MULTIMODAL SOUND PRESSURE FIELD IN A CYLINDRICAL DUCT

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Introduction

Active control systems are used to reduce transmitted energy at the end of ducts. When high order modes propagate, the sound field becomes very complex and effective attenuation is more difficult to obtain. The first step in developing an active control system model is to accurately define the interior pressure field of the duct.

This paper presents a model able to predict the interior pressure field of a finite length circular duct when high order modes can no longer be neglected. The studied duct is close at one end and open at the other. As a first step, theoretical and experimental maps of the sound field in the circular duct are presented for zero modal impedance coefficients.

I. The model

A- Hypothesis. The sound pressure is calculated for a circular duct with a finite length (L), and an infinite flange at the open extremity. A velocity point source of strength Q_0 is located at (r_0,θ_0,z_0).

The pressure outside the duct ($z \geq 0$) is given by the Helmoltz integral :

$$P(r,\theta,z) = j\omega\rho \cdot \int_{r=0}^{a} \int_{\vartheta=0}^{2\pi} G(r,\theta,z;r',\theta',z) \cdot V^+(r',\theta',z) \cdot r'dr'd\theta \quad (1)$$

$V^+(r',\theta',z)$ is the axial velocity at the open end of the duct and G is the Green function of the semi-infinite space :

$$G(r,\theta,z;r',\theta',z) = \frac{e^{jkh}}{2\pi h}, \text{ with } h = r'^2 + r^2 - 2rr'\cos(\theta-\theta') + z^2$$

The pressure inside the duct ($z \leq 0$) is given by the following modal decomposition before and after the point source (r_0,θ_0,z_0) :

$$P^-(r,\theta,z) = \sum_{m=-\infty}^{+\infty} e^{jm\theta} \cdot \sum_{n=0}^{\infty} [A^-_{mn}e^{jk_z z} + B^-_{mn}e^{-jk_z z}] \cdot \Psi_{mn}(k_{mn}r) \quad (2)$$

$$P^+(r,\theta,z) = \sum_{m=-\infty}^{+\infty} e^{jm\theta} \cdot \sum_{n=0}^{\infty} [A^+_{mn}e^{jk_z z} + B^+_{mn}e^{-jk_z z}] \cdot \Psi_{mn}(k_{mn}r) \quad (3)$$

P^+ and P^- are the generalized solutions of the Helmoltz equation (4) respecting the boundary condition (5) on the duct walls :

$$(\Delta + k^2)P = 0 \quad (4)$$

$$\frac{\partial P}{\partial r} = 0 \quad (5)$$

Equation (5) gives : $\Psi_{mn}(k_{mn}) = \dfrac{J_m(k_{mn}r)}{N_{mn}}$ (6)

where k_{mn} are determined using the boundary condition (5). The radial modes are orthogonal so it is convenient to choose a normalizing factor N_{mn} such that :

$$k^2 \int_0^a r \cdot \Psi_{mn}(k_{mn}r) \cdot dr = 1 \quad (7)$$

and $N_{mn} = ka \cdot \left\{ \frac{1}{2} \cdot \left(1 - \frac{m^2}{k^2_{mn}a^2}\right) J^2_m(k_{mn}a) \right\}^{1/2}$ (8)

The dispersion relation gives : $k_z^2 = k^2 - k^2_{mn}$ (9)

The continuity of the pressure and the velocity inside the duct are:

$$P^+(r,\theta,z_0) = P^-(r,\theta,z_0) \quad (10)$$

$$V^+(r,\theta,z_0) - V^-(r,\theta,z_0) = Q_0 \cdot \delta(r-r_0) \cdot \delta(\theta-\theta_0) \quad (11)$$

B- Impedances-Pressure field. The acoustic pressure and the velocity at the open end of the duct can be expressed as :

$$P^+(r,\theta,0) = \sum_{m=-\infty}^{+\infty} e^{jm\theta} \cdot \sum_{n=0}^{\infty} P^+_{mn} \cdot \Psi_{mn}(k_{mn}r) \quad (12)$$

$$V^+(r,\theta,0) = \sum_{m=-\infty}^{+\infty} e^{jm\theta} \cdot \sum_{n=0}^{\infty} V^+_{mn} \cdot \left(\frac{k_z}{\rho\omega}\right) \cdot \Psi_{mn}(k_{mn}r) \quad (13)$$

A new expression of the pressure is found by subsituting equation (13) in equation (1) and by eliminating the function G as it is described in [1].

