Active control of acoustic radiation using strain sensors

Patrice Masson, Alain Berry and Jean Nicolas
GAUS, Université de Sherbrooke,
Sherbrooke, Qué, J1K 2R1

INTRODUCTION

Sensing approaches used in active structural acoustic control (ASAC) are mainly concerned with monitoring acceleration, velocity or displacement of the structure, typically by means of accelerometers [1] or PVDF [2], thereby eliminating the need for far-field acoustic sensor(s). The radiation of the structure is estimated from this information, either directly [3] or from a model [4] of the radiating structure. The direct approach has the advantage of not being dependent on the accuracy of any model.

A new strategy is herein presented, where the estimation of acoustic radiation involves monitoring the *strain field* of the structure at discrete points. Such an information is *directly* given by fiber optics strain sensors, for example, so that significant gain is expected at this level. Two approaches are presented using the second derivative of the displacement, or strain, information. The ASAC is performed in the wavenumber domain and the cost function is defined as the radiated acoustic power.

COST FUNCTION IN THE WAVENUMBER DOMAIN

The acoustic radiation from a simply supported rectangular plate excited by a point disturbance will be considered (see Figure 1). A piezoelectric patch (PZT) is used for the control.

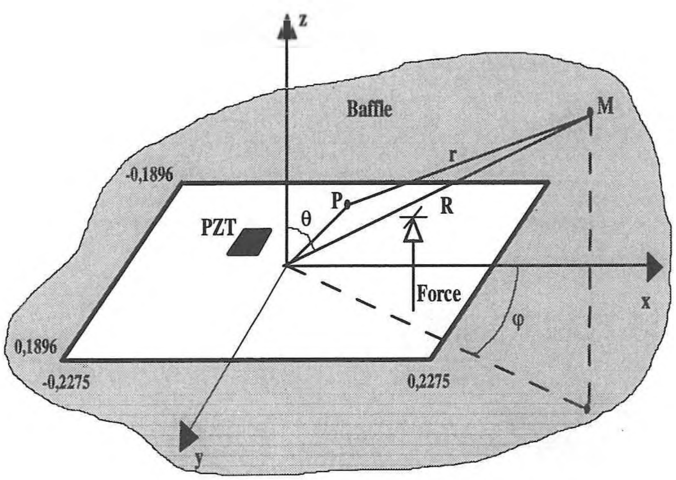

Figure 1 : Situation considered (dimensions in meters)

Assuming harmonic displacement, a far-field approximation to pressure is given by:

$$p(R,\theta,\varphi) = -\frac{\omega^2 \rho_o}{2\pi} \frac{e^{-ikR}}{R} \tilde{w}(\lambda,\mu) \tag{1}$$

where λ and μ are the x and y components of the structural wavenumber, ω is the angular frequency , k is the acoustic wavenumber, ρ_o is the density of the fluid and $\tilde{w}(\lambda,\mu)$ is the Fourier transform of the normal displacement field $w(x,y)$.

The resulting pressure at a point M is the sum of a primary pressure field p_p associated with the disturbance and a secondary pressure field p_s associated with the control:

$$p(R,\theta,\varphi) = p_p(R,\theta,\varphi) + V_s\, p_s(R,\theta,\varphi) \tag{2}$$

where V_s is the control voltage (complex) applied to the PZT.

The cost function is taken to be the radiated acoustic power Π which is given in the wavenumber domain by:

$$\Pi = \frac{1}{2\rho_o \omega}\left[\alpha_{pp^*} + V_s^* \alpha_{ps^*} + V_s\, \alpha_{p^*s} + |V_s|^2\, \alpha_{ss^*}\right] \tag{3}$$

where the α 's are obtained by integration in the supersonic region (A and B are associated with p and s):

$$\alpha_{AB^*} = \int_{-k}^{k} \int_{-\sqrt{k^2-\lambda^2}}^{\sqrt{k^2-\lambda^2}} \frac{\left(p_A\, p_B^*\right)}{\sqrt{k^2-\lambda^2-\mu^2}}\, d\mu \; d\lambda \tag{4}$$

PRESSURE FROM DISCRETE STRAIN INFORMATION

Two formulations are proposed in order to express the pressure in Eq. (4) in terms of the discrete strain information. The first formulation is obtained by integrating Eq. (1) twice by parts while the second utilizes a finite difference scheme to estimate the displacement field to be Fourier transformed in Eq. (1).

1. Integration by parts of Fourier transform

Integrating twice by parts the Fourier transform in Eq.(1) gives:

$$\tilde{w}(\lambda,\mu) = \sum_{i=1}^{2} I_i - \frac{1}{\lambda^2}\, \mathcal{FT}\left[\frac{\partial^2 w(x,y)}{\partial x^2}\right] \tag{5}$$

where I_i is a boundary term associated with an edge i parallel to the y axis and $\mathcal{FT}$ denotes the discrete Fourier transform. The displacement and the rotation at the corners, as well as the second derivatives of the displacement may appear in the boundary terms. However, depending on the edge conditions, some of these terms can be simplified or expressed in terms of strain measurements along the edges.

2. Finite differences approach

Using a central finite difference scheme, the displacement at discrete $p \times q$ points (separated in the x direction by a distance h) on the plate can be *reconstructed* from the strain measurements by:

$$\frac{1}{h^2}\begin{bmatrix} -2h & -2 & 2 & & & \\ & 1 & -2 & 1 & & \\ & & & \ddots & & \\ & & & 2 & -2 & 2h \end{bmatrix} \begin{bmatrix} w'_{1,1} & \cdots & w'_{1,q} \\ w_{1,1} & \cdots & w_{1,q} \\ \vdots & & \vdots \\ w_{p,1} & \cdots & w_{p,q} \\ w'_{p,1} & \cdots & w'_{p,q}\end{bmatrix} = \begin{bmatrix} w''_{1,1} & \cdots & w''_{1,q} \\ \vdots & & \vdots \\ w''_{p,1} & \cdots & w''_{p,q}\end{bmatrix} \qquad (6)$$

This linear system can be solved if two conditions on w or w' are given as boundary conditions. The corresponding system is then obtained:

$$[A_{p,p}][w_{p,q}] = [w''_{p,q}] - [A_{p,2}][w_{2,q}] \qquad (7)$$

SIMULATION RESULTS

A first series of simulations was performed to validate the radiated sound power calculation with respect to the approach used. Figure 2 presents the radiated acoustic power as obtained with the two approaches, using a 16 × 16 array of sensors, and how it compares with a reference analytic curve (EOLE). The finite differences approach seems better on the whole range, up to 1300 Hz. Other results with 6 × 6 sensors showed good agreement up to 500 Hz with the finite differences approach.

Optimal control simulations were also conducted to assess the performance of our control scheme. Figure 3 presents the effect of active control on the radiated power using the finite differences approach and one 0,06 m × 0,04 m PZT located at $x = -0{,}08$ m, $y = 0{,}07$ m. As a result of the control, the reduction of the amplitudes in the supersonic region of the wavenumber spectrum is presented in Figure 4.

CONCLUSION

Strain sensors showed to be both simple and effective to evaluate the radiated sound power. For a simply supported plate, with 16 × 16 strain sensors, the finite differences approach allows good agreement with the analytic reference up to 1300 Hz. Both approaches were validated within optimal control situation and good performance was obtained. Future work includes the validation for other boundary conditions and the use of more than one actuator. The use of cubic splines as an alternative to finite differences will be investigated.

ACKