A Hybrid Approach for Vibrational Analysis of Coupled Structures

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1) Introduction

Mechanical coupling of structures along continuous boundaries induces serious difficulties for vibrational analysis. Analytical, numerical or experimental methods alone are not efficient enough to tackle the problem.

There are a vast amount of methods that use theoretical or experimental methods to predict the vibrational behavior of coupled systems. Statistical methods (e.g. SEA) are often used to calculate power flows between coupled structural elements and the vibration levels of each element of the system when the modal density is high [1]. Deterministic theoretical solutions have also been presented for different combinations of academic structures such as beams, plates and shells. The majority of these methods are based on a substructure synthesis approach which divide the whole structure into different substructures and try to obtain a general solution of the whole structure using solutions of all substructures [2]. The classical modal analysis CMA [3] and mobility power flow [4] are other proposed methods in the literature. CMA is based on the modal analysis of the global structure and no assumption is made about either the nature of the coupling or the excitation type. The main objective of the mobility power flow method is to develop the appropriate expressions for the vibrational power flow through coupled substructures by means of structural mobility functions. Finite element method is a general method which can be used for all types of complex structures. The method is however too expensive when applied to three dimensional problems and/or high frequency domains.

This paper presents a hybrid method that combines analytical, numerical and experimental methods to derive the dynamic response of a complex structure. A complex coupled structure is divided into two categories: a master structure and auxiliary substructures. In general, one is interested in the behavior of the master structure while the influence of each auxiliary structure is introduced by the compliance functions at the junction. This approach allows one to change the characteristics of one or more elements of the system or to add or eliminate elements, without discarding measurements or calculations which have been done for other elements of the system.

2) Line coupling

In a general three dimensional problem, after dividing the whole structure into main and auxiliary structures, there exist six components of load for each contact point. The substructure compliance matrix at contact point defines the relations between each component of the deformation vector to each component of the load vector. The problem is more complicated in the case of a line or surface contact between two or more structures. As shown in figure 1, a linear junction can however be presented as a combination of separate coupling points. Therefore each component of the substructure compliance matrix is a square matrix with dimension N_p, and each component of the load or deformation vector is a vector of dimension N_p, where N_p is the number of contact points.

$$\begin{Bmatrix} D_j^1 \\ D_j^2 \\ .. \\ .. \\ D_j^N \end{Bmatrix} = \begin{bmatrix} \beta_{11}^{ij} & \beta_{21}^{ij} & .. & .. & \beta_{N1}^{ij} \\ \beta_{12}^{ij} & \beta_{22}^{ij} & .. & .. & \beta_{N2}^{ij} \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ \beta_{1N}^{ij} & \beta_{2N}^{ij} & .. & .. & \beta_{NN}^{ij} \end{bmatrix} \begin{Bmatrix} F_i^1 \\ F_i^2 \\ .. \\ .. \\ F_i^N \end{Bmatrix} \tag{1}$$

where $i, j = 1, N_p$ and N=6 for a general three dimensional problem. The compliance matrix β can be defined as:

$$\beta_{ij}^{km} = \frac{D_j^m}{F_i^k} \quad , \quad [\eta] = [\beta]^{-1} \tag{2}$$

where F_i^k is the excitation at point "i" in the direction of "k" and D_j^m is the measured deformation at point "j" in the direction "m".

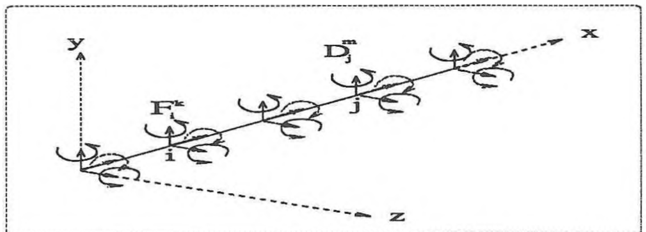

Figure 1: Distribution of forces and moments due to coupling along a common edge.

In reality, there are many problems that can be treated more easily. As an example, let's analyze the force vibration of an L-shape plate with simply supported edges. After dividing the L-shape plate into main and auxiliary structures, one is able to study the auxiliary structure by applying a distributed moment M_x at its junction. It is the only coupling load because of the pinned joint between two plates that avoids the in-plane wave transmission. By applying a moment M_x at point "i" and measuring or calculating the rotations θ_x of all points along the junction, one can write $\beta_{ij}^{R_x} = \theta_j / M_i$. Hence

$$\theta_j = \sum_i \beta_{ij}^{R_x} M_i \qquad\qquad M_j = \sum_i \eta_{ij}^{R_x} \theta_i \tag{3}$$

