A Model on Self-Damping of Stranded Cables in Random Transverse Vibrations

A. Leblond[1], L. Cloutier[1], C. Hardy[2] and L. Cheng[1]
[1]Département de Génie Mécanique, Université Laval, Ste-Foy, Québec, G1K 7P4.
[2]Institut de Recherche d'Hydro-Québec, Varennes, Québec, J3X 1S1.

1. Introduction

Predicting the across-wind response of overhead conductors to alternating forces caused by vortex shedding mechanism (aeolian vibrations) requires knowledge of conductor self-damping characteristics. Conductor self-damping represents the capacity of the conductor to dissipate energy internally during motion. Energy dissipation is mainly related to frictional damping due to small relative movements between adjacent wires.

Methods of measurement of conductor self-damping have been described in a IEEE Guide [1]. Conductor self-damping is usually determined by means of a laboratory test span by measuring the energy dissipated by the conductor vibrating in a principal mode. However, such event involving a pure sinusoidal vibration is rarely observed on a test span excited by natural wind action [2]. Most recordings indicate a combination of two or more frequencies.

Conductor self-damping is a non-linear phenomenon since loss factor depends on the conductor vibration amplitude. Consequently, the energy dissipated is not so readily determined as the superposition principle cannot be used when several modes are excited in the conductor. This paper presents a model to evaluate the energy dissipated by the conductor undergoing sinusoidal as well as multi-modal vibrations.

2. Theoretical approach

The variation of conductor curvature plays a dominant role in the dissipation mechanism. It is proposed here to develop a model based on the conductor curvature and its time-wise variations.

Usually, the mean power dissipated by the conductor undergoing sinusoidal vibrations is expressed as a function of antinodal amplitude, frequency and mechanical tension, but not directly in terms of curvature. The present model assumes that the instantaneous power dP dissipated by any infinitesimal element dx of the conductor is related to its curvature $y''(x,t)$, the time rate of change of curvature $\dot{y}''(x,t)$ and the static longitudinal stress σ_{al} in the outer layer of the conductor according to the following equation:

$$dP = C\left|y''(x,t)\right|^{\alpha}\left|\dot{y}''(x,t)\right|^{\beta}\sigma_{al}^{\gamma}\,dx \qquad (1)$$

where C is a proportionality factor. Quantities α, β and γ are exponents to be determined experimentally. Absolute values of $y''(x,t)$ and $\dot{y}''(x,t)$ are used to make sure the instantaneous power dP is always positive. For sinusoidal vibrations, the energy dW dissipated per cycle in the element dx is given by:

$$dW = \int_0^T dP \qquad (2)$$

where T is the period of vibrations. Hence, the total energy W dissipated per cycle in a span of length L (neglecting end effects) is:

$$W = \int_0^L dW \qquad (3)$$

Introducing equations (1) and (2) in (3), one finds:

$$W = C\,\sigma_{al}^{\gamma}\int_0^L\int_0^T \left|y''(x,t)\right|^{\alpha}\left|\dot{y}''(x,t)\right|^{\beta}\,dt\,dx \qquad (4)$$

Equation (4) has been calibrated to the similarity laws of Noiseux [3] recently modified by Hardy and Leblond [4] to be applicable to sinusoidal vibrations of multi-layer electrical stranded conductors. On this basis, equation (1) can be expressed as:

$$dP = K\frac{EI}{\rho_{al}^{2.44}}\left|y''(x,t)\right|^{1.69}\left|\dot{y}''(x,t)\right|^{0.75}\sigma_{al}^{-0.32}\,dx \qquad (5)$$

where K is a proportionality factor, EI is flexural rigidity of the cable $\left(\cong {EI_{max}}/{2}\right)$ in Nm^2, ρ_{al} is aluminum density in kg/m^3, σ_{al} is expressed in Pa and, $y''(x,t)$ and $\dot{y}''(x,t)$ are expressed in m^{-1} and $m^{-1}\,s^{-1}$ respectively.

Figure 1 shows instantaneous power dP as a function of spatial coordinate x and time t for sinusoidal vibration of a half-wavelength of the conductor. Positions $x=0$ and $x={\lambda}/{2}$ correspond to nodes of vibration. Times $t=0$ and $t={T}/{4}$ correspond to zero and maximum displacements of the conductor respectively. One can see that dP is maximum at $x={\lambda}/{4}$ which is the position of the antinode, somewhere between $t=0$ and $t={T}/{4}$. The value of t corresponding to this maximum is solution of the following equation:

$$\cos^2\left(\frac{2\pi t}{T}\right)=\frac{0.75}{1.69}\sin^2\left(\frac{2\pi t}{T}\right) \qquad (6)$$

which leads to $t = 0.156\,T$. The total volume included between surface dP and the x-t plane represents the energy dissipated in a half-wavelength of the conductor in a quarter cycle of vibration, as stated by the double integration in equation (4). At time t, the instantaneous power dP is zero at a vibration node. It is also zero at $t=0$, corresponding to zero curvature of the conductor and at

$t = \frac{T}{4}$, which corresponds to zero time rate of change of conductor curvature.

