# Relationship of Underwater Acoustic Intensity Measurements to Beamforming

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**Introduction**

The standard method of determining the directional properties of underwater acoustic fields is to coherently combine the weighted pressure field measurements made by spatially distributed sensors. An alternative approach is to make measurements of other properties of the acoustic field, in addition to pressure, at single points in space. For example, measurements of acoustic particle velocity can be combined multiplicatively with those of pressure to determine the vector acoustic intensity. The purpose of this presentation is to discuss the relationship between the information obtained on the directionality of the sound field by the standard beamforming techniques and vector acoustic intensity.

**I. Standard vs Single Point Beamforming**

The link between the standard approach to beamforming and beamforming using single point measurements is provided by the Taylor series expansion of the acoustic pressure field. The expansion in space about the measurement point $\underline{x}_o$ is:

$$p(\underline{x}, t) = p(\underline{x}_o, t) + \nabla p(\underline{x}_o, t) \cdot \Delta \underline{x} + \frac{1}{2} \Delta \underline{x}^T \begin{bmatrix} \text{Matrix of} \\ \text{2nd Derivatives} \end{bmatrix}_{\underline{x}_o} \Delta \underline{x} + \cdots \quad (1)$$

where $\Delta \underline{x} \equiv \underline{x} - \underline{x}_o$. What Eq. (1) says is that the measurement of acoustic pressure and its higher-order spatial derivatives at a single point in space is equivalent to the measurement of acoustic pressure in a volume about the measurement point. Therefore, the techniques used in beamforming with spatially distributed pressure measurements can be applied to single point measurements of pressure and its spatial derivatives. In particular, since acoustic particle velocity at a given frequency is proportional to the first order spatial gradient of pressure, then high resolution beamforming techniques can be used with simultaneous pressure and particle velocity measurements. Application of minimum variance (Capon) beamforming techniques to these types of single point measurements has been discussed previously [Acoust. Soc. Am. meetings, May, 1992 and May, 1993]. Some examples using real ocean data also will be given in this presentation.

The distance, $\Delta \underline{x}$, that the pressure field can be extrapolated from the measurement point with a given error and with an expansion to a given order is dependent upon the degree of spatial variability of the pressure field. The maximum spatial variability in any direction at a given frequency cannot exceed that determined by the acoustic wavelength, at least for non-evanescent acoustic fields. (Otherwise, the velocity of the energy flow required to support the spatial structure would exceed the speed of sound in the fluid, which is physically impossible). Therefore the effective spatial aperture of a single point array is related to the acoustic wavelength, rather than being determined by a fixed inter-element spacing as with conventional arrays. Thus, single point arrays are frequency-adaptive; their effective aperture decreases with increasing frequency. The result of this property is that the plane wave response (or beampattern) for single point arrays is independent of frequency. This result also implies that no grating lobes exist, i.e., spatial aliasing cannot occur since a sampling in space is not being performed.

The phenomenon of superdirectivity is directly related to the Taylor series expansion of the pressure field. That is, superdirectivity arises for a spatially distributed hydrophone array when the directivity index is maximized as a function of the element weights, and the interelement spacing becomes smaller than half the acoustic wavelength [Pritchard, J. Acoust. Soc. Am., Nov, 1954]. One can show that, as the ratio of the interelement spacing to the acoustic wavelength approaches zero, the weights for a "linear point array" approach the finite difference approximations to the spatial derivatives of pressure given in Eq. (1). The instability that results when the weights become large and of opposite sign can be avoided by the use of alternative transduction methods suggested by the physical interpretation of the spatial derivatives of pressure, e.g., when the measurement of a component of acoustic particle velocity replaces the measurement of pressure at two closely-spaced points.

Note that high resolution beamforming techniques have been applied to single point measurements in other fields, e.g., the estimation of ocean surface gravity wave directional spectra from "pitch-and-roll" buoys [Oltman-Shay and Guza, J. Phys. Oceano., Nov, 1984].

**II. Acoustic Intensity and Beamforming**

The central quantity in array processing is the data cross spectral (or cross correlation) matrix. For simultaneous acoustic pressure and particle velocity measurements at a single point, the cross-spectral matrix is:

$$[S(f)] \equiv \begin{bmatrix} S_p(f) & z_x S_{px}(f) & z_y S_{py}(f) & z_z S_{pz}(f) \\ \cdots & z_x z_x^* S_x(f) & z_x z_y^* S_{xy}(f) & z_x z_z^* S_{xz}(f) \\ \cdots & \cdots & z_y z_y^* S_y(f) & z_y z_z^* S_{yz}(f) \\ \cdots & \cdots & \cdots & z_z z_z^* S_z(f) \end{bmatrix}$$

where the symbol "*" indicates complex conjugation. Conversion factors, indicated by $z_x$, $z_y$, and

$z_z$, must be defined in order to convert the terms involving particle velocity into units of pressure. Typically, the conversion factors are set equal to the $\rho_o c$ of the medium. The output autospectrum of the single point beamformer under application of the conventional beamforming method is:

$$D_B(f) = \underline{e}^H\,[S(f)]\,\underline{e}$$

where the plane wave steering vector, $\underline{e}$, is given in terms of the direction cosines as:

$$\underline{e}^H = \frac{1}{2}[\,1\ \cos(\beta_x)\ \cos(\beta_y)\ \cos(\beta_z)\,]$$

