ACTIVE NOISE CONTROL SIMULATIONS IN A CAVITY-BACKED FLEXIBLE PLATE SYSTEM

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Introduction

Manufacturers today are facing international competition and complete customer satisfaction is quickly becoming the focus of all the different echelons within an organisation. The transport industry is a good example of a competitive market where a product's acceptance is primordial and the stakes considerable. Among attributes of importance to customers, studies have found that acoustic discretion is among the first three. Most transport means are plagued by a high levels of noise in the passenger environment. In order to achieve reasonable noise reductions, manufacturers are confronted with choices which usually requires non-negligible space, adds weight, increases cost and complexity of manufacturing tasks (the more so if in their interest lie in the low frequency range). Recently, with the advent of technological advances in computing power, digital data aquisition and signal processing techniques, the possibility of attacking the noise problematic with counter noise measures has developed and commercial systems are being tested on aircraft [1,2] and automobiles [3] to eliminate undesirable tonal components. In transport vehicles, noise is produced by vibration and acoustic sources. The excitation produced by such sources eventually reaches the passenger compartment through many types of paths. One of these paths (structureborne) leads to noise radiation inside the passenger cavity by flexible plate type structures. Analytical and experimental investigations have been initiated on a cavity-backed simply supported plate system which has for principal goal the comprehension of the fundamental principles and the determination of the applicability of active noise control techniques for structureborne noise. The present work aims at the development of a rapid and complete tool to simulate the efficiency of active control schemes for low to mid range frequencies.

In the 1980's, publications began to appear on the active suppression of enclosed sound fields. Rigorous work has been conducted by Nelson et al. [4] on the active control of sound inside a rigid paralellepipedic enclosure. Their work was stimulated by propeller aircraft application and culminated by the actual design and testing of an active control system. Fuller et al. [2] have published extensively on similar applications but their studies concentrated on flexible cylinders and plates with control of interior sound field generated acoustically or structurally by point force actuators, acoustic drivers and piezoelectric devices. Pan et al. [5] have furthered the concepts to a cavity-backed flexible plate system. They have simulated and experimentaly studied the control of the cavity sound field due to a primary excitation (acoustic drivers exciting the flexible plate) by secondary point force actuators.

The research conducted by the authors is concerned with the simulation of active noise control within a plate-cavity system by the action of primary vibrational inputs acting on the flexible plate. Comparisons between vibration and acoustic control strategy as well as the influence of sensor and actuator numbers are pursued. Simplicity of the geometric model as well as the understanding of the nature and characteristics of modal behaviour account for the choice of the physical model. Simulated control schemes include the use of vibration control inputs (point, line, surface forces and line moments) acting on the flexible plate and acoustic control acting within the cavity (speakers). The following discussion will be limited to the development of the analytical basis and the comparison of results obtained in a case studied by Pan et al. [5].

I. Model development

The analytical model used in this study was developed using a modal summation approach similar to the ones used by Nelson et al. [4] and Pan et al. [5]. The model to be considered is shown in Figure 1 and consists of a rigid-walled paralellepipedic cavity having at one face (z=0) a simply supported flexible vibrating plate. Also shown are the elements which are modelled analytically, they are point, line and surface forces, line distributed moments and surface mounted acoustic velocity sources (speakers). By using the modal summation approach, the pressure contribution due to the plate *in vacuo* and cavity modal responses are evaluated for both controlled and uncontrolled conditions. The goal is to obtain an expression for the pressure within the cavity (where control is desired) in terms of primary (known) and control excitations. Minimization of the quadratic pressure is the method used to control the sound field inside the cavity. The analytical development is summarized in the following lines. Initially, the plate's movement equation is obtained in terms of its displacement W and is given by (a list of symbols used is given in Appendix I):

$$\mu(\partial^2 W(x,y)/\partial t^2) + D\nabla^4 W(x,y) = F^{tot}(x,y) \qquad (1)$$

