AN ANALYTICAL APPROACH FOR THE FREQUENCY RESPONSE OF A MULTILAYER DISC

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Introduction :

The vibro-acoustic design of multilayer structures is becoming more and more important but also quite difficult to model due to the complexity of such structures. The first step to get over it is to find a simple yet accurate formulation to treat the case of a constrained viscoelastic layer.

Multilayer discs are commonly used in many mechanical systems : grinding wheels, gears, circular saw blades, ... This paper presents a model of such an arrangement and the determination of both its natural frequencies and mean quadratic velocity. For this purpose, a multilayer disc excited by a harmonic point force is used. The determination of the free and forced frequency responses is achieved through a variational approach combined with a Rayleigh-Ritz type approximation.

Statement of the problem :

A multilayer disc, clamped at its inner radius and free at its outer radius, is considered (figure 1). Each layer is described by its thickness, specific mass, Poisson's ratio and complex Young's modulus. The classical assumptions of Reissner[1] and Mindlin[2] for thin plates are used :

i) negligible axial traction-compression : $\sigma_{zz} = 0$;
ii) negligible torsion around the axis : $\tau_{r\theta} = 0$;
iii) pure bending;
iv) linear shear deformation;
v) each layer is made of a homogeneous and isotropic material.

Hence, the allowed motion of the disc comprises five *elementary displacements* :

i) a membrane effect described by two longitudinal displacements : u_r, u_θ;
ii) a pure bending described by a transversal displacement : w;
iii) a linear shear described by two agles : ϕ_r, ϕ_θ.

Therefore, the displacement field can be written :

$$(a)..\begin{cases} U_r^{cn}(L,t) = u_r^{cn}(L,t) + (R_n - z)\cdot\left[\partial w/\partial r(L,t) + \phi_r^{cn}(L,t)\right] \\ U_\theta^{cn}(L,t) = u_\theta^{cn}(L,t) + (R_n - z)\cdot\left[\partial w/\partial\theta(L,t) + \phi_\theta^{cn}(L,t)\right] \\ U_z^{cn}(L,t) = w(L,t) \end{cases}$$

where : - cn stands for layer n;
- R_n is the distance between the top surface and the neutral fiber of the n^{th} layer;
- L is defined as the triplet (r,θ,R_n).

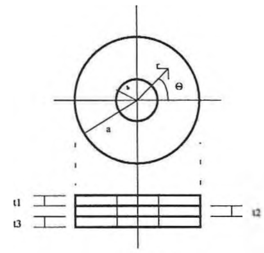

Figure 1

Methodology :

To calculate both free and forced vibration of the multilayer disc, the variational approach is used. The Hamilton's functional of the system is given by :

$$(b)..H = \iiint_{Vol} [T - V + W]\cdot dVol$$

where : - T is the sum of the kinetic energy of *each* layer;
- V is the sum of the deformation energy of *each* layer;
- W is the work applied to the entire disc;
- Vol is the total volume of the disc.

Using the continuity between the layers, the kinetic and deformation energies of each layer are written in terms of the displacement field of the *first* layer. To do so, the following assumptions are made, Guyader[3] :

i) continuity of the slopes throughout the layers :
- dw/dr and $dw/d\theta$ equal a constant;

ii) physical cohesion of the disc at the *interface* :
- the continuity of the displacement U_r, U_θ, U_z;
- the continuity of the shear stresses τ_{rz}, $\tau_{\theta z}$.

These continuity conditions can be written as a transfer matrix between the elementary displacements of the n^{th} layer (cn) and those of the *first* layer (c1) :

$$(c)..\begin{Bmatrix} u_r^{cn} \\ u_r^{cn} \\ \partial w/\partial r \\ \partial w/\partial\theta \\ \phi_r^{cn} \\ \phi_r^{cn} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & K_2 & 0 & 0 \\ K_3 & 0 & K_4 & 0 & 1 & 0 \\ 0 & K_3 & 0 & K_5 & 0 & 1 \end{bmatrix} \cdot \begin{Bmatrix} u_r^{c1} \\ u_r^{c1} \\ \partial w/\partial r \\ \partial w/\partial\theta \\ \phi_r^{c1} \\ \phi_r^{c1} \end{Bmatrix}$$

where :

$\cdot K_{1,2} = C_{55,66}^{c1}/C_{55,66}^{cn}$

$\cdot K_3 = 1/2\cdot(t_n + 2\cdot t_{n-1} + 2\cdot t_{n-2} + \ldots + 2\cdot t_2 + t_1)$

$\cdot K_{4,5} = C_{55,66}^{c1}/2\cdot\left[(t_n/C_{55,66}^{cn}) + 2\cdot(t_{n-1}/C_{55,66}^{c(n-1)}) + \ldots + 2\cdot(t_2/C_{55,66}^{c2}) + (t_1/C_{55,66}^{c1})\right]$

C^{cn}_{ij} being the coefficient (i,j) of Hook's matrix (stress-strain relations) satisfying the previous assumptions.

