A formulation for the vibro-acoustic behavior of a rectangular plate with constrained-layer damping.

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1. Introduction

In an attempt to reduce the resonant bending of a structure, intensive research on plates vibrations have been done over the years. It is well known that flexural vibrations in a plate can be damped by application of a viscoelastic layer constrained by a thin, elastic layer. Furthermore, the addition of a spacer between the plate to be damped and the viscoelastic layer have been found to enhance the damping performance. Kerwin [1] has shown that energy from such a structure is dissipated when shear deformation is induced in the viscoelastic layer. The spacer and the constraining layer enhance the damping effectiveness by inducing additional shear deformation in the viscoelastic layer.

The purpose of this work is to develop a rapid but rigorous tool to help acoustics engineers understand and predict the vibro-acoustic behavior of a constrained-layer damping of a plate. A rectangular four layered simply supported baffled plate is considered. In addition, the plate is assumed to be semi-complex in the sense that it can support added masses, stiffeners and several types of excitation (i.e point, line, surface forces and moment). The problem is formulated using a variational approach and solved by the Rayleigh-Ritz method. The modeling of the stiffeners is based on an equivalent orthotropic layer. Since the plate is assumed to radiate in air, added mass due to fluid loading is ignored. However, possible cross modal coupling due to stiffening or the type of the excitation is accounted for. This is done using a novel method for evaluating the radiation impedance matrix based on multipoles expansions of Green's kernel [2]. The numerical evaluation of the radiated power is done easily from the radiation impedance matrix.

2. Theoretical model

Several studies have been devoted to modeling plates constrained damping [3,4,5]. The most comprehensive is based on the Reissner-Mindlin's hypothesis which assumes that each layer could have pure bending, shear deformation and traction-compression effects [5]. These models yield an accurate representation of constrained damping but need long computational time. In addition, most existing studies have been limited to the vibrational problem. The model developed herein is inspired from the work done by M.R. Garrison et al. [3]. It is a simplification in comparison with the Reissner-Mindlin's model in the sense that it uses the appropriate assumptions for each layer. The displacement field of the elastic outer layer is considered to allow for pure bending (Love-Kirchhoff's assumptions) and traction-compression effects.

$$\begin{aligned} u_i(x,y,z,t) &= u_i^0(x,y,t) - z\frac{\partial w(x,y,t)}{\partial x} \\ v_i(x,y,z,t) &= v_i^0(x,y,t) - z\frac{\partial w(x,y,t)}{\partial y} \\ w_i(x,y,z,t) &= w(x,y,t) \end{aligned} \tag{1}$$

where u_i and v_i are the transverse displacements of the outer layers (i = 1 for the plate to be damped and i = 3 for the constraining layer). w is the normal displacement of the plate.

For the viscoelastic inner layer it is assumed that it can support both bending, traction-compression and shear deformation. The displacement field in the viscoelastic layer is completly defined by the continuity of the displacement at its junction with the constraining layers. Consequently the displacement field in the vicoelastic layer is obtained by linear interpolation between the two outer layers displacements and hence it is expressed in term of the assumed displacement of the outer layers .

$$\begin{aligned} u(x,y,z,t) &= \frac{z}{h_2}\left[u_3^0 + \frac{1}{2}\frac{\partial w}{\partial x}(h_3+h_1+2h_s) - u_1^0\right] \\ &\quad + \frac{1}{2}\left[u_3^0 + \frac{1}{2}\frac{\partial w}{\partial y}(h_3-h_1-2h_s) + u_1^0\right] \\ v(x,y,z,t) &= \frac{z}{h_2}\left[v_3^0 + \frac{1}{2}\frac{\partial w}{\partial x}(h_3+h_1+2h_s) - v_1^0\right] \\ &\quad + \frac{1}{2}\left[v_3^0 + \frac{1}{2}\frac{\partial w}{\partial y}(h_3-h_1-2h_s) + v_1^0\right] \\ w(x,y,z,t) &= w(x,y,t) \end{aligned} \tag{2}$$

h_1, h_2, h_3 and h_s represent respectively the thicknesses of the plate to be damped, the viscoelastic layer, the constraining layer and the spacer.

Since the spacer is assumed to be rigid in shear and to have no bending stiffness, its motion follows the motion of the plate to be damped. Furthermore, in order to allow for the equivalent orthotropic modeling of stiffeners, the four layers are assumed to be orthotropic thus the stress field is deduced from the displacement field using an orthotropic matrix of elasticity, which satisfies the plane stresses hypothesis for each layer.

Once the displacement field is defined, the problem is formulated using a variational approach. The functional of Hamilton is written as

$$H(\vec{u}) = \int_{t_0}^{t_1} (T - V + W)\,dt \tag{3}$$

where T is the kinetic energy (rotary inertia is neglected), V the potential energy, W is the work of the external forces and $\vec{u}$ represents the unknown displacements.

