FREQUENCY AND THE SENSATION OF SOUNDS

Willy Wong and Kenneth H. Norwich
Institute of Biomedical Engineering and Departments of Physiology and Physics
University of Toronto, Toronto, Canada M5S 1A4

Psychoacoustics deals with the biological response to an auditory stimulus. This biological response, however, depends on the characteristics of this physical stimulus in a complex manner. For example, it would be an oversimplification to state that loudness of a pure tone depends solely on the amplitude of a sound pressure wave, and that pitch depends solely on the frequency. The physical quantities sound intensity, sound duration and frequency combine in a subtle manner to produce the psychoacoustic quantities loudness and pitch. In an attempt to isolate the effects of each physical variable, experiments have been conducted in the past to determine the relationship between:

a) loudness and intensity, with frequency held constant;
b) pitch and frequency, with intensity held constant.

The relationship between loudness/pitch and intensity/frequency has also been investigated using experiments on auditory discrimination. Discrimination experiments involve determining how much change in intensity (or frequency) is required to elicit a change in biological response. For example, a tone of 100 Hz can probably not be distinguished from a tone of 101 Hz at all relevant sound intensities, as long as the intensity is held constant between the two tones. Commonly one calculates the Weber Fraction, which is the change in intensity/frequency divided by the reference intensity/frequency, required to produce a change in sensation. The Weber fraction is calculated over a range of intensities (or frequencies). There are two types of Weber Fraction with which we shall be concerned, corresponding to Experiments a) and b) above:

c) fractional changes in intensity, with frequency held constant;
d) fractional changes in frequency, with intensity held constant.

We shall return to these experiments after giving a little theoretical background.

We have been developing, over a number of years, an entropic or informational approach towards quantifying human sensation. A sequence of papers have been published detailing our advances of this method [1,2]. A single, master equation of three parameters has been derived relating the variables biological response, constant sensory stimulus intensity, and time since onset of the sensory stimulus. This equation accounts quantitatively for nearly all experimental results relating the three sensory variables. The published form of this equation deals only with steady sensory inputs (constant intensity over time), but the equation functions over almost all sensory modalities. In particular, we are now interested in its predictive abilities in auditory psychophysics.

The entropy equation, in its simplest form, may be written as follows:

$$F = \tfrac{1}{2} k \ln(1 + \beta I^n / t), \qquad (1)$$

where F is biological response, I is auditory stimulus intensity, and t, the time since onset of stimulus, must be $\leq t_{max}$. The exponent, n, to which I is raised, can be identified with the Steven's exponent appearing in the power law of sensation. For audition, this exponent has a mean value of about 0.3 if I is measured in units of sound intensity. The response F, may be either impulse firing rates in auditory ganglion cells or loudness, with appropriate change in time scale. The derivation of this equation is given in [1,2,3]. Notice that the time variable divides into the intensity variable in the numerator. For constant I, this has the effect of decreasing the "effective intensity" over time and, consequently, decreasing the response F, as seen by Equation (1). A monotonic decreasing F with time corresponds to adaptation to a stimulus of constant intensity. In audition, F would represent either loudness decreasing over time, or the decrease of the firing rate in the primary sensory afferent neurons attached to the hair cells. This equation has been applied to the experiment of Yates et al. on the guinea pig auditory ganglion cells. The parameters k, β and n were obtained from curve fitting the data of Yates et al. [4]; the analysis can be found in reference [3].

For magnitude estimation experiments, the sound stimulus is applied for a constant duration of time, say t'. Equation (1) then takes on the following form:

$$F = \tfrac{1}{2} k \ln(1 + \beta I^n / t'). \qquad (2)$$

By setting $\gamma = \beta / t'$, we obtain

$$F = \tfrac{1}{2} k \ln(1 + \gamma I^n). \qquad (3)$$

Notice that, for small values of γI^n, we can expand the right hand side of Equation (3) in a Taylor series of the form $\ln(1 + x) \approx x$ to get

$$F = (\tfrac{1}{2} k \gamma) I^n. \qquad (4)$$

This equation is recognized as the power law of sensation. One can now appreciate why the parameter n in Equation (1) can be identified with the Steven's exponent in the power law of sensation. If we now let γI^n become large, we can approximate Equation (3) with the form $\ln(1 + x) \approx \ln(x)$ to obtain

$$F = (\tfrac{1}{2} kn) \ln(I) + const. \qquad (5)$$

which is the logarithmic law of sensation. Although it has been observed that, for most of the physiological range of I, *both* the power and the logarithmic laws hold to a high degree of approximation, both laws systematically deviate from the data at,

respectively, larger and smaller intensities. Equation (3) is a more general law of sensation embracing both the power and the logarithmic laws. Equation (3) describes all published data of the loudness-intensity type whether or not they conform to a straight line when plotted in a semilog or full log plot. To wit, consider the magnitude estimate data of Luce and Mo [5], which do not conform to a straight line with either type of graph, but are fitted well by Equation (3). The analysis of the data can be found in reference [3].

Returning to Equation (3), taking the derivative of F with respect to I and replacing the differentials with finite differences, one can arrive at an expression for the Weber Fraction describing discrimination experiments. Reference [3] provides the derivation of the equation, which has the following form,

$$\frac{\Delta I}{I} = \frac{2\Delta F / k}{n}(1 + \gamma^{-1}I^{-n}).\qquad(6)$$

Equation (6) can be applied to analyze the experiments of Riesz [6], who performed auditory intensity discrimination experiments (Experiment c) on human subjects. In fact, the equation he used to describe the data matches Equation (6) term for term. Although Riesz did not derive the equation he used, offering the equation only as an empirical fit for the data, we have now been able to derive the equation theoretically. There are various other auditory experiments involving intensity embraced by the seminal Equation (1), including the SDLB effect and some of Békésy's results.

We now wish to incorporate a frequency variable into Equation (1) in order to account for frequency effects in the sensation of sounds. Notice that, if we were to utilize Equation (1) to describe pitch sensation, the time variable becomes "extraneous", in the sense that one does not adapt to the frequency of sound. In other words, the time variable, in pitch sensation, is used for something other than adaptation. We can associate the inverse of time with frequency. For a pure tone, inverse time will simply be the frequency of the tone. For a more complex tone, Schouten's theory of hearing postulates that a non-linear filter in the ear allows the ear to pick up the fundamental frequency of oscillation of a complex tone (what Schouten calls a *residue*) [7]. The modified equation would now have the following form, where f is the relevant frequency:

$$F = \tfrac{1}{2}k\ln(1 + \beta I^n f).\qquad(7)$$

The work of Linsay and Norman [8] provides an excellent of test of Equation (7). At a fixed sound intensity, they determined experimentally how pitch changes with frequency (Experiment b). Furthermore, the empirical equation they used to fit their data is identical to Equation (7),

$$F = 2410.\ln(1 + 1.6 \times 10^{-3} f),\qquad(8)$$

providing some confirmation of the validity of our inverse time equation.

An additional equation can be derived to account for frequency discrimination experiments (Experiment d). The derivation is mathematically identical to the derivation of Equation (6) and the Weber Fraction function for frequency takes on a similar form to it:

$$\frac{\Delta f}{f} = 2\frac{\Delta F}{k}[1 + (\beta I^n f)^{-1}].\qquad(9)$$

This equation provides good prediction of the data of Shower and Biddulph [9], who did experiments to determine the Weber Fractions of frequency as a function of frequency for constant sound intensity. The equation also predicts that $\Delta f / f$ will diminish for increasing I as shown by Shower and Biddulph.

In summary, while experiments a) through d) have been analyzed by the experimenters themselves, their method of analysis is often empirical or applicable only to their own experiments. We now offer a unified approach to the study of auditory psychophysics by proposing that a *single* equation can account for experiments a) to d). To do so, we have reinterpreted a variable in our original equation so that it can now account for frequency effects in the sound stimulus.

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