EVALUATION OF APPROPRIATE SAMPLE SIZE FOR MEASUREMENT OF VIBRATION LEVELS INDUCED BY RAIL TRANSIT VEHICLES

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1 INTRODUCTION

Building vibration levels induced by rail transit vehicles under usual operational conditions can vary from one vehicle to another. In order to obtain meaningful vibration levels, measurements of building and (or) ground vibrations should be performed for an appropriate number of vehicle pass-bys. In a recent rail transit system vibration study by the authors, the number of vehicle pass-bys for a particular vehicle population was chosen so that the mean vibration level of the sampled vehicles is reasonably close to the true mean of vibration levels induced by all vehicles in the population. This paper presents statistical analysis procedures, based on Kreyszig (1979) and Ang & Tang (1975), that can be used to determine an appropriate sample size to achieve a certain confidence range of the true mean vibration level, e.g. ±3 dB of the sample mean, for a certain confidence level, e.g. 95%. The results obtained using these statistical procedures are reported for subway train and streetcar populations of the rail transit system studied by the authors. In addition, verification of the appropriateness of the calculated sample sizes is presented.

2 CONFIDENCE INTERVAL AND LEVEL

Supposing that a sample of (n) vehicle pass-bys is used, the mean vibration level is calculated as

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad (1)$$

and the standard deviation is calculated as

$$s = \left(\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^{1/2} \qquad (2)$$

In this study, vibration signals induced by rail transit vehicles were processed individually to obtain 1/3 octave band and overall frequency-weighted vibration levels using the computer program TOAP-Version 4 (software for 1/3 octave analysis and frequency weighting using recursive digital filtering on PCs; see Al-Hunaidi et al. (1992) for details). Rms levels for 1/3 octave bands in the range from 10 to 125 Hz and for the frequency-weighted vibration signals were calculated using linear time integration with a 1 second integration time. For each vehicle pass-by, only maximum rms levels for each 1/3 octave band and the frequency-weighted signal were retained. The vibration level (x_i) in Eqs. 1 and 2 is the maximum rms vibration level of the i^{th} vehicle pass-by. Assuming that the vibration levels (x_i) induced by the population of each type of vehicle, e.g. subway trains or street cars, to have a normal density distribution, there is a certain chance (γ) that the true mean of vibration levels (μ) is in the range

$$\bar{x} - k \le \mu \le \bar{x} + k \qquad (3)$$

where

$$k = s.c / \sqrt{n} \qquad (4)$$

and c is the solution of the equation

$$F(c) = \frac{1}{2}(1+\gamma) \qquad (5)$$

found from t-Distribution tables with (n-1) degrees of freedom. In this study, the confidence level was taken equal to 95%. If the true standard deviation of the population (σ) is known, Eq. (4) becomes (for a 95% confidence level):

$$k = 1.96\,\sigma / \sqrt{n} \qquad (6)$$

When the sample size (n) is large, e.g. >20, the standard deviation of the sample (s_{large}) is a good estimation of the standard deviation of the total population (σ). Hence using s_{large} as an approximation for σ, the sample size required for a 95% confidence interval (k) can be calculated using Eq. (6).

In order to determine how accurate is the estimated standard deviation of the total population (taken as the standard deviation of a very large sample s_{large}), the following equation can be used to calculate the 95% confidence interval for the true standard deviation (σ):

$$\frac{s_{large}^2}{1+1.96\sqrt{2/(n_{large}-1)}} \le \sigma^2 \le \frac{s_{large}^2}{1-1.96\sqrt{2/(n_{large}-1)}} \qquad (7)$$

3 GOODNESS OF FIT: CHI-SQUARE TEST

The hypothesis of normal distribution for the vibration levels induced by the population of a type of vehicles can be tested using the chi-square test. Only overall frequency-weighted levels were used in this test. The test is performed as follows:

- Subdivide the vibration levels x_j into K intervals $I_1, I_2, \ldots, I_K$ such that each interval contains at least 5 values. Determine the number b_j of values in each interval.
- Using the normal distribution function

$$F(x_j) = \phi\left(\frac{x_j - \bar{x}}{s}\right) \qquad (8)$$

calculate the probability p_j that a vibration level assumes a value in the interval I_j (note: $\phi(z)$ is the distribution function of the normal distribution with mean equal to 0 and variance equal to 1. It is tabulated under normal distribution in most statistics textbooks). Then compute the number of vibration levels theoretically expected in each interval, i.e.

$$e_j = np_j \qquad (9)$$

where

$$p_j = \phi\left(\frac{x_j - \bar{x}}{s}\right) - \phi\left(\frac{x_{j-1} - \bar{x}}{s}\right) \qquad (10)$$

- Compute the deviation

$$\chi_o^2 = \sum_{j=1}^{K} \frac{(b_j - e_j)^2}{e_j} \tag{11}$$

- Choose a significance level α (e.g. 2.5% or 5%)
- Determine the solution for c in the following equation

$$P(\chi^2 \le c) = 1 - \alpha \tag{12}$$

from chi-square distribution tables with $K - r - 1$ degrees of freedom, where r is the number of estimated parameters (e.g. r=2 if the mean and standard deviation are estimated). In Eq. 12, P designates the probability of the event $\chi^2 \le c$.

