NUMERICAL MODELLING OF FIRE STOPS AT THE INTERSECTION OF FLOORS AND LOAD BEARING PARTY WALLS

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Introduction

This is a companion paper to a previously published paper[1] which presented a numerical model for the transmission via a fire stop that coupled two corner joints. The present paper expands upon the previous work allowing for the modelling of transmission via a fire stop coupling two 'tee' joints. The three plates of the 'tee' joints could then be the floor, upper and lower load bearing party walls. The fire stop could be formed by running the floor decking under the upper load bearing party walls. Figure 1 shows a mechanical representation of the system as applied to a double wood stud construction.

Joint Equations

In this paper we will restrict ourselves to a model of simple bending waves and the effects of beams at the joint (in the form of joist headers, sole and head plates) will be ignored. Such a model will tend to predict stronger transmission than would a more complete model, especially at high frequencies. However, the simplified model will enable us to rank order fire stops of various stiffness and to examine the effectiveness of various treatments to the plates.

In deriving the equations for the joint it is assumed that each 'tee' is pinned (i.e., there is no translational motion) and the fire stop that couples the 'tee' joints has only stiffness. If the plates of the 'tee' joints are rigidly connected the following equations of motion can be written,

$$\phi_1 = \phi_2,\ \phi_2 = -\phi_4,\ \phi_3 = -\phi_5, \text{ and } \phi_5 = -\phi_6 \quad [1a]$$

$$M_1 - M_2 - M_4 - M_f = 0 \text{ and}$$

$$M_3 - M_5 - M_6 + M_f = 0, \quad [1b]$$

where φ is the angular rotation, and M is the moment. The subscript indicates the arm of the joint (as per Figure 1) while the subscript 'f' refers to the fire stop material that connects the two 'tee' joints. If the bending wave enters the joint from plate 1 then the following equations define the transverse displacement, ξ, in each arm,

$$\xi_1 = \left(e^{-ik_1 x\cos\theta_1} + T_1 e^{+ik_1 x\cos\theta_1} + T_{n1} e^{+k_{n1}x}\right) e^{-ik_1 y\sin\theta_1}, \quad [2a]$$

$$\xi_2 = T_2 e^{-ik_2 z\cos\theta_2} + T_{n2} e^{-k_{n2}z}, \quad [2b]$$

$$\xi_3 = T_3 e^{-ik_3 x\cos\theta_3} + T_{n3} e^{-k_{n3}x}, \quad [2c]$$

$$\xi_4 = T_4 e^{ik_4 z\cos\theta_4} + T_{n4} e^{k_{n4}z}, \quad [2d]$$

$$\xi_5 = T_5 e^{-ik_5 z\cos\theta_5} + T_{n5} e^{-k_{n5}z}, \quad [2e]$$

$$\xi_6 = T_6 e^{ik_6 z\cos\theta_6} + T_{n6} e^{k_{n6}z}, \quad [2f]$$

where T is the amplitude of the bending wave on the plate indicated by the subscript, the 'n' of the subscript indicates the near field or evanescent wave, k is the wave number for traveling waves, k_n is the wave number for the evanescent waves and θ is the angle of propagation. The displacements given in Equation[2] are related to Equation[1] by,

$$\phi_1 = \frac{d\xi_1}{dx} \text{ and } M_1 = -B_1\left[\frac{d^2\xi_1}{dx^2} + \mu\frac{d^2\xi_1}{dy^2}\right], \quad [3]$$

where B is the bending stiffness of the plate and μ is Poisson's ratio. Equations[1], [2] and [3] represent a set of six simultaneous equations involving the twelve unknown coefficients. Six additional equations are obtained by using the following boundary conditions for pure rotation at the pinned 'tee' joints:

$$T_1 + T_{n1} = -1,\ T_2 + T_{n2} = 0,\ T_3 + T_{n3} = 0,$$

$$T_4 + T_{n4} = 0,\ T_5 + T_{n5} = 0, \text{ and } T_6 + T_{n6} = 0 \quad [4]$$

Now a set of six simultaneous equations can be created to describe the amplitude of the bending waves in each of the six plates. For practical application normal incidence will be considered as a closed-form solution can be obtained. It was shown in a previous study[2] that the normal incidence results for this type of joint over estimate the transmission by about 1.8 dB. We will also assume that plates 2, 4, 5 and 6 are similar (i.e., constructed of the same material). While plates 1 and 3 are similar. Thus the following simplifications can be made,

$$k_2 = k_{n2} = k_4 = k_{n4} = k_5 = k_{n5} = k_6 = k_{n6}, \quad \text{and}$$

$$k_1 = k_{n1} = k_3 = k_{n3}, \quad \text{also}$$

$$B_2 = B_4 = B_5 = B_6, \text{ and } B_1 = B_3. \quad [5]$$

Using the general relationship between the joint transmission coefficient and the amplitude,

$$\tau_{12} = \chi^2\psi\left|T_{12}\right|^2 \quad [6]$$

where,

$$k_2 = k_1\chi, \text{ and } B_2 k_2 = \psi B_1 k_1 \quad [7]$$

one obtains Equations [8], [9], and [10] for the joint transmission coefficient where,

$$\delta = \frac{B_f}{B_1 k_1}. \quad [11]$$

The joint transmission loss, R, for the path from plate i to plate j though the fire stop joint is given by,

$$R_{ij} = 10\log\frac{1}{\tau_{ij}} \quad [12]$$

In the special case that all the plates are made of the same material (both χ and ψ are unity) and it can be seen that in the limit that the fire stop is infinitely stiff, the joint transmission loss from plate 1 to all other plates is 12.6 dB. As the stiffness of the fire stop tends to zero, the joint transmission loss from plate 1 to plates 3, 5, and 6 tends to infinity, while the joint transmission loss to plates 2 and 4 tends to 9.5 dB (i.e., exactly that for a 'tee' joint).

Fire Stop Modelling

The effective bending stiffness of the fire stop material is given by,

$$B_f = \frac{Et^3}{(1-\mu^2)}\frac{1}{d} \qquad [13]$$

where E is Young's Modulus, μ is Poisson's ratio, and d is the span of the fire stop. (Typically this is about 25 mm since this is the distance between plates in a double stud wall.)

By examining equations [8], [9],[10], [11] and [13] it can be seen that the transmission across the fire stop can be minimized by,

1. Reducing the apparent stiffness of the fire stop. This can be done by using a thin material, increasing the span, d, between the sole plates, and using a material that has a low Young's Modulus (E);
2. Increasing the bending stiffness of the source plate. (For the case shown this is plate 1. Plate 3 received the same treatment for symmetry.)

Reducing the apparent stiffness of the fire stop may not be practical if the assembly has already been built. However, it is possible to increase the bending stiffness of plates 1 and 3 by adding a concrete topping directly to the exposed floor decking.

To illustrate the potential improvement to the sound isolation Figure 2 shows the predicted joint transmission loss, R in dB, before and after plates 1 and 3 receive a 38 mm thick concrete topping (surface density 91 kg/m^3).

Plates 1 and 3 (floor decking) are 16 mm thick oriented strand board (OSB), plates 2, 4, 5, 6 (party walls) are 16 mm gypsum board. Before the topping the maximum attenuation from one dwelling to the next across the fire stop was between 10 and 15 dB (over the normal building acoustics range 100-4000 Hz). This would most certainly represent a serious flanking path especially if the party walls offer a high degree of sound isolation potential (i.e. double stud construction).

However, the model indicates that increasing the bending stiffness of the source plate by adding a topping can greatly reduce transmission across the fire stop from plate 1. The joint transmission reduced by at least 15 dB, and at least 20 dB for frequencies where the flanking surfaces can efficiently radiate.