In order to study pure bending, shear deformation and traction-compression effects, five degrees of freedom are needed, hence there are five unknown displacements. These unknown displacements are written in terms of series of sine functions that allow for simply supported boundary conditions. The coefficients of the sine functions are solved for using the Rayleigh-Ritz approach which leads to the classical system of linear equations

$$[M]\{\ddot{a}\} + [K]\{a\} = \{f\} \tag{4}$$

where $[M]$ and $[K]$ represent respectively the mass and stiffness matrices which are deduced from the kinetic and potential energy. $\{f\}$ is the generalized vector of forces and $\{a\}$ is the vector of the coefficients of the unknown displacement field.

3. Results

The developed model has been validated by comparison with a model based on Reissner-Mindlin's theory for the three layers[6], and good agreement has been found for the different vibro-acoustic indicators.

In order to show the effect of the constrained-layer damping, the results (mean square velocity, acoustic ratiated power, radiation coefficient) of the present model (without a spacer) are compared with the predicted behavior of an undamped plate. All of the results are obtained using a primary layer which is 0.533 m long, 0.203 m wide, 1.9 mm thick and whose density is 2762 kg/m^3 (aluminum). The Young's modulus is 68.9 GPa and the loss factor is 0.005. The thickness of the viscoelastic layer is 0.1 mm, the Young's modulus is 3.45 MPa, the loss factor is 0.1 and the density is 1024 kg/m^3. The constrained layer is made with aluminum whose properties are the same as the primary layer and a thickness of 0.25 mm.

Figure 1 shows the effect of constrained damping on the mean square velocity. This effect is more pronounced at lower frequencies and around the resonances. The radiation efficiency is shown in Fig. 2. As expected it is unchanged by the treatment, because it only characterizes the primary layer. Finally, the radiated power decreases mainly due to the decrease of the mean square velocity as seen in Fig. 3.

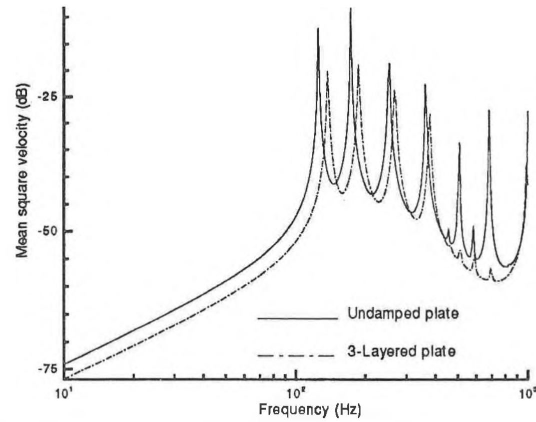

fig. 1 Mean square velocity, comparison between an undamped plate and the same plate with constrained layer damping.

4. Conclusion

The proposed model allows for an accurate modeling of the physics with minor computational effort. This is important since the main objective of the study is the development of a simple and accurate model for viscoelastic damping. Furthermore, the model combines well with a novel integral approach for the calculation of the radiated field thus allowing for the investigation of the effect of viscoelastic damping on the radiated field.

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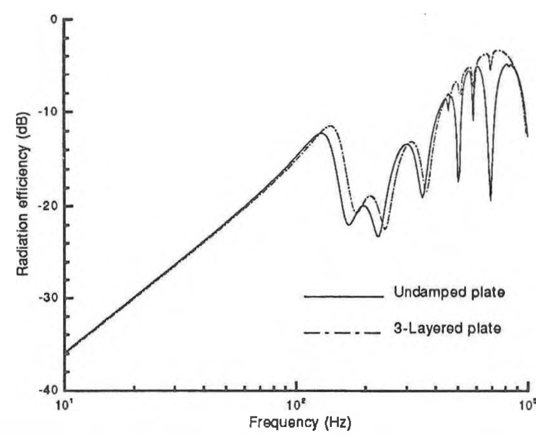

fig. 2 Radiated coefficient, comparison between an undamped plate and the same plate with constrained layer damping.

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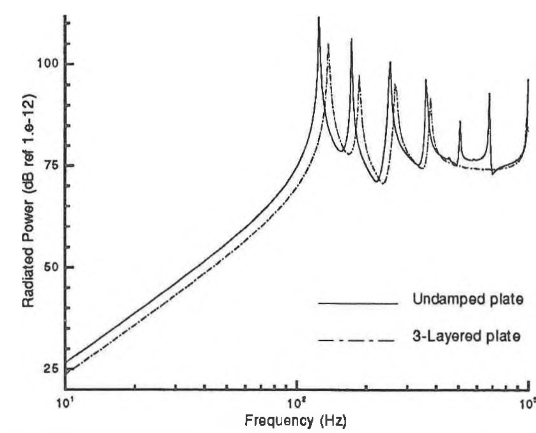

fig. 3 Radiated power, comparison between an undamped plate and the same plate with constrained layer damping.