Modelling Azimuthal and Vertical Directionality of Active Sonar Systems for Undersea Reverberation

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1. Introduction

Traditional methods for modelling undersea reverberation where either the transmitter or receiver has azimuthal directionality approximate the azimuthal component by sector coverage. Often, the effects from other than a main lobe are ignored. In some cases, the width of the sector is chosen such that the azimuthally-integrated response is the same for the sectored coverage as the original sonar in the horizontal plane. Unfortunately, should the sonar have both azimuthal and vertical directivity (which is often the case), accounting for only the main lobe or using a simple azimuthal integration may lead to erroneous reverberation estimates. Historically, inclusion of the full directionality or beam pattern has been too computationally costly to consider, but with the proliferation of modern high-speed computers, this is no longer the case. The full directionality can and in many cases should be included.

This azimuthal-integration concept is not new or difficult to understand; yet, it is often ignored when considering beam patterns. The concept has been used in calculating an effective beamwidth or in calculating the scattering area. Ellis[1] considered an "effective vertical beam pattern" by assuming that either the transmitter or the receiver was azimuthally independent. Urick[2] considered the case where the transmit angle and the receive angle are the same. i.e. backscatter. This omits the "hybrid paths" shown by Ellis and Franklin[3] to be important in bottom reverberation estimates. Even if hybrid paths were included, the technique would not properly model the high-angle paths which dominate the reverberation at shorter ranges. Urick also indicates that it is generally too difficult to analytically perform the integration even for this simplified case. The following sections describe the equations and techniques for the azimuthal integration.

2. Theory

We restrict the problem to consider azimuthally symmetric environments with source and receiver horizontally co-located. However, they may be vertically displaced. Without loss of generality, we will assume that all of the scatterers lie on the same boundary, the seafloor. Reverberation effects from other scatterers may be computed and added to obtain the total reverberation. The reverberation intensity from an ocean boundary as a function of time may be written as[1]:

$$R(t) = I_0 \int_{A(t)} \sum_{s,r} H_s(\rho,\theta_s)\, B_s(\theta_s,\phi) \bullet H_r(\rho,\theta_r)\, B_r(\theta_r,\phi)\, S(\theta_s',\theta_r')\, dA, \quad (1)$$

where

t	time,
I_0	source intensity,
dA	elemental area,
$A(t)$	area contributing to reverberation at time t,
r,s	receiver and source, respectively,
H_s, H_r	propagation loss to and from the scattering area,
ρ	radial component in cylindrical coordinates,
ϕ	azimuthal angle,
θ	vertical angle measured from the horizontal at the sonar,
θ'	grazing angle at the seafloor,
B_s, B_r	source and receiver beam patterns, and
S	scattering function.

Note that due to refraction effects, θ and θ' are not necessarily equal. The sumation over r and s indicate the inclusion of all combinations of relevent transmit and receive paths. i.e. hypbrid paths are included.

Furthermore, if one substitutes $dA = \rho\, d\rho\, d\phi$, the two-dimensional integration may be reduced and the sonar directivity (beam pattern) isolated:

$$R(t) = 2\pi I_0 \int_{\rho(t)} \sum_{s,r} H_s(\rho,\theta_s)\, H_r(\rho,\theta_r)\, S(\theta_s',\theta_r') \bullet D(\theta_s,\theta_r)\, \rho\, d\rho, \quad (2)$$

where D is defined by

$$D(\theta_s,\theta_r) = \frac{1}{2\pi} \int_0^{2\pi} B_s(\theta_s,\phi) B_r(\theta_r,\phi)\, d\phi\ . \quad (3)$$

$D(\theta_s,\theta_r)$ is an azimuthally-integrated beam pattern product function. In agreement with Urick, even in the case where $\theta_s = \theta_r$, this integral can not be generally solved using analytic tools. Of course, if one removes the azimuthal dependence, the integral is trivial.

Ellis considered the case in which either B_s or B_r was azimuthally independent. For discussion purposes, consider B_s as being azimuthally independent; a case which may be easily characterized using existing reverberation models. B_s may be entered in its usual form, while the B_r is replaced by its "effective vertical beam pattern." Ellis defined the "effective vertical beam pattern" to be the azimuthally-integrated beam pattern similar to equation 3, but without B_s.

As long as one is willing and able to modify an existing reverberation model, Ellis' restriction is not necessary. Eigenray-based numerical models may be easily modified to accept $D(\theta_s,\theta_r)$ as input or alternatively to accept B_s and B_r and perform a numerical integration to arrive at $D(\theta_s,\theta_r)$. The NUSC Generic Sonar Model[4] has been modified to include a "Simpson's Rule" integration of the beam pattern product in order to produce the results in section 4.0.

3. Example Sonar System

To demonstrate this technique, we will use a simple sonar consisting of an 8-element horizontal line array receiver, spaced at 1/2 wavelength with rectangular weighting. Receiver sidelobes are cut-off at -35 dB.

For demonstration purposes, we shall choose three sources. An omni-directional source which has azimuthal and vertical independence; a four-element (1/2 wavelength spacing) vertical line array which gives vertical dependence while having azimuthal independence and a third which consists of the four-element vertical array rotated to be collinear with the receiving array. This yields a source with both vertical and azimuthal dependence. It is possible to properly model the first two source/receiver options using Ellis' technique (but not Urick's), but the third source requires the solution presented in the previous section. For simplicity, we shall choose the transmit parameters to be the same as those used by Ellis; namely a 0.33 s rectangular CW pulse at a frequency of 315 Hz. The source level, I_0, is 220 dB re 1 μPa at 1m. Transmitter sidelobes are cut-off at -20 dB.

4. Calculated Reverberation

To demonstrate the technique, we again turn to Ellis to supply the environment. We will assume a typical deep-water environment with no surface duct and a water depth of 5300 m. The source and receiver are collocated at 175 m.

Figure 1 shows the calculated reverberation including fathometer returns for each of the source options and the line array receiver. Each of the line-array sources is steered in the broadside direction. In each case, the receiver is steered in the broadside direction.

The results for the omni directional source show significant increases over the smoothly decaying reverberation due to the fathometer returns. The results for the horizontal source show the same maximum levels of the fathometer returns as with the omni source, with slightly lower levels in the smoothly decaying region. The fathometer-return levels are the same since the horizontal line array is steered in the broadside direction where it has unity response in the vertical direction. The fathometer-return levels for the vertical line source show a significant decrease since the array is steered in the horizontal direction (broadside) and does not insonify the higher angle paths to the seafloor which dominate the reverberation.

Analyzing these differences between source directionalities would be extremely difficult or impossible without considering the full azimuthal directionality of both the receiver and transmitter. Using alternate techniques that ignore the 2D dependence may produce misleading results.

5. Discussion

This paper shows a relatively straight-forward technique for including both azimuthal and vertical directionality in reverberation calculations for active sonar systems. Though the technique is not new or difficult to understand, this azimuthal integration concept is often ignored.

The resulting equations are easily incorporated in almost any reverberation model. Without any modification, most reverberation models accounting for vertical directionality may be used to model systems where either the source or receiver has azimuthal independence. The difficulty arises when both source and receiver have azimuthal and vertical dependencies. Though generally not difficult, changes are required in most numerical models to account for this case.

The included example indicates how the technique may be applied to the analysis of complicated sonar systems. Other than in very special cases, the azimuthal and vertical directionality of the source and receiver should be included to properly model reverberation.

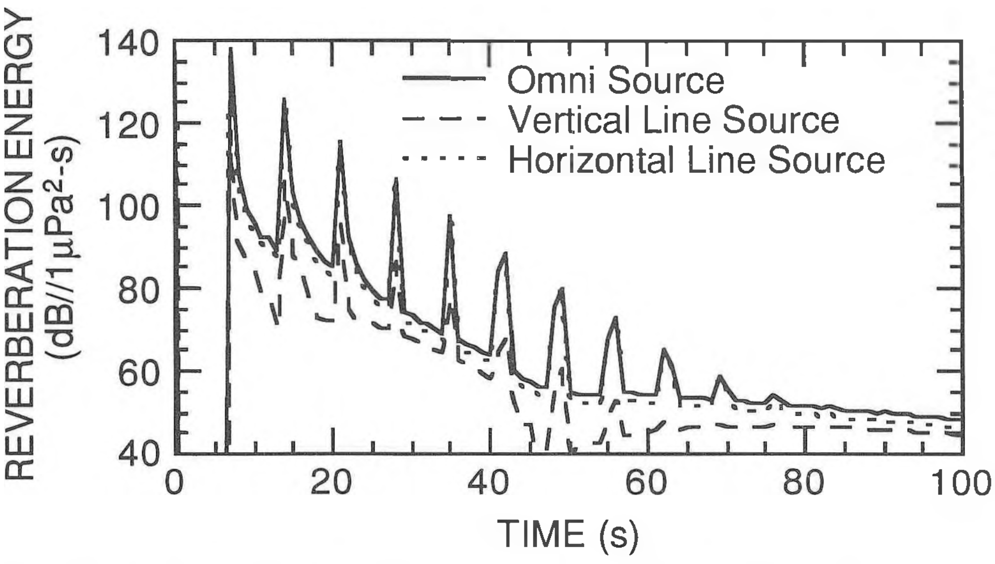

Figure 1 *Example Reverberation versus Time Plot for Line Array Receiver and Three Sources.*

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[1] D.D. Ellis, "Effective Vertical Beam Patterns for Ocean Reverberation Calculations," IEEE JOE, Vol. 16, No. 2, April 1992.

[2] R.J. Urick, Principles of Underwater Sound, 3rd ed. New York; McGraw-Hill, 1983.

[3] D.D. Ellis, J.B. Franklin, "The importance of hybrid ray paths, bottom loss, and facet reflection on ocean bottom reverberation," in Progress in Underwater Acoustics, H.M. Merklinger, Ed. New York; Plenum, 1987, pp. 75-84.

[4] H. Weinberg, "Generic SONAR Model," in Oceans '82 Proceedings, pp. 201-205, 1982.