Equations (12) and (13) introduce a relation between the modal pressure amplitudes , the modal velocity amplitudes and the modal impedance coefficients :

$$P^+_{mn} = Z_{mn} V^+_{mn} \cdot \left(\frac{k_z}{\rho\omega}\right) \quad (14)$$

The coupling impedances are neglected.

Then, the generalized modal impedance becomes :

$$Z_{mn} = \int_0^{\pi/2} \sin\phi \cdot D^2_{mn}(\sin\phi) \cdot d\phi - j\int_0^{\infty} \cosh\xi \cdot D^2_{mn}(\cosh\xi) \cdot d\xi \quad (15)$$

where $D_{mn}(\tau) = k^2 a \left\{ \dfrac{\tau k \cdot J_{m+1}(\tau ka) J_m(k_{mn}a) - k_{mn} J_m(\tau ka) J_{m+1}(k_{mn}a)}{N_{mn} \cdot (\tau^2 k - k^2_{mn})} \right\}$ (16)

Now, the interior problem has to be solved to find the modal amplitudes. The equations (10) and (11) have to be used to find the pressure before and after the source.

Finally, the pressure inside the duct is given by :

$$P^-(r,\theta,z) = \frac{k^2 r_0 Q_0 \rho\omega}{2k_z} \sum_{m=-\infty}^{+\infty} e^{jm(\theta-\theta_0)} \sum_{n=0}^{\infty} \frac{\left(e^{jk_z z_0} + R_{mn}e^{-jk_z z_0}\right)}{R_{mn}e^{2jk_z L} + 1} \times \quad (17)$$
$$\left[e^{jk_z z} + e^{-jk_z(z+2L)}\right] \cdot \Psi_{mn}(k_{mn}r_0) \cdot \Psi_{mn}(k_{mn}r)$$

$$P^+(r,\theta,z) = \frac{k^2 r_0 Q_0 \rho\omega}{2k_z} \sum_{m=-\infty}^{+\infty} e^{jm(\theta-\theta_0)} \sum_{n=0}^{\infty} \frac{\left(e^{jk_z(z_0+2L)} + e^{-jk_z z_0}\right)}{R_{mn}e^{2jk_z L} + 1} \times \quad (18)$$
$$\left[e^{jk_z z} + R_{mn}e^{-jk_z z}\right] \cdot \Psi_{mn}(k_{mn}r_0) \cdot \Psi_{mn}(k_{mn}r)$$

with the modal reflection coefficient :

$$R_{mn} = \frac{Z_{mn}\left(\frac{k_z}{\rho\omega}\right) + 1}{Z_{mn}\left(\frac{k_z}{\rho\omega}\right) - 1} \quad (19)$$

C- Theoretical results. This paragraph presents the preliminary results obtained by using zero modal impedance coefficients. It means that the exterior pressure was taken to be zero. The figure 1 shows the theoretical sound field map in a section of the duct.
The duct used for predictions is 3,30 meters long and has a radius of 0,147 m. The theoretical cut-on frequencies of this duct are given by the boundary condition (5) (table 1). For all tests, the source is located at 120 degrees, at 0.147 m on the radius and at 0.42 m of the closed extremity

Modes	Cut-on frequencies
00	0
10	671
20	1112
01	1396
30	1541

Table 1 : Cut-on frequencies of the duct used for measurements.