Conclusions

The simple model for transmission across a fire stop connecting two 'tee' joints has shown that fire stops should not be formed from continuous surfaces. However, structural requirements may force connections between dwellings by continuous surfaces. In such cases, greatly increasing the bending stiffness of the exposed surfaces can significantly improve sound isolation. This may be a practical solution for both new constructions and retro-fit situations.

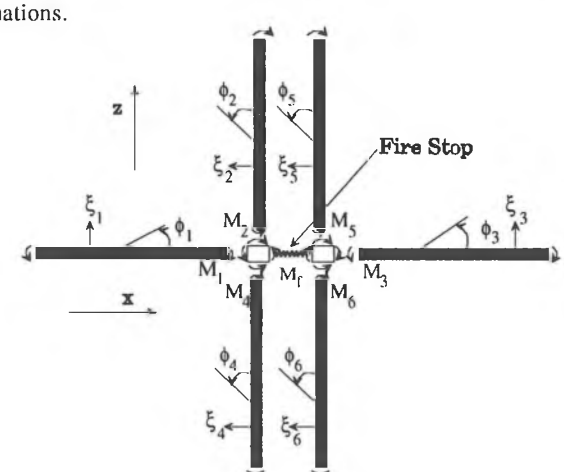

Figure 1: Mechanical representation of the joint.

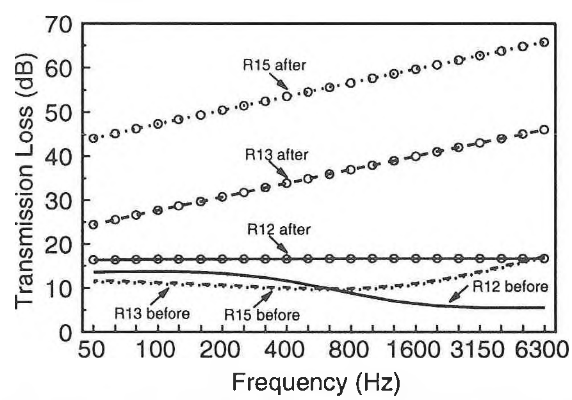

Figure 2: Predicted transmission loss for the various plates with and without a 38 mm thick concrete topping applied to the OSB floor decking.

$$\frac{1}{\tau_{12}} = \frac{1}{\tau_{14}} = \frac{2\psi\left(\delta^2 - 2\delta\psi - 4\delta + 2\psi^2 + 8\right)}{\left(8 + 9\delta^2\psi^2 + 8\psi^4 + 40\psi^3 + 66\psi^2 + 40\psi + 9\delta^2 + 18\delta^2\psi - 12\delta - 12\delta\psi^3 - 42\delta\psi^2 - 42\delta\psi\right)} \qquad [8]$$

$$\frac{1}{\tau_{15}} = \frac{1}{\tau_{16}} = \frac{2\psi\delta^2}{\left(8 + 9\delta^2\psi^2 + 8\psi^4 + 40\psi^3 + 66\psi^2 + 40\psi + 9\delta^2 + 18\delta^2\psi - 12\delta - 12\delta\psi^3 - 42\delta\psi^2 - 42\delta\psi\right)} \qquad [9]$$

$$\frac{1}{\tau_{13}} = \frac{2\delta^2}{\left(8 + 9\delta^2\psi^2 + 8\psi^4 + 40\psi^3 + 66\psi^2 + 40\psi + 9\delta^2 + 18\delta^2\psi - 12\delta - 12\delta\psi^3 - 42\delta\psi^2 - 42\delta\psi\right)} \qquad [10]$$

[1] Nightingale, T.R.T., Craik, R.J.M., "Numerical modelling of wood frame joints having fire stops," Canadian Acoustics, Vol. 29 No. 3, pp. 29-30. 1994.

[2] "Sound transmission through framed buildings", Internal Report IRC-IR-672, Institute for Research in Construction, National Research Council Canada, May 1995.