Separation of Acoustic Multipaths in Saanich Inlet

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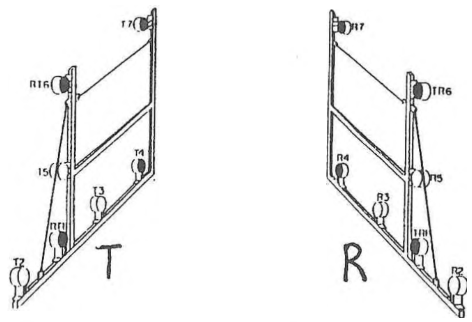

Figure 1: Acoustic array configuration deployed at 35 metres during the experiment.

1 Introduction

Acoustical scintillation measurements can provide a basis for determining the properties of ocean structure in coastal waters and can be used as a tool to remotely measure oceanographic processes (e.g. current and turbulent structure) which differ markedly from processes in the open ocean. A scintillation experiment in 1989 was carried out in a coastal environment in British Columbia, Canada in order to relate acoustic characteristics to the oceanography.

The experimental site was Saanich Inlet which is a deep (220 m), quiescent (maximum currents 10 $cm \cdot s^{-1}$) and stratified fjord. Our goal is to compare acoustic propagation through this relatively undisturbed, coastal environment with previous measurements in a turbulent tidal channel (Farmer *et.al.* [2]).

Figure 1 shows the two dimensional square array configuration used (2 metre spacing between darkened transducers). The arrays were deployed at 35 metres depth. Each transducer is directional with a beam width of 10 degrees at -3dB.

The 67 kHz acoustic signal used in the Saanich Inlet experiment made use of a 127 bit phase modulated pseudo-random-noise (PRN) code so as to improve the signal to noise ratio. The bit width of the code was chosen so that multipaths separated in arrival time by about $300\mu s$ ($20 cycles$) could be distinguished. The matched filter output of the PRN code produces a well known triangular peak shape.

The transmitter array cycles through all four transducers 5 times each second. The incoming signals at the receiver are filtered, amplified and then separated into in-phase (I) and quadrature (Q) components. Each is then correlated with a stored PRN template of the transmitted signal. The matched filter output shows a series of peaks corresponding to different signal paths. The amplitude ($A = \sqrt{I^2 + Q^2}$) and

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(b

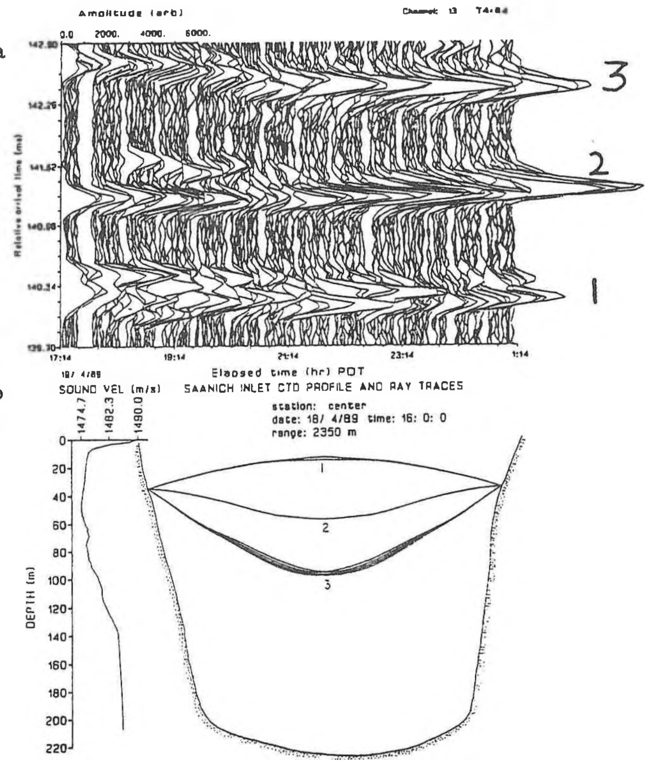

Figure 2: (a) Acoustic amplitude measured as a function of relative arrival time and as a function of elapsed time. (b) Averaged sound speed profile with the corresponding acoustic eigenrays.

arrival time (τ) are determined using a maximum likelihood procedure. The phase ϕ is then defined as $\arctan[\frac{Q(\tau)}{I(\tau)}]$. The arrival time τ is used to resolve the 360^o phase ambiguity.

2 Multipath Analysis

The acoustic observations in Saanich Inlet show multipath propagation conditions. Figure 2(a) shows the measured amplitude (in arbitrary units) as a function of relative arrival time (3 ms total) and as a function of elapsed time. The figure represents 8 hours of sub-sampled data (75 second intervals). Three distinct acoustic paths are present and they correspond to the eigenrays shown in figure 2(b). These rays are obtained using a simple range independent ray tracing algorithm.

The first set of arrivals is refracted upwards into the near surface because of the shallow thermocline. The second arrival, although appearing to have a well defined peak, is actually a superposition of several overlapping multipaths. It is this path that will be studied in closer detail. The third set of arrivals clearly show phase variations related to the internal tide that exists at 100 metres and beyond.

In order to separate the multipaths observed in the preliminary analysis, a maximum likelihood estimation algorithm is developed following Ehrenberg *et.al.* [1]. The mathematical model for the received signal $r(t)$ is,

$$r(t) = \sum_{i=1}^{N} A_i s(t - \tau_i) + n(t) \tag{1}$$

where A_i and τ_i are the amplitude and arrival times for the i^{th} path and $n(t)$ is the noise. The signal $s(t)$ is known since it is the matched filter output (c.f. Menemenlis and Farmer [3]):

$$s(t - \tau) = \sum_{n=0}^{9} a_n \left(\frac{t-\tau}{\tau_p}\right)^{2n}, \tag{2}$$

where a_n are known coefficients and τ_p is the half-width of the correlation peak (= 1 bit = 3 samples). This function is triangular with a rounded apex.

The maximum likelihood estimation is derived as follows: minimize

$$Q = \sum_t \left[r(t) - \sum_{i=1}^{N} A_i s(t - \tau_i)\right]^2, \tag{3}$$

$$= \sum_t r(t)^2 - 2\left[\sum_{j=1}^{N} A_j C(\tau_j) - \frac{1}{2}\sum_{j=1}^{N} A_j \sum_{k=1}^{N} A_k B(\tau_j, \tau_k)\right], \tag{4}$$

with respect to A_i and τ_i. Minimizing Q implies that the second term on the right of the last equation should be maximized. That is,

$$\text{maximize} \quad P = \mathcal{C}^T \mathcal{A} - \frac{1}{2}\mathcal{A}^T \mathcal{B} \mathcal{A} \quad \text{w.r.t. } A_i \text{ and } \tau_i, \tag{5}$$

where $C(\tau_j) = \sum_t r(t)s(t - \tau_j)$ is the cross covariance between the received and modelled signal, and $B(\tau_j, \tau_k) = \sum_t s(t-\tau_j)s(t-\tau_k)$ is the auto covariance between the modelled signals. This maximization problem is written in matrix form where

$$\mathcal{A}^T = (A_1, A_2, ..., A_N), \tag{6}$$

$$\mathcal{C}^T = (C(\tau_1), C(\tau_2), ..., C(\tau_N)), \tag{7}$$

$$\mathcal{B} = \begin{bmatrix} B(\tau_1,\tau_1) & B(\tau_1,\tau_2) & ... & B(\tau_1,\tau_N) \\ B(\tau_2,\tau_1) & B(\tau_2,\tau_2) & ... & B(\tau_2,\tau_N) \\ \cdot & \cdot & \cdot & \cdot \\ B(\tau_N,\tau_1) & B(\tau_N,\tau_2) & ... & B(\tau_N,\tau_N) \end{bmatrix}. \tag{8}$$

Maximizing with respect to each of the A_i yields,

$$\mathcal{A} = \mathcal{B}^{-1}\mathcal{C}. \tag{9}$$

Substituting into equation [5] gives the following maximization problem,

$$\text{maximize} \quad [\frac{1}{2}\mathcal{C}^T \mathcal{B}^{-1} \mathcal{C}] \quad \text{w.r.t. } \tau_i. \tag{10}$$

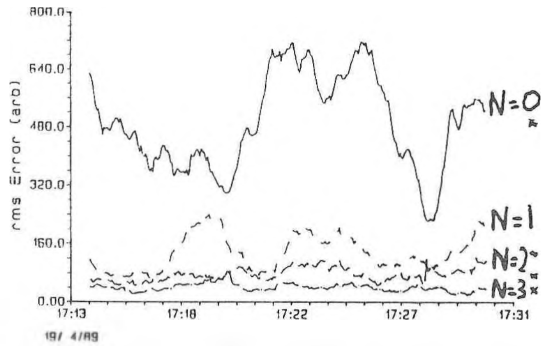

Figure 3: Integration of the noise for an assumed number of paths N. The integrated signal level is shown for $N = 0$. Averages are shown as an $*$ on the far right side.

Therefore, to determine the maximum likelihood estimate, equation [10] must first be maximized with respect to the arrival time estimates. This equation is in a quadratic form and is a function of N independent variables and so maximization occurs over an N dimensional space. Powell's quadratically convergent method [4] is used for this procedure. The resulting set of arrival time estimates are then used in equation [9] to obtain the amplitude estimates.

In deriving the maximum-likelihood estimate it is assumed that the number of paths N is known and fixed. The number of paths chosen is $N = 3$. This is because the arrival time integration of the noise calculated for $N = 0, 1, 2, 3$ multipaths (see figure 3) gives the correct noise level for $N = 3$.

3 Conclusions

Now that we have separated the overlapping multipaths, we are in a position to use the amplitude and phase measurements in a variety of ways to contribute to our understanding of acoustic propagation in this environment. For example, the phase difference between vertical and horizontal receivers can be used to detect the angle of arrival of the acoustic waveform. Cross correlation techniques can be implemented in order to determine the current component perpendicular to the direction of propagation as well as give some indication of the coherence length scales.

References

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