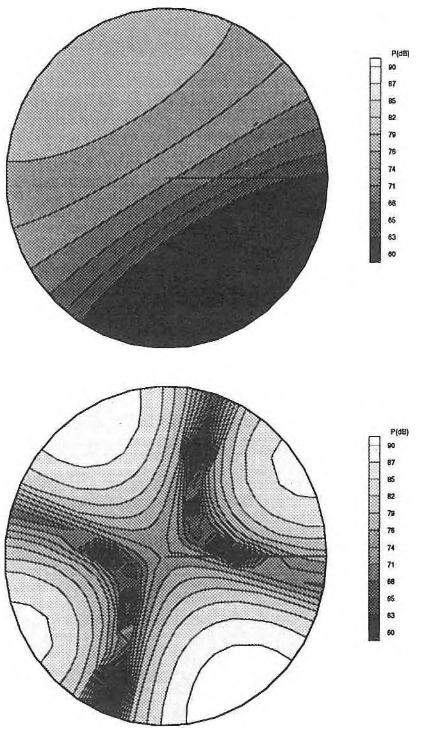

Fig 1 : Theoretical maps of the sound field at 1000 Hz and 1200 Hz at 2,95 m of the noise source in the duct.

The modes propagate according to the table 1. Hence, the first graphic is a superposition of modes (0,0) and (1,0) while for the second one the (2,0) mode is also present.

II. Pressure field measurements

A- Experimental setup. This second paragraph describes the measurements of the sound field in the same circular duct as used previously. The duct is placed in a semi-anechoïc room. To measure the sound pressure, five microphones are placed on a radius. This set of microphones can rotate around the axis of the duct by 10 degre steps. On a given section, this represents 210 measurement points.
The duct used for measurements is the identical to the duct used for the predictions. The source is also similarly located.

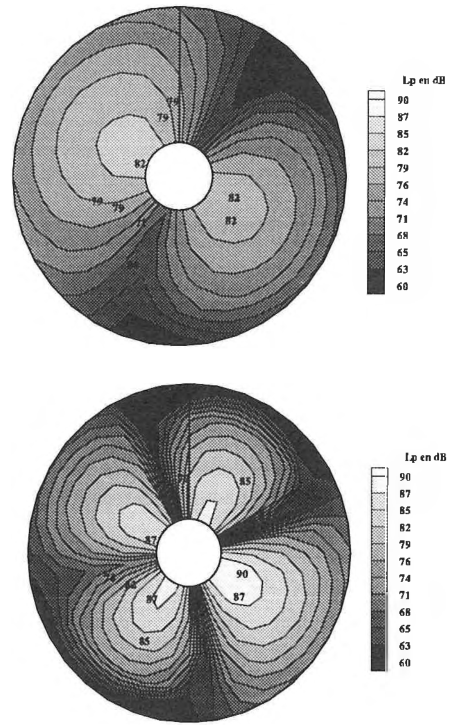

Fig 2 : Experimental maps of the sound field for 1000 Hz and 1100 Hz at 2,95 m of the noise source.

B- Experimental results. The fig 2 shows experimental maps of sound field for 1000 Hz and 1200 Hz. The modes propagate according to the table 1. In practice, the cut-on frequencies of modes do not really exist. We can say that the mode influence increase as the frequency is approaching its cut-on frequency.

III. Comparaison between theoretical and experimental results

It is important to note that when high modes propagate the sound field is the result of modes superposition as the equations (2) and (3) suppose it. Thus it is difficult to identify each mode independantly.
If we compare theoretical map (fig 1) and experimental map (fig 2), we can see two important differences :

i) the nodal lines are not exactly at the same place,

ii) the maximum and the minimum levels do not have the same repartition.

The zero modal impedance coefficients may be the cause of these differences. Further studies using non zero impedance coefficients with theory shown here will soon be conducted.

Conclusion

A model of the interior pressure field of finite length circular duct when high order modes propagate has been presented. The studied duct is closed at one end and baffled at its other open end. Theoritical and experimental results have shown important differencies for a zero modal impedance coefficient hypothesis. Theoretical cases with non zero modal impedance coefficients, calculated with the theory presented will now be computed.

[1] Zorumski W. E., *Generalized radiation of impedances coefficients of circular and annular ducts*, (1973) J. acoust. Soc. Am., vol 54 (6), p1667-1673.