where $F^{tot}(x,y) = \sum F^p(x,y) + \sum F^c(x,y) + P^p(x,y,0) + P^c(x,y,0)$

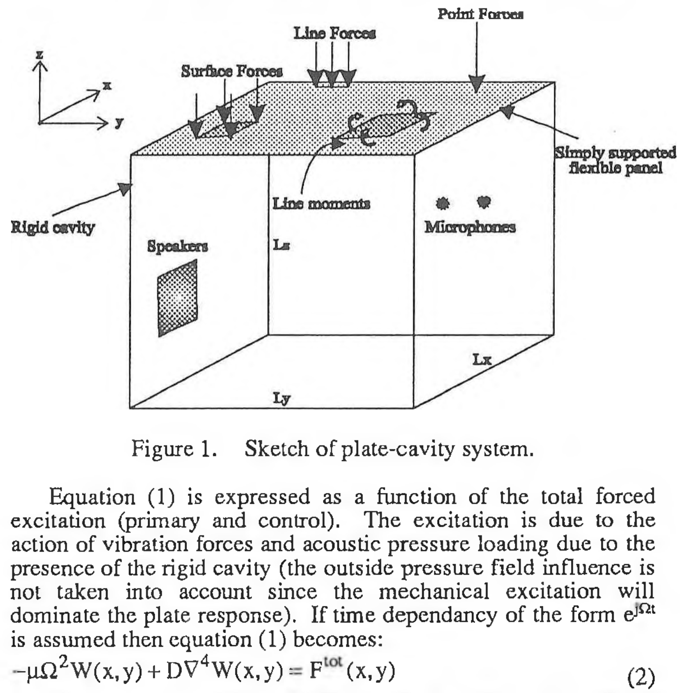

Figure 1. Sketch of plate-cavity system.

Equation (1) is expressed as a function of the total forced excitation (primary and control). The excitation is due to the action of vibration forces and acoustic pressure loading due to the presence of the rigid cavity (the outside pressure field influence is not taken into account since the mechanical excitation will dominate the plate response). If time dependancy of the form $e^{j\Omega t}$ is assumed then equation (1) becomes:

$$-\mu\Omega^2 W(x,y) + D\nabla^4 W(x,y) = F^{tot}(x,y) \qquad (2)$$

Expressing the plate displacement in terms of a modal summation basis:

$$W(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} w_{mn}(x,y) \qquad (3)$$

where $w_{mn}(x,y) = \sin(m\pi x/Lx)\sin(n\pi y/Ly)$

is the modal shape function for simply supported conditions. Substituting equation (3) in (2), integrating the result over the plate surface and using the orthogonality property of *in vacuo* plate modes we obtain an expression for the coupled plate-cavity system:

$$A_{mn}(\omega_{mn}^2 - \Omega^2)(\mu LxLy/4) = (\sum F^p(x,y) + \sum F^c(x,y))w_{mn}(x,y)$$
$$+\rho_o\Omega^2 \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\int_0^{Lx}\int_0^{Ly}(P^p(x,y,0)+P^c(x,y,0))\,dydx \qquad (4)$$

The pressure terms inside equation (4) can be expressed in all points as a function of the plate displacement and a Green function developed for the modelled cavity by using the Kirchoff-Helmoltz Theorem. The pressure is given by:

$$P(M) = -\iint_{S_c}\frac{\partial P(M_d)}{\partial n}.G(M,M_d)dS_c - \iint_{S_p}\frac{\partial P(x,y,0)}{\partial z}.G(M,(x,y,0))\,dS_p \qquad (5)$$

The first integral corresponds to the contribution of acoustic velocity sources and the partial derivative represents a velocity distribution. The second surface integral corresponds to the contribution of the plate movement and the partial derivative represents the acoustic/mechanic velocity continuity condition on the plate-cavity interface, which is given by the following expression: $\rho_o\Omega^2 W(x,y)$. The Green function is developed for a rigid walled paralellepipedic cavity. It's development brings:

$$G(M,M_d) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{r=0}^{\infty}\frac{\Phi_{pqr}(M).\Phi_{pqr}(M_d)}{\alpha_{pqr}(\omega_{pqr}^2 - \Omega^2)} \qquad (6)$$

where $\Phi_{pqr}(x,y,z) = \cos(p\pi x/Lx)\cos(q\pi y/Ly)\cos(r\pi z/Lz)$

is the modal shape function representation for a rigid cavity. The A_{mn} terms can be found by substituting equations (5) and (6) in (4), evaluating the surface integral at the plate-cavity interface and solving the resulting linear system. Using a viscous damping model for both plate and cavity, the pressure expression for an arbitrary point within the cavity space is found to have the following form for the case of surface force excitation and control (no acoustic velocity terms):

$$P(x,y,z) = -\rho_o c_o^2\Omega^2.\frac{LxLy}{\pi^2}\sum_{mn=1}^{\infty}\sum_{pqr=1}^{\infty}\left[\frac{(A_{mn}^p(\omega)+A_{mn}^c(\omega)).\Phi_{pqr}(x,y,z)}{\alpha_{pqr}(\omega_{pqr}^2 - \Omega^2 + j\eta_c\omega_{pqr}^2)}\right].$$
$$\left(\frac{1}{(m+p)_{odd}}+\frac{1}{(m-p)_{odd}}\right)\left(\frac{1}{(n+q)_{odd}}+\frac{1}{(n-q)_{odd}}\right) \qquad (7)$$

