New Design Criterion for Walking Vibrations

by D.E. Allen
Institute for Research in Construction, National Research Council Canada, Ottawa, Ontario, Canada. K1A 0R6

The modern trend to lighter, more flexible floor systems with less damping has resulted in more complaints from walking vibrations. A new design criterion for walking vibration of wide applicability to all floor systems with natural frequencies less than 10 Hz has recently been developed (Allen and Murray, 1993). The new criterion is intended to replace the criterion contained in Appendix G of the CSA Standard CAN/CSA S16.1-M89. Recent experience has shown that the S16.1 criterion has limited application.

The new design criterion, plotted for use in practice in Figure 1, is given by

\[ \beta W \geq K \exp(-0.35 f_0) \]  

where:
- \( f_0 \) = fundamental natural frequency of the floor structure (Hz)
- \( W \) = weight of a floor panel representing its fundamental mode of vibration (kN)
- \( \beta \) = damping ratio of the floor system (Table 1)
- \( K \) = a constant (Table 1)

The basis for Equation (1), described in Allen and Murray (1993), is resonance. Resonance occurs when a harmonic component of a repeated force, such as footsteps, corresponds to the natural frequency of the floor structure. The peak resonance acceleration for any harmonic force is equal to the peak harmonic force divided by the equivalent mass \((W/2g)\) times twice the damping ratio \((2\beta)\). Walking produces significant harmonic forces at approximately 2, 4, 6 and 8 Hz, i.e., the first four harmonics. Resonance occurs if the floor frequency is equal to any of these harmonics, and this is often the case if the fundamental frequency of the floor is less than 10 Hz. The harmonic force, however, decreases with increasing harmonic. Also there are other factors reducing the real situation as compared to pure resonance, such as the number of footsteps near mid-span and the fact that the person annoyed is some distance from the walker. A more important factor is human reaction to vibration which depends very much on the use and occupancy of the floor. All these factors are combined together in the constant \( K \) given in Table 1 and used in Figure 1 (Allen and Murray, 1993).

To apply the design criterion (Figure 1 or Equation 1), the designer needs to estimate the parameters \( \beta, f_0, \) and \( W \). The damping ratio, \( \beta \), can be estimated with the help of Table 1, which applies to typical concrete deck and steel floor systems. The other two parameters require more care:

**Natural Frequency.** Natural frequency, \( f_0 \), is a function of floor stiffness and mass. The stiffness of the floor structure, however, is determined by the flexibility of the floor joists or beams, plus the flexibility of the supporting girders. A useful formula for design of simply-supported joist-and-girder floor systems is

\[ f_0 = \frac{18}{\sqrt{\Delta_j + \Delta_g}} \]  

where \( \Delta_j \) and \( \Delta_g \) are the deflections (in mm) of the joist and girder under the weight that they support. Composite action can often be assumed for most concrete-deck-steel-floor systems but there is a reduction for certain systems, such as concrete decks separated from girders by joist shoes (Allen and Murray, 1993).

**Weight of Equivalent Panel:** In the case of a simply-supported footbridge the panel weight is simply equal to the weight of the suspended footbridge, i.e.,

\[ W = w B L \]  

where \( w \) is the distributed mass weight (in kPa), \( L \) is the span, and \( B \) is the width of the panel. For simple one-way joist, beam or girder systems on rigid supports the equivalent panel is defined by the span \( L \) with width, \( B \), determined from

\[ B = C \left( \frac{D_y}{D_x} \right)^{1/4} L \]  

where \( C \) = constant.
= 2.0 for joists or beams in most areas,
= 1.0 for joists or beams beside interior openings,
= 1.6 for girders supporting joists on top,
= 1.8 for girders supporting beams connected to webs.

\[ D_y = \text{flexural rigidity (per unit width) transverse to the direction of the span} \]

\[ D_x = \text{flexural rigidity (per unit width) in the direction of the span}. \]

When flexible joists or beams rest on flexible girders, the equivalent weight is determined from the interaction formula:

\[ W = \frac{\Delta_j}{\Delta_j + \Delta_g} W_j + \frac{\Delta_g}{\Delta_j + \Delta_g} W_g \]  \hspace{1cm} (5)

where the subscripts \( j \) and \( g \) refer to the joist and girder panels respectively.

More guidance on estimating these parameters is contained in Allen and Murray (1993).