Applying the minimum variance approach introduced by Capon [Proc. IEEE, 1969], the single point beamformer autospectrum becomes [D'Spain *et al*, Oceans 92 Conf., Nov, 1992]:

$$D_C(f) = \left[\underline{e}^H\,[S(f)]^{-1}\,\underline{e}\right]^{1}$$

One part of the link between acoustic intensity and single point beamforming is now clear. That is, the active acoustic intensity at a given frequency is equal to the real part of the cross spectrum between pressure and particle velocity, and the three orthogonal components of this cross spectrum are the off-diagonal elements in the first row and column of the data cross spectral matrix.

Physical interpretations can be provided for the other quantities in the data cross spectral matrix, i.e., its trace is proportional to the total acoustic energy density and the properties of the 3-by-3 particle velocity submatrix are describable in terms of the polarization of acoustic particle motion. Relationships among the various elements of this matrix can be derived from the basic acoustic principles of conservation of mass and momentum [D'Spain *et al*, J. Acoust. Soc. Am. Mar, 1991].

The second part of the link between single point beamforming and acoustic intensity can be obtained by integrating the conventional beamformer output autospectrum weighted by the look direction vector, $D_B(f)\hat{l}$, over all look directions, $\hat{l}$. The result, for purely real conversion factors, is the vector sum of the active intensity components, $C_{pj}(f)$. For example, for zero elevation angle, the integration over azimuth $\theta$ gives:

$$\frac{1}{2\pi}\int_0^{2\pi} D_B(f)\hat{l}\,d\beta = z_x C_{px}(f)\hat{x} + z_y C_{py}(f)\hat{y}$$

where $\hat{l} = \cos(\theta)\hat{x} + \sin(\theta)\hat{y}$ and when the conversion factors, $z_x$, $z_y$, are purely real.

In summary, beamforming decomposes the sound field into spatial frequency components. The decomposition is performed at first order in the acoustic variables where the principle of superposition is valid. Acoustic intensity, on the other hand, measures the total, net flow of acoustic energy at a single point in space. It accounts for the full interaction of all the wave field's spatial spectral components.

## III. MPL's Vertical DIFAR Array

MPL's Swallow floats, freely drifting sensor systems that measure both acoustic pressure and the three components of acoustic particle velocity, thereby permitting the calculation of acoustic intensity, are discussed elsewhere at this meeting [Desharnais and D'Spain, this meeting; also D'Spain *et al*, IEEE J. Ocean Engin., Apr, 1991]. Another MPL system capable of making underwater acoustic intensity measurements is the vertical DIFAR array [Nickles *et al*, Oceans 92 Conf., Nov, 1992]. The array is composed of 16 elements, with an inter-element spacing of 15 m. Each of the elements contains three orthogonally-oriented geophones to measure acoustic particle velocity and a hydrophone to measure pressure. The array elements also contain a flux-gate compass for measuring the orientation of the horizontal geophones, a programmable high frequency data acquisition for acoustic element localization, a preamp with programmable gain from 0 to 120 dB, and a 16-bit A/D converter that provides additional dynamic range. Data collected by this array will be used to illustrate some of the preceeding points of this presentation.

## IV. Extension of Single Point Beamforming and Energetics to Higher Order

A physical interpretation of the term at second order in the Taylor series expansion in Eq. (1) can be made by realizing that in an acoustic field in an otherwise stationary fluid, the spatial gradient of the acoustic particle velocity is equal to the acoustic rate of strain. Referring to the second rank strain rate tensor as $\dot{\varepsilon}$, the following relation is true:

$$p\,\dot{\varepsilon} = \nabla(p\underline{v}) + \frac{\partial}{\partial t}\left\{\frac{1}{2}\rho_o(\underline{v})(\underline{v})\right\}$$

It can be shown that the trace of $p\dot{\varepsilon}$ yields the conservation of acoustic energy equation. For stationary, stochastic fields, in the frequency domain:

$$S_{p\dot{\varepsilon}}(f) = \nabla \underline{C}_{p\underline{v}}(f)$$

Therefore, the cross spectrum between the acoustic pressure and the acoustic rate of strain provides information on the local spatial heterogeneity of the active acoustic intensity.

The second order Taylor series term also improves the performance of the single point beamformer. Whereas the beamformer at first order has a maximum directivity index (DI) of 6 dB, a main lobe width of 105 deg, and can distinguish two sources in azimuth, the second order beamformer has a maximum DI of 9.5 dB, a main lobe width of 65 deg, and can distinguish four distinct sources in azimuth.

Proposed modifications to MPL's vertical DIFAR array to permit approximate measurements of the acoustic rate of strain will be presented.

## V. Summary Recommendations

With simultaneous measurements of acoustic pressure and particle velocity, single point beamforming is most applicable in cases where the directionality of a source of interest is to be estimated in the presence of another loud, interfering source. The use of the active acoustic intensity vector is appropriate when the number of sources is large, as in ocean ambient noise studies.

## References

A complete set of references for this presentation is available.