STATISTICAL ENERGY ANALYSIS APPLIED TO LIGHTWEIGHT CONSTRUCTIONS PART 1: SOUND TRANSMISSION THROUGH FLOORS

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This is the first of three papers on the application of statistical energy analysis (SEA) to a lightweight wood frame construction. In this paper the basic theory behind SEA will be presented by examining a simple model for sound propagation through the floor/ceiling assembly shown in Figure 1.

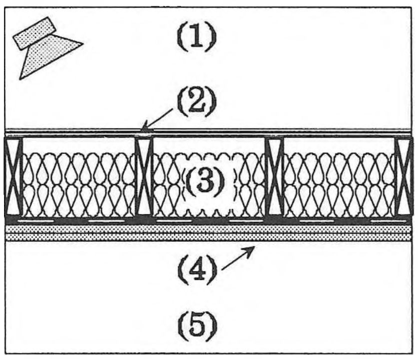

Figure 1: Sketch of the floor/ceiling assembly. (1): source room, (2): 15.9 mm OSB decking, (3): 235 mm deep cavity with two layers of 89 mm batt insulation, (4): 2 layers 12.7 mm type X gypsum board mounted on resilient channels, (5): receive room.

SEA enables the prediction of energy contained in individual elements (or sub-systems) of a complete system when power is input in one or more of the sub-systems. The basic principle of SEA requires that the power, W, into a sub-system is equal to the power lost either through transmission, radiation, or internal losses.

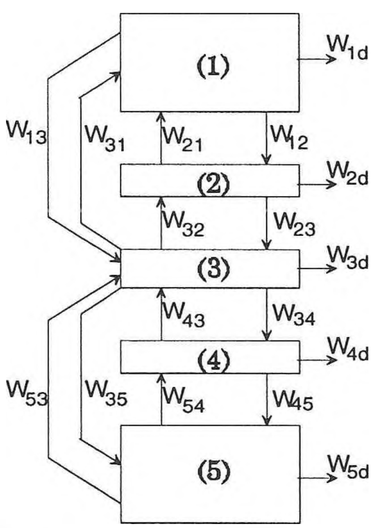

Figure 2: Sub-system diagram for the SEA model of the modelled floor/ceiling assembly.

Figure 2 shows a sub-system diagram with the paths of power flow illustrated by the arrows. It has been assumed that the resilient channels remove any coupling between the floor decking and the gypsum board ceiling via the joists. There are arrows showing power flow directly from room to cavity or via versa apparently without involving the floor decking or the gypsum board ceiling. These represent the non-resonant paths of energy transport that are essentially independent of the damping of the element through which energy passes.

The following set of equations can be written describing the partition of energy between the sub-systems,

$$\begin{bmatrix} -\eta_1 & \eta_{21} & \eta_{31} & 0 & 0 \\ \eta_{12} & -\eta_2 & \eta_{32} & 0 & 0 \\ \eta_{13} & \eta_{23} & -\eta_3 & \eta_{43} & \eta_{53} \\ 0 & 0 & \eta_{34} & -\eta_4 & \eta_{54} \\ 0 & 0 & \eta_{35} & \eta_{45} & -\eta_5 \end{bmatrix} \bullet \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \end{bmatrix} = \begin{bmatrix} -W_1/\omega \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad [1]$$

where the power flow between sub-systems i and j is given by

$$W_{ij} = E_i \omega \eta_{ij}, \quad [2]$$

E_i is the energy stored in i, ω is the angular frequency, and η_{ij} is the coupling loss factor (CLF) between i and j. The CLF is defined as the fraction of energy transmitted in one radian cycle. Also, the concept of a total loss factor (TLF) was introduced to simplify the set of equations,

$$\eta_i = \eta_{id} + \sum_j \eta_{ij}, \quad [3]$$

where j indicates the sub-systems to which i is connected and η_{id} is the internal loss factor. The TLF is the faction of energy transmitted, lost due heat and radiation. The TLF is easily measured as it is a simple function of the reverberation time, T, and frequency, f,

$$\eta_i = \frac{2.2}{fT_i} \quad [4]$$

Thus the energy in any one of the sub-systems can be found by solving a set of equations involving CLF's and TLF's.

CLF's

The CLF will depend on i and j and how they are coupled. It is beyond the scope of this paper to derive the individual CLF equations but rather they will just be presented to indicate their dependencies. Craik has previously provided a detailed examination[1].

Resonant transmission from a room volume to a plate (i.e., (1) to (2) and (5) to (4)) is given by,

$$\eta_{12} = \frac{5556\, S_2 f_{c2} \sigma_2}{V_1 \rho_{s2} f^3}, \quad [5]$$

where S_2 is the surface area of the decking, f_{c2} is its critical frequency, ρ_{s2} is its surface density, and f is the frequency. The radiation factor, σ, can be calculated from various equations[2] depending on the accuracy required.

Resonant transmission from a plate to a room (i.e., (2) to (3), and (4) to (5)) is given by,

$$\eta_{45} = \frac{66\sigma_4}{\rho_{s4} f}. \quad [6]$$

If the method of energy transport between rooms/cavities is non-resonant transmission then the coupling loss factor can be determined from the non-resonant transmission coefficient, τ_{ij}, for the element separating the rooms or cavities. For example, the CLF between the source room (1) and the floor cavity (3) for non-resonant transmission is,

$$\eta_{13} = \frac{13.7 S_3 \tau_{13}}{f V_1}, \quad [7]$$

where τ_{13} is the transmission coefficient of the floor decking.