REFERENCE


Table 1. Values of \( K \) and \( \beta \) for use in Eqn. (1)

<table>
<thead>
<tr>
<th></th>
<th>( K )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Offices, residences, churches</td>
<td>58</td>
<td>0.03*</td>
</tr>
<tr>
<td>Shopping Malls</td>
<td>20</td>
<td>0.02</td>
</tr>
<tr>
<td>Footbridges</td>
<td>8</td>
<td>0.01</td>
</tr>
</tbody>
</table>

* 0.05 for full height partitions, 0.02 for floors with few non-structural components (ceilings, ducts, partitions, etc.) as can occur in churches.

Figure 1. New Design Criterion for Walking Vibrations
FORCED VIBRATION OF A STEEL CANTILEVER BEAM WITH THICK VISCOELASTIC DAMPING LAYER

D.C. Stredulinsky and J. P. Szabo
Defence Research Establishment Atlantic, P.O. Box 1012, Dartmouth, Nova Scotia B2Y 3Z7

Introduction
The control of radiated noise is important for naval applications. DREA has been conducting research on the application of elastomeric materials to anechoic, decoupling and vibration damping tiles for ship hulls and machinery vibration isolation systems. As well, over the past twenty years DREA has developed, in-house and through contract, the general purpose finite element (FE) code VAST [1] for vibration and strength analysis of complicated structures. Recently a direct frequency response method was incorporated in VAST [2, 3] to allow modelling of frequency dependent dynamic mechanical properties. DREA has also developed methods for measuring the dynamic mechanical properties of elastomeric materials in the frequency domain [4].

This paper considers the vibration of a cantilever beam with a thick layer of viscoelastic damping material bonded to one surface. The measured forced response is compared to numerical results obtained using the VAST direct frequency response method in conjunction with measured dynamic mechanical properties for the damping material. Predictions of the composite system loss factor, made using VAST and independently using a code PREDC, are also compared to the experimental data. PREDC is a computer program obtained from University of Dayton, Ohio which employs analytical equations for free and constrained damping treatments of beams and rectangular plates.

The VAST Direct Frequency Response Method
The VAST direct frequency response method assumes a steady state harmonic forcing function and results in the following system of equations for the displacement of nodes in the FE model

\[
([K] + j[K']) - \omega^2 [M] \{\delta\} = \{F(\omega)\}
\]

where \([K]\) and \([K']\) are the real and imaginary parts of the global stiffness matrix (generally frequency dependent), \([M]\) is the mass matrix, \([F(\omega)]\) is the complex load vector, \(\{\delta\}\) is the complex nodal displacement vector, \(\omega\) is the forcing frequency and \(j = \sqrt{-1}\). This system of equations was solved at each specified forcing frequency to obtain the amplitude and phase for each component of the nodal displacement vector \(\{\delta\}\).

The dynamic mechanical properties for groups of elements in the VAST FE model can be represented using a complex Young's modulus \(E^*(\omega) = E(\omega)(1 + j\eta(\omega))\) where \(\eta\) is the material damping or loss factor. The frequency dependence of \(E(\omega)\) and \(\eta(\omega)\) can be specified using tables of frequency weighting values with linear interpolation between values or by using quadratic polynomials over a specified frequency range.

The Cantilever Beam Description
The cantilever beam considered is shown in Figure 1. The steel beam was 9.5 mm thick and clamped between steel blocks at one end. A 27 mm thick layer of EAR Isodamp C-1002 viscoelastic damping material was bonded to the upper surface of the beam. The Young's modulus \(E = 2.07 \times 10^5\) MPa, Poisson's ratio \(\nu = 0.3\) and density of 7870 kg/m\(^3\). The Young's modulus \(E\) and loss factor \(\eta\) for the damping material, shown in Figure 2, were determined using a direct stiffness method [4]. The fitted curves, required for the PREDC program, were also used to generate tables of frequency weighting values for the FE analysis. The density of the damping material was 1280 kg/m\(^3\).

Viscoelastic damping materials typically have Poisson's ratios near 0.5 (incompressible) in the 'rubbery' region, decreasing through a transition region to a value of 0.3 in the 'glassy region' [6]. The PREDC code assumed a Poisson's ratio of 0.5. The VAST FE code presently cannot consider a truly incompressible material so that a Poisson's ratio of 0.47 was used in the FE analysis.

The Beam Forced Vibration Response
The measured forced response was reported in reference [5] both for the bare beam and the beam with damping layer. A vibration exciter was used to apply a load at the centre-line, 9 mm from the tip, normal to the bottom surface of the beam. The applied force was measured with a force transducer and the acceleration of the top surface of the beam measured with an accelerometer placed at several locations along the centre-line.

The forced response of the damped beam was predicted using the direct frequency response method in VAST Version 7.1. Loss factors extracted from the bare beam experiment were used for steel in the FE analysis of the damped beam.