II. Control scheme and numerical results

Evaluation of the pressure equation at one point composed of the contribution of primary and control excitations can then be performed. Multiplying equation (7) with its complex conjugate form (separated into its primary and control constituents) and integrating the result over the cavity volume yields the average volumetric squared pressure as shown in equation (8):

$$\langle P_{ave}^2\rangle = 1/V_c.\iiint_{V_c}(P^p(M).P^p(M)^* + P^p(M).P^c(M)^* +$$
$$P^c(M).P^p(M)^* + P^c(M).P^c(M)^*)dV_c \qquad (8)$$

The expansion and rearrangement of this cost function into a quadratic form (having for unknown the point force actuation amplitude and phase) allows its minimization by using the unique minimum property of quadratic functions. A comparison has been attempted with a case found in Pan et al. [5] where the primary excitation was due to a plane acoustic wave acting perpendicular to the plate surface and where the control was achieved with a centered point force. The present model only accounts for vibration forces, thus the acoustic plane wave is replaced by an equivalent surface force. Qualitative comparison of the results obtained in figure 2 and the literature shows that the two sets of curves agree extremely well. This was somewhat predictable since very similar methods were used. Figure 2 shows the noise reduction achieved (NR is defined as the logarithmic ratio of the average volumetric quadratic pressure within the cavity to the quadratic surface force applied on the plate). The plane surface force and the point force actuator destructively interfere optimally in this particular case where control is applied in the plate center (both primary and control forces excite odd-odd plate modes only).

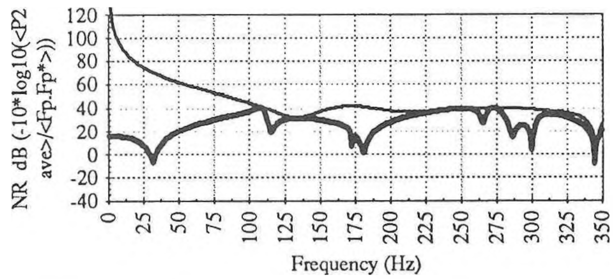

Figure 2. Noise reduction achieved with point force vibration control (___ with control, ____ without control).

As can be observed plate modes dominate the spectrum. Vibration control at the plate is found to be beneficial and the mechanism responsible for the reduction observed is termed "modal suppression". This phenomena is characterized by the reduction in modal vibration amplitudes and consequently the noise radiated. The first plate mode (efficient radiator) is well attenuated at 30 Hz while higher order less efficient radiator odd-odd modes are relatively less controlled. The comparison validates the exactness of the model and will allow further studies to be conducted with the use of multiple point forces for control and excitation. Investigation of control mechanisms and control optimization as well as other developments concerning the acoustic control aspects will be presented in the conference.

References

[1] Elliott, S.J.; Nelson, P.A.; Stothers, I.M.; Boucher, C.C.; Evers, J.F. and Chidley, B.,"In-flight experiments on the active control of propeller-induced cabin noise," AIAA conference paper, AIAA-89-1047 (1989).

[2] M. A. Simpson, T.M. Luong, C.R. Fuller and J.D. Jones,"Full-scale demonstration tests of cabin noise reduction using active vibration control," Journal of Aircraft **28**(3), 1990.

[3] L. O'Connor,"Putting a lid on noise pollution," Mechanical Engineering June 1991, 46-51.

[4] P. A. Nelson, A. R. D. Curtis, S. J. Elliott and A. J. Bullmore,"The active minimization of harmonic enclosed sound fields, Part I: Theory,"J. Sound Vib. **117**(1), 1-13.

[5] Jie Pan, C. H. Hansen and D. A. Bies,"Active control of noise transmission through a plate into a cavity: I. Analytical study," J. Acoust. Soc. Am. **87**(5), 2098-2108.

Appendix I

A_{mn}	plate modal amplitude	α_{pqr}	modal coupling coef.
c	speed of sound (m/s)	η	damping coef.
D	flexural rigidity (N.m)	μ	surface mass (kg/m^2)
L	plate dimensions (m)	ρ	volume density (kg/m^3)
F	force (N)	ω_{mn}	plate nat. angular freq.
m,n	plate modal indices	ω_{pqr}	cavity nat. angular freq.
p,q,r	cavity modal indices	Ω	angular frequency
P	int. cavity pressure (Pa)	∇^4	2nd order Laplace oper.
S	surface (m^2)	$\vec{n}$	normal vector to surface
V	volume (m^3)	M	point of evaluation
x,y,z	general system coordinates	M_d	disturbance point
	Subscripts		**Superscript**
c	cavity	p	primary excitation
p	plate	c	control excitation
o	air characteristics		