To find the disc eigenfrequencies and mean quadratic velocity, the method of assumed mode is used. A polynomial set of admissible trial function with unknown amplitudes is used. Introducing the displacement field [Eq.(a)] in the functional [Eq.(b)] and using the variational principle, one arrives to a system of linear equations for the unknown amplitudes.

Results :

All the results presented in this section have been computed for a 150 mm outer radius and 25 mm inner radius disc submitted to a 1 N force applied at a radius of 140 mm. Furthermore, every calculation has been conducted using 8 nodal diameters and 8 nodal circles.

Figure 2 shows the mean quadratic velocity of two identical discs : one is made of a single steel layer, the other is made by artificially subdividing the disc into three layers. This figure shows the accuracy of the model used.
Figure 3 displays a comparison between a disc made of one elastic layer placed between two free viscoelastic layers and an equivalent one layer disc. The theory behind the determination of the characteristics of the equivalent disc is taken from Mezache[4]. The agreement between the two approaches is excellent.
Finally, figure 4 presents the mean quadratic velocity of two identical steel base discs, the second having a supplementary 0.25 mm thick viscoelastic (3M ISD 112) layer constrained by a 0.5 mm thick steel layer. This figure shows the importance of damping achieved through a constrained viscoelastic treatment.

Conclusions :

- A new formulation for multilayer disc has been developed;
- an application to a three layer disc has been successfull;
- the efficiency of a constrained viscoelastic layer has been demonstrated, even, and surprisingly, in the low frequency range.

Acknoledgement :

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References :

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Influence of the number of layers

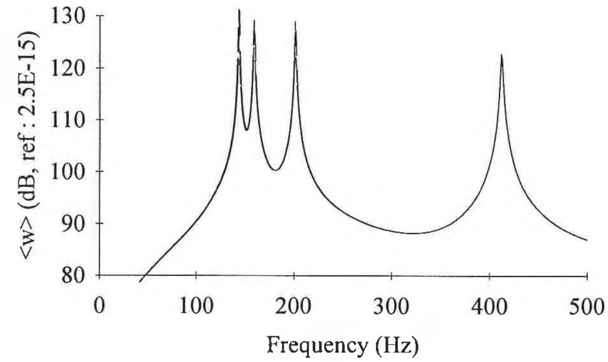

Figure 2

Comparison with an equivalent disc

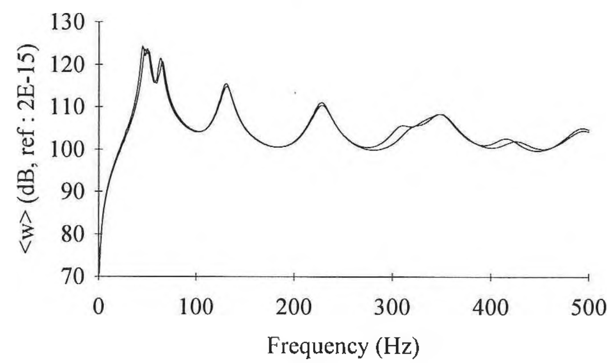

Figure 3

Influence of a constrained visco layer

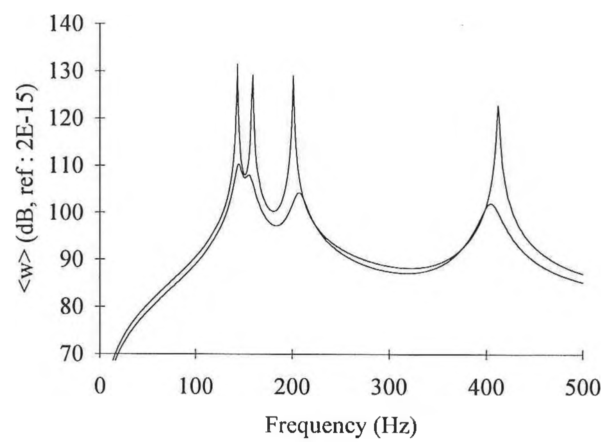

Figure 4