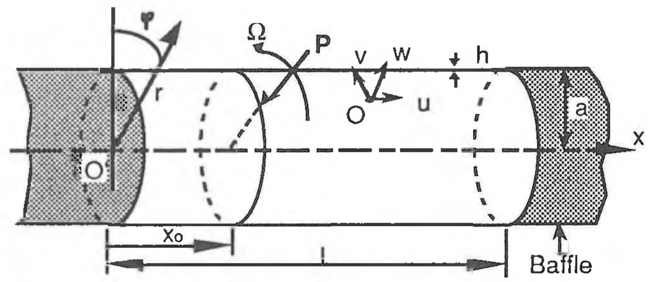

Fig. 1: Schematic of the cylindrical shell excited by a circumferentially moving radial point force

$$\left\{\begin{aligned}P(M,t)&=\sum_{N=-\infty}^{\infty}P(M)P(t)\\P(M)&=\frac{P}{2\pi a\cdot L}\delta(x-x_o)\cdot e^{-jN\varphi}\\P(t)&=e^{jN\Omega t}\end{aligned}\right\} \tag{4}$$

Integrating $P(M)$ in (2) and developing on the modes of a simply supported *in vacuo* circular cylindrical shell, one can minimize this energy function with respect to the modal amplitudes and obtain the expression of the generalized force vector:

$$P^{\alpha}_{Nnmj}=\frac{L}{2}\delta^{\alpha}_{Nn}P\sin\left(\frac{m\pi x_o}{L}\right) \tag{5}$$

where

$$\delta^{\alpha}_{Nn}=\left\{\begin{array}{ll}0 & n\neq N\\0 & n=N=0,\alpha=0\\2 & n=N=0,\ \alpha=1\\-j & n=N\neq 0,\ \alpha=0\\1 & n=N\neq 0,\ \alpha=1\end{array}\right\}$$

Applying the same development for the three first energy terms of equation (1) gives finally, for an $e^{jN\Omega t}$ rotational excitation, the following modal equation of motion:

$$m_{nmj}\left(\omega^2_{nmj}(1-j\eta)-(N\Omega)^2\right)a^{\alpha}_{nmj}-j(N\Omega)\sum_{q=1}^{\infty}\sum_{k=1}^{3}Z_{nmq}\,a^{\alpha}_{nqk}=P^{\alpha}_{Nnmj} \tag{6}$$

where $N\Omega$ represents the N^{th} harmonic of the rotational speed, n the circumferential order, m and q the longitudinal orders, j and k the type of mode (torsional, radial, axial), ω_{nmj} the eigen-angular frequencies, a^{α}_{nmj} the modal amplitudes, η the structural damping, and Z_{nmq} the modal radiation impedances.

For a better understanding of equation (6), let's neglect Z_{nmq}. Then, one can observe that maxima for modal amplitudes will occur when

$$\Omega = \Omega_c = \frac{\omega_{nmj}}{N}; \quad N=n \qquad (7)$$

where Ω_c is named the critical speed. In fact, there are as many critical speeds as eigen-angular frequencies.

2. NUMERICAL RESULTS

The results for a 0.003 m thick steel shell with a length of 1.2 m and a radius of 0.48 m are presented in Figs. 2, 3 and 4 for two different rotational speeds (25 Hz and 75 Hz), for the first longitudinal order ($m=1$), and for the type of mode (torsional, radial, axial) having the lowest eigen-angular frequency.

Fig. 2 represents critical speeds versus circumferential orders. As one can see, the first critical speed occurs at 28 Hz, for the fifth circumferential order and the first longitudinal order (i.e. mode (5,1)).

Because the 25 Hz rotational speed is very close to the first critical speed of 28 Hz associated with the mode (5,1), the quadratic velocity amplitude presents a significant single peak for the 25 Hz fifth harmonic (i.e. 125 Hz or the fifth '+' in Fig. 3). Since only frequencies around mode (5,1) (75-250 Hz) are very excited, a low sound power will be radiated (see Fig. 4).

If the rotational speed is increased up to 75 Hz, one can predict, by looking at Fig. 2, that a first peak will occur at its third harmonic (mode (3,1)) and a second at its sixteenth harmonic (mode (16,1)). The result predicted is verified in Fig. 3. The bandwidth excited is now very large and the final result will be an important increase of the radiated sound power (see Fig. 4).

For the 75 Hz rotational speed, the previous results include only the first longitudinal order. If the m first longitudinal orders are included, the quadratic velocity and the radiated sound power will be radically different because more than two critical speeds will occur.

For the 25 Hz rotational speed, including m longitudinal modes will not change appreciably the curves because no other critical frequency will occur.

Finally, as one can see, on Fig. 3 and 4, or by the mean of equation (6), to obtain the quadratic velocity and the radiated sound power at 2000 Hz, for a radial force rotating at 25 Hz, the circumferential order has to be equal to 80 (80th harmonic). In that case, we need to ensure that thin shell theory is still applicable by using the following criteria:

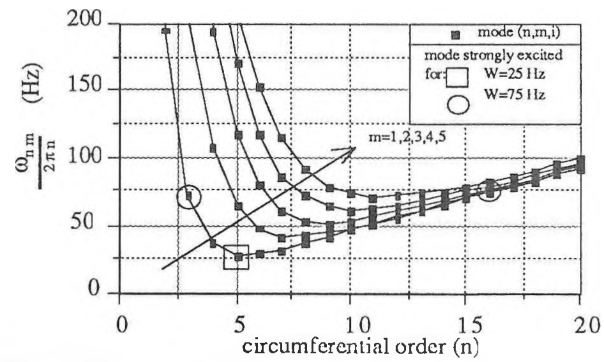

Fig. 2: Critical speed versus circumferential orders

$$n < 0.2\,\pi\left(\frac{a}{h}\right) \qquad (8)$$

3. CONCLUSION

The model developed in the case of a simply supported cylindrical shell has allowed us to draw some preliminary conclusions useful in design such as the low level of sound radiation when the force rotational speed is lower than the critical frequency associated with the first mode. The use of the variationnal approach will allow the integration of more complex parameters such as stiffeners, visco-elastic layers, internal pressure and heavy fluid.

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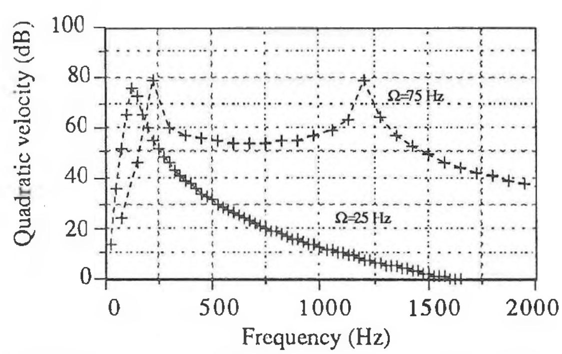

Fig. 3: Quadratic velocity (each '+' corresponds to an harmonic of the 25 or 75 Hz rotational speed)

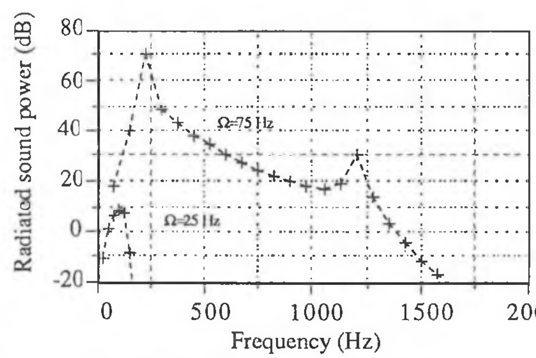

Fig. 4: Radiated sound power (each '+' corresponds to an harmonic of the 25 or 75 Hz rotational speed)