A variational formulation of a rectangular thin plate using Rayleigh-Ritz approach has been performed to study the main structure. A polynomial decomposition for lateral displacement of the plate is considered and artificial springs connected at the edges allow one to define different types of boundary conditions. The potential and kinetic energies as well as the potential energy due to boundaries are derived and the energy terms due to work done by external forces and moments are included. The only new term to be considered, is the effect of the auxiliary structure. This energy term can be defined as

$$E_{cp} = \int_{-b}^{b} M_x \theta_x dx \tag{4}$$

where

$$\theta_x = \frac{\partial W}{\partial y} \quad and \quad W = \sum_{i,j} a_{ij} \left(\frac{x}{b}\right)^i \left(\frac{y}{h}\right)^j \tag{5}$$

with "b" is half the width of the plate along the x axis, and "h" is half the length along the y axis. The line of junction is x=[-b,b]. Replacing the moment distribution by rotational deformations in equation 4 and using the second equation in 3 , E_{cp} is easily obtained. Experimentally , it is easier to measure the β matrix and E_{cp} can be derived by inverting β. This however leads to two problems: Firstly, by increasing the number of contact points, β matrix approaches singularity; secondly, the nature of the η matrix is such that the regression analysis is not permitted. It is simply due to the fact that the value of the compliance at a single point will change with the number of contact points chosen. Figure 2 shows the variations of the central line of β and η matrices. For the β matrix, it means that the rotational moment is applied at point x=0 and the responses are measured at different contact points along the junction. The dashed and solid lines are used for 5 and 9 contact points respectively. It is clear that by increasing the number of contact points, the inversion process complexifies and the regression analysis on η matrix leads to incorrect estimation of η at other points. To avoid β matrix inversion, a polynomial decomposition is considered for the moment distribution along the junction:

$$M_x(x, y_0) = \sum_i \sum_j b_{ij} \left(\frac{x}{b}\right)^i \left(\frac{y_0}{h}\right)^j \tag{6}$$

where y_0 indicates the y coordinate of the junction which, for simplicity, is taken parallel to the x axis. The matrix β can be transformed to a two dimensional function $\beta(x, \zeta)$ using a polynomial regression technique

$$\beta(x, \zeta) = \sum_k \sum_l C_{kl} x^k \zeta^l \tag{7}$$

where x and ζ indicate the excitation and the observation points respectively. Using equations 3 ,5 ,6 ,7 and some mathematical operations, one obtains a matrix relation between coefficients a_{ij} and b_{ij}. The dynamic energy due to the auxiliary structure can then be computed by equation 4 . The other kinetic and potential energy terms are all related to the main structure and can be simply derived.

3) Results

As a practical application, the force vibrations of a system composed of two coupled perpendicular thin plates has been studied. Both plates have same dimensions and material properties. The excitation is a unit lateral load. Figure 3 indicates the variation of lateral displacement amplitude *vs* frequency. The substructure compliance matrix has been obtained by the same variational procedure as the main structure using 9 contact points. For more complex structures, the compliance matrix can either be calculated by numerical methods or measured experimentally. The results are compared by a finite element solution using the IDEAS package with 12 and 24 vibrational modes. It is clear that at higher frequencies, higher number of modes must be used to obtain valuable results. A sensibility analysis has been performed regarding possible errors in measuring or calculating compliance components of the substructures. It is however out of

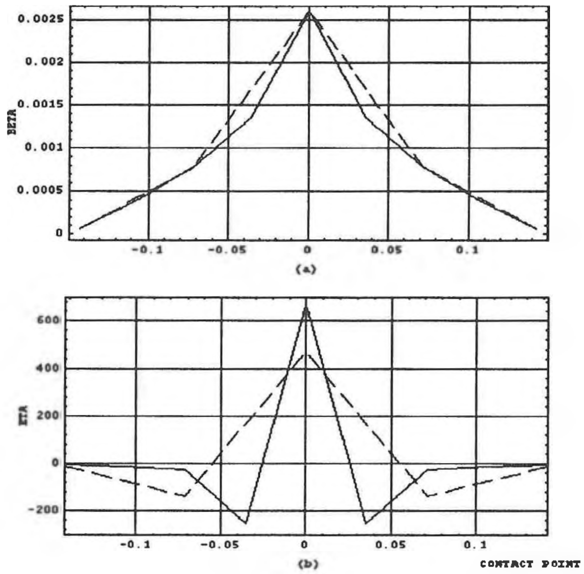

Figure 2: Compliance variation and its inverse along the junction with 5 and 9 contact points

the scope of this brief report. Also, an analysis has been conducted to investigate the role of the number of measurement points and the suitable degree of regression in different frequency ranges. It will help to minimize the measurement and calculation time and cost to obtain a desired precision.

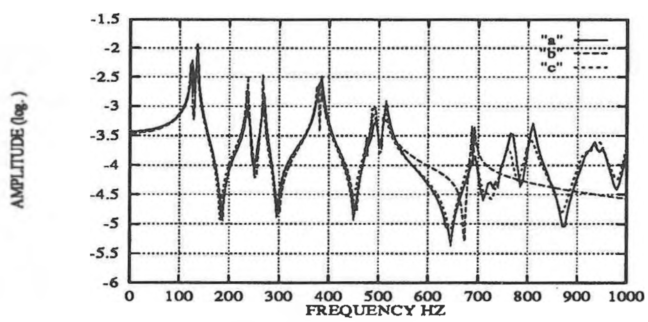

Figure 3: Lateral vibration of two coupled perpendicular plates a) Hybrid method b) Finite element using first 12 modes c) Finite element using first 24 modes.

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