CLF's associated with joints occur commonly when modelling complete building assemblies,

$$\eta_{ij} = 0.1365\left(\frac{hC_{Li}}{f}\right)^{\frac{1}{2}} \frac{L}{S_i} \tau_{ij} \quad [8]$$

where C_{Li} is longitudinal wave speed for sub-system i, L is the length of the joint, S_i is the surface area of the sub-system i, and τ_{ij} is the power transmission coefficient between i and j.

The CLF between two sub-systems can be measured using the power balance between the two sub-systems,

$$\eta_{12} = \frac{E_2 \eta_2}{E_1}, \quad [9]$$

where η is the loss factor. In writing this equation it has been assumed that the power input into sub-system 2 from all other subsystems other than 1 can be ignored. The estimates of the energy contained in a plate can be obtained from the measured space-time averaged surface velocity $\langle v^2 \rangle$ given,

$$E_{plate} = \langle v^2 \rangle \rho_s S, \quad [10]$$

where ρ_s is the surface density, and S is surface area. Alternately, if the sub-system is a room or cavity then the energy can be estimated from the space-time average sound pressure $\langle p^2 \rangle$ given,

$$E_{room/cavity} = \frac{\langle p^2 \rangle V}{\rho_o c_o^2}, \quad [11]$$

where V is the room volume, ρ_o is the density of air, and c_o is the speed of sound in air.

Transmission through the floor cavity

The model of Price and Crocker[3] was used to predict the transmission through cavity. It assumes that transmission from a room into the cavity of a partition wall or floor is the same as transmission into a small room from a much larger room. Both resonant and non-resonant transmission are possible. Transmission out of the partition cavity into the receive room then follows from the consistency relationship,

$$n_{room} \eta_{room,cavity} = n_{cavity} \eta_{cavity,room} \quad [12]$$

where n is the modal density.

Measured and Predicted Results

Often it is easier, and even more accurate, to use the measured TLF for a similar or nominally identical sub-system in a similar construction than it is to assume that the TLF is the sum of the CLF's. Given in Table 1 are the measured regression fit equations for the total loss factors of some common lightweight building elements,

Sub-system	Density (kg/m^3)	Total loss factor
Gypsum board of a wood stud wall	751	$0.5 f^{-0.37}$
16 mm OSB floor decking	451	$0.4 f^{-0.37}$
Wood joist floor cavity 1/2 to 3/4 full of absorption	n/a	$3.0 f^{-0.34}$

Table 1: Computed TLF's from measured data for some common building elements. Values will vary depending on specific factors such as joint types amount and type of cavity absorption, etc.

The five sub-system model shown in Figure 1 represents a very simplified model for the floor ceiling assembly. The effect of the joists have been ignored, and it is assumed that the resilient channel prevents structure borne transmission from the decking to the gypsum board ceiling. The measured TLF's listed in Table 1 were used in the model. All CLF's were calculated according the equations given. Figure 3 shows that despite the very simple model the measured and predicted results are in good agreement.

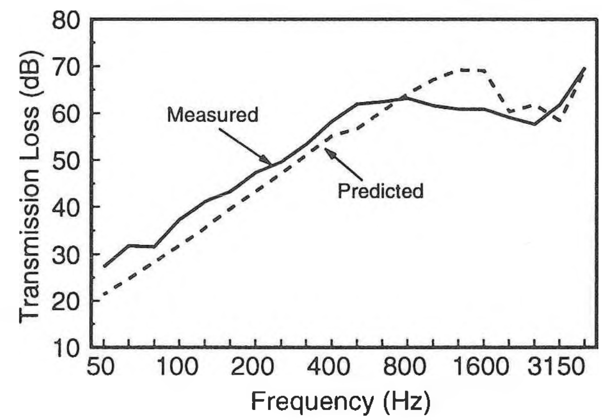

Figure 3: Measured and predicted transmission loss.

The SEA prediction tends to overestimate the efficiency of the coupling between the room/cavities and the wall surfaces, thus underestimating the transmission loss. This is especially true at the critical frequencies (2000 and 3000 Hz). More accurate models of the radiation efficiency which controls the degree of coupling can be used but with significantly more computation time.

Conclusions

The predicted transmission loss of the simplified model agrees well with measured results despite the many assumptions (the resilient channels remove any physical coupling from floor decking to the gypsum board ceiling via the joists, and the joists have no effect). The model showed that transmission through a cavity can be modelled using the method of Price and Crocker.

1 Craik, R.J.M., "The noise reduction of flanking paths," Applied Acoustics, Vol. 22, pp. 163-175, 1987.

2 Leppington, F.G., Broadbent, E.G., Heron, K.H., "Acoustic radiation from rectangular panels with constrained edges," Proc. Royal. Soc., A393, pp. 67-84. 1984.

3 Price, A.J., Crocker, M.J., "Sound transmission through double partitions using statistical energy analysis," JASA, No 47, pp. 683-693, 1970.