Finally, if the displacement $y(x,t)$ of the conductor is known (one or more frequencies), it is possible to calculate the total energy W dissipated in the span between times 0 and t:

$$W = K \frac{EI}{\rho_{al}^{2.44}} \sigma_{al}^{-0.32} \int_0^L \int_0^t \left|y''(x,t)\right|^{1.69} \left|\dot{y}''(x,t)\right|^{0.75} dt\, dx \qquad (7)$$

3. Results

The integration time t in equation (7) is taken as the elapsed time for a traveling wave to move from one end of the span to the other, where it is reflected, and to return to the same point. Under this consideration, the relative phases between each vibrational mode excited in the conductor has negligible effect on the integral result. Integral in equation (7) is solved numerically using Monte Carlo method [5]. Had the self-damping phenomenon been linear, the total energy dissipated during the conductor motion would be the sum of the separate energies dissipated by each of the modes present. In this fictitious case, it has been verified that the results of the present model calibrated on this basis would be the same as those obtained through application of the superposition principle.

4. Conclusion

The present model permits the evaluation of the energy dissipated by the conductor undergoing general transverse vibrations. This model could be integrated in any computer program performing aeolian vibration calculations.

5. Acknowledgments

The authors gratefully acknowledge Dr. Denis Noiseux for many helpful discussions. This work has been supported by Hydro-Québec.

6. References

[1] "IEEE Guide on Conductor Self-Damping Measurements", IEEE Standard 563-1978.

[2] Transmission Line Reference Book, "Wind-Induced Conductor Motion", Electric Power Research Institute, Palo Alto, CA, 1980, Section 1.4.

[3] Noiseux, D.U., "Similarity Laws of the Internal Damping of Stranded Cables in Transverse Vibrations", Proc. of the 1991 IEEE PES, Trans. & Distr. Conf., Dallas, September 1991.

[4] Hardy, C., Leblond, A., Goudreau, S., Cloutier, L.J., "Review of Models on Self-Damping of Stranded Cables in Transverse Vibrations", To be presented to the International Symposium on Cable Dynamics, Liège (Belgium), 19-20 Oct. 1995.

[5] Press, W.H., Teukolsky, S.A., Vetterling, W.T., Flannery, B.P., "Numerical Recipes in Fortran: The Art of Scientific Computing", Second Edition, Cambridge University Press, 1992.

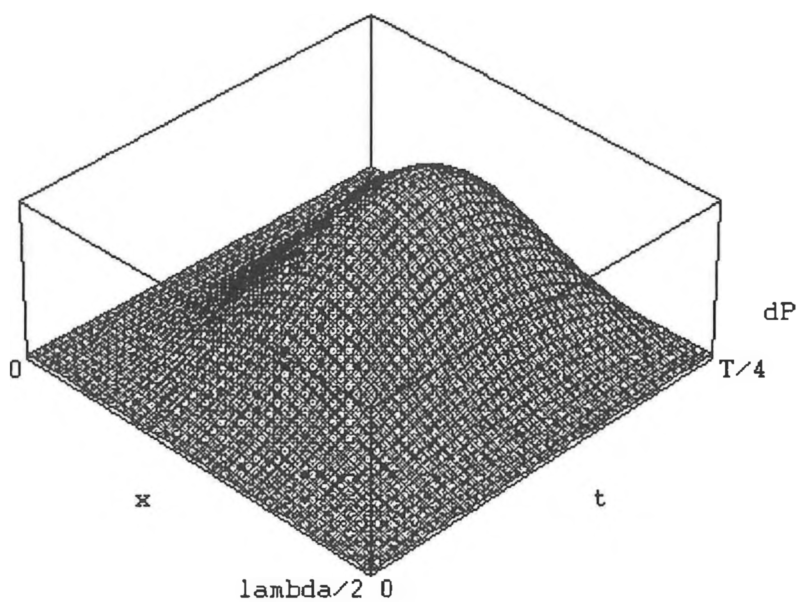

Figure 1: Instantaneous power dP dissipated in a half-wavelength of the conductor as a function of coordinate x and time t.