Reciprocal Travel Time Scintillation Analysis

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1 Introduction

During a study of the arctic boundary layer, Menemenlis and Farmer [4] used acoustical reciprocal transmissions to obtain line-averaged velocity measurements along 200 m horizontal paths in the mixed layer beneath ice. The present discussion is motivated by a desire to interpret the observed high frequency velocity fluctuations in terms of the advection and evolution of turbulent velocity fine structure.

Kaimal *et al.* [2] discussed the problem of line-averaging in the context of extending the useful range of sonic anemometers to scales shorter than the acoustic paths. They derived transfer functions that relate measured and ideal one-dimensional power spectra. We investigate the asymptotic behaviour of the spectral transfer functions, as the length of the measuring baseline is increased, and derive analytic expressions.

The spectral transfer functions are shown to vary with dimensionless wavenumber and with angle between the measuring baseline and the mean velocity. The analysis is extended to anisotropic and inhomogeneous flows. Finally, some experimental data taken in the boundary layer beneath ice is compared with the theory.

2 Theory

Consider a stationary and homogeneous random velocity field advected by the mean flow past a long measuring baseline. The spectral transfer function, $T(k_1\ell,\theta,p)$, is defined as the ratio between the line-averaged one-dimensional spectrum and the ideal streamwise one-dimensional spectrum. It indicates the spectral attenuation due to line-averaging as a function of streamwise wavenumber k_1, pathlength ℓ, orientation of the measuring baseline relative to the mean velocity vector θ, and spectral slope $-p$.

Kaimal *et al.* [2] solved the spectral transfer function numerically for an isotropic inertial subrange, $p = 5/3$, and a measuring baseline perpendicular to the mean flow, $\theta = 90°$. When the measuring baseline is long compared to the scales of interest, an analytic solution can be found,

$$T(k_1\ell,\theta,p) = \frac{K(p)\sin^p\theta}{k_1\ell}\left(\frac{(p+1)(\cos^4\theta+\sin^4\theta)+1}{2}\right),$$

where $K(p) = 2\Gamma(1/2)\Gamma(p/2+1/2)/\Gamma(p/2)$.

2.1 Axisymmetric Turbulence

McPhee [3] reports that heat and momentum flux under ice is typically caused by turbulent eddies that are of order 10-20 m in horizontal extent and a few meters in vertical extent. Therefore, in a given wavenumber range, the flow is more likely to be axisymmetric about the vertical axis rather than isotropic.

Herring [1] introduced a simple and useful formalism for describing axisymmetric turbulence in terms of an anisotropy parameter a, where $0 \le a \le 1$, and $a = 1/2$ implies isotropy. Once again, when the measuring baseline is long compared to the scales of interest, an analytic solution can be found,

$$T_a = \frac{K(p)\sin^p\theta}{k_1\ell}\left(\frac{b(\cos^4\theta+\sin^4\theta)+2a\cos^2\theta\sin^2\theta}{(b+ap)\cos^2\theta+(a+bp)\sin^2\theta}\right)(p+1),$$

where $b = (p+2)(1-a)$. For $a = 0$, *i.e.* when all the kinetic energy is contained in the horizontal mode of motion, the spectral transfer function is at most 16% higher than for the isotropic case.

2.2 Anisotropic Turbulence

Consider a baseline of length ℓ that is split in two pieces of length α and β so that $\ell = \alpha + \beta$. The line-averaged velocity $\tilde{u}$ is related to $\tilde{u}_\alpha$ and $\tilde{u}_\beta$, the velocities averaged along α and β respectively, by $\tilde{u} = (\alpha\tilde{u}_\alpha + \beta\tilde{u}_\beta)/\ell$. We define $\tilde{F}_\alpha(k_1)$ and $\tilde{F}_\beta(k_1)$ to be the line-averaged one-dimensional spectra associated with $\tilde{u}_\alpha$ and $\tilde{u}_\beta$ respectively. For $k_1\alpha$ and $k_1\beta$ sufficiently large, $\tilde{u}_\alpha$ and $\tilde{u}_\beta$ are statistically independent and in that wavenumber range

$$\tilde{F}(k_1) = \frac{\alpha^2\tilde{F}_\alpha(k_1) + \beta^2\tilde{F}_\beta(k_1)}{\ell^2}.$$

Assuming homogeneity, this equation can only be satisfied when $\ell\tilde{F}(k_1) = \alpha\tilde{F}_\alpha(k_1) = \beta\tilde{F}_\beta(k_1)$, *i.e.* the measured spectra are inversely proportional to the averaging length. Using k_1 to non-dimensionalize ℓ, we conclude that $\tilde{F}(k_1) \propto F(k_1)/\ell k_1$, *i.e.* for a sufficiently long baseline, the measured spectral slope is one unit less than the true spectral slope.

2.3 Inhomogeneous Turbulence

We use the same notation and paradigm as above, but now α spans an isotropic region with one-dimensional spectrum $F_\alpha(k_1)$ and similarly $F_\beta(k_1)$ is the spectrum associated with

β. Using the spectral transfer functions for isotropic turbulence derived earlier, we obtain

$$\tilde{F}(k_1) = T(k_1\ell, \theta, p)\left(\frac{\alpha F_\alpha(k_1) + \beta F_\beta(k_1)}{\ell}\right),$$

i.e. at sufficiently high wavenumbers, line-averaged measurements provide a true weighted average of the turbulent kinetic energy spectrum.

3 Experiment

During a coordinated study of the boundary layer beneath ice in the Arctic, Menemenlis and Farmer [4] deployed an acoustical instrument to measure path-averaged horizontal current and vorticity. A triangular acoustic array of side 200 m was used to obtain reciprocal transmission measurements at 132 kHz, at 8, 10 and 20 m beneath an ice floe. Pseudo random coding and real-time signal processing provided precise acoustic travel time and amplitude for each reciprocal path.

Mean current along each acoustic path is proportional to travel time difference between reciprocal transmissions. Horizontal velocity normal to the acoustic paths is measured using scintillation drift. The instrument measures horizontal circulation and average vorticity relative to the ice, at length scales characteristic of high frequency internal waves in the region. The rms noise level of the measurements is less than 0.1 mm/s for velocity and 0.01 f for vorticity, averaged over one minute. Except near the mechanical resonance frequency of the moorings, the measurement accuracy is limited by multipath interference.

The sensitivity of the path-averaging acoustical current meter is such that it allows detection of kinetic energy at frequencies associated with the advection and evolution of turbulent velocity fine structure. In addition to the acoustical instrument, clusters of high resolution mechanical current meters were deployed roughly at the center of the acoustic array by McPhee [3].

The underside surface of the ice is irregular and contains keels that extend down to 10 m depth in the vicinity of the acoustic array. Because of the passage of internal waves and tides under the ice camp, steady current flow in one direction is rarely achieved for periods long enough to obtain good statistical averages. For these reasons, we expect departure of observed properties from the predictions made by the theory.

Fig. 1, is a spectral comparison of line-averaged and locally measured horizontal velocity fluctuations at 20 m depth on April 13, 1989. The spectra are a six hour average, during a period when the mean flow relative to the ice was 15 cm/s in a northward direction. The line-averaged data corresponds to a sonic path that forms angle $\theta = 76°$ with the mean flow. A straight line with $-5/3$ spectral slope is fitted to the high wavenumber region of McPhee's data. A second line with $-8/3$ slope is also drawn based on the analytic transfer function derived earlier. This prediction is seen to fall well within the 95% confidence interval of the line-averaged spectrum.

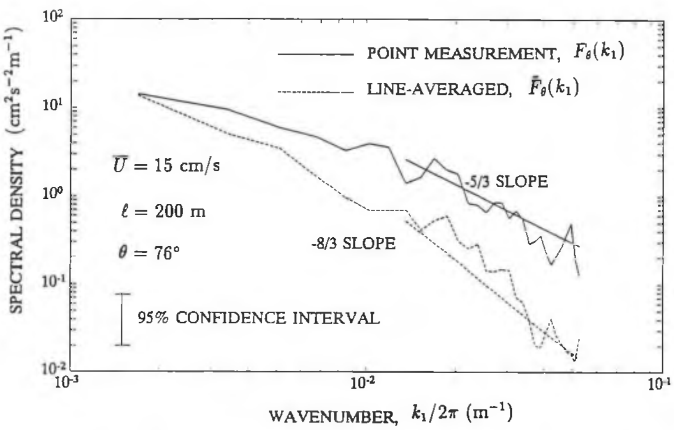

Figure 1: Comparison of energy spectra for point and line-averaged horizontal velocity measurements obtained 20 m beneath floating ice in the arctic boundary layer, on April 13, 1989.

4 Conclusions

For measuring baselines long compared to the scales of interest analytic expressions for the transfer function between true and line-averaged one-dimensional spectra have been derived for isotropic and axially symmetric turbulence. Line-averaged measurements are most sensitive to velocity fine structure when the mean velocity is perpendicular to the measuring baseline. The error incurred by assuming isotropy instead of axial symmetry is unlikely to exceed 16%.

For homogeneous turbulence and a sufficiently long baseline, the slope of line-averaged measurements is one unit less than the true spectral slope, irrespective of the form of the spectral density tensor. At sufficiently high wavenumbers, line-averaged measurements through regions of locally homogeneous turbulence provide a true weighted average of the kinetic energy spectrum.

This theory explains high frequency fluctuations observed during an experimental study of the arctic boundary layer beneath ice, where reciprocal acoustical transmissions were used to obtain line-averaged velocity measurements along 200 m horizontal paths.

References

[1] J. R. Herring. Approach of axisymmetric turbulence to isotropy. *The Physics of Fluids*, 17(5):859–872, May 1974.

[2] J. C. Kaimal, J. C. Wyngaard, and D. A. Haugen. Deriving power spectra from a three-component sonic anemometer. *Journal of Applied Meteorology*, 7:827–837, Oct. 1968.

[3] M. G. McPhee. Turbulent heat flux in the upper ocean under sea ice. *Journal of Geophysical Research*, 97(C4):5365–5379, April 15, 1992.

[4] D. Menemenlis and D. M. Farmer. Acoustical measurement of current and vorticity beneath ice. *Journal of Atmospheric and Oceanic Technology*, in press.