- If $\chi_o^2 \le c$, do not reject the hypothesis.

4 CALCULATION PROCEDURE

In step form, the procedure used in this study to determine the appropriate sample size (n) was as follows.

Step 1

At selected locations, one for subway trains and another for streetcars, measurement of vertical ground vibration level for about 100 vehicle pass-bys was performed. It is assumed that operational conditions for transit vehicles at other locations along the rail transit system are similar to those at the selected locations. This is a reasonable assumption, except perhaps for a sloping rail track. For each 1/3 octave band and the overall frequency-weighted signal, the standard deviation of the vibration levels in these large samples is calculated using

$$s_{large} = \left(\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x}_{large})^2 \right)^{1/2} \tag{13}$$

where

$$\bar{x}_{large} = \frac{1}{n} \sum_{i=1}^{n} x_i \tag{14}$$

The 95% confidence range of the true standard deviation is calculated using Eq. 7.

Step 2

The assumption of normal distribution for the measured vibration levels is verified for the overall weighted vibration level using the chi-square test with a 2.5% significance level as described above.

Step 3

The appropriate sample size (n) for each 1/3 octave band is determined as that which yields a confidence range defined in Eq. 3 with k = 3 dB and a 95% confidence level. The size (n) is calculated using Eq. 6, but using s_{large} instead of σ, i.e.

$$n = \left(1.96\, s_{large} / k\right)^2 \tag{15}$$

The appropriate sample size is taken as the maximum of the values obtained for the various 1/3 octave frequency bands or overall frequency weighted signal.

Step 4

The appropriateness of the sample size (n) calculated in Step 3 is verified using a few randomly selected samples from the large sample described in Step 1. This is done by checking if the confidence range obtained with these samples and calculated using Eq. 4 for a 95% confidence level is within the range used in Step 3.

Note: For measurements to be performed after deciding on the sample size, it is suggested that the confidence interval be calculated using Eq. 4 for the 95% confidence level and reported for each 1/3 octave band at each measurement station.

5 RESULTS

The statistical procedures described in the previous sections were applied to determine appropriate sample sizes for measurement of vibration levels induced by subway train and streetcar populations of a major rail transit system.

Subway Trains

The assumption of normal distribution for vibration levels was checked using overall frequency-weighted rms vibration levels induced by 99 subway train pass-bys. Vibration signals were measured in the vertical direction on the ground in front of a residential home. Calculations for the chi-square test, described in Section 3, yielded χ_o^2=5.349. Finding the solution for c in Eq. 12 as 11.14, using 4 degrees of freedom and 2.5% significance level, the hypothesis of normal distribution for vibration levels induced by the subway train population was not rejected. The required sample size for a 95% confidence interval with k equal to 3 dB was then calculated using Eq. 15 for each 1/3 octave band in the range from 10 to 125 Hz and for the overall frequency weighted signal. The maximum required sample size occurred at the 1/3 octave band with a 16 Hz centre frequency and was equal to 9. The corresponding sample size was equal to 2 for the 40 Hz 1/3 octave band, which was the predominant frequency at the location of measurements, and for the overall frequency-weighted vibration level. This is much lower than the sample size required for 16 Hz. An explanation for this is that ground vibration at the 16 Hz frequency was caused by several sources of vibration in addition to subway trains, e.g. traffic on adjacent roads. To be on the conservative side, the recommended sample size was taken to be equal to 10 subway trains.

Streetcars

The assumption of normal distribution for vibration levels was checked using overall frequency-weighted rms vibration levels induced by 101 streetcar pass-bys. Vibration signals were measured in the vertical direction on the street curb. Calculations for the chi-square test yielded χ_o^2=9.247. Finding the solution for c in Eq. 12 as 9.35, using 3 degrees of freedom and 2.5% significance level, the hypothesis of normal distribution for vibration levels induced by the streetcar population was not rejected. The maximum required sample size occurred at the 1/3 octave band with a 10 Hz centre frequency and was equal to 6. Hence, the recommended sample size was equal to 6 streetcars.

6 VERIFICATION OF SAMPLE SIZE

Verification of the sample size recommended in the previous section was performed using arbitrarily selected samples at different times during the period in which the large samples were acquired. For subway trains, the error was well below 3 dB, and for streetcars the error slightly exceeded 3 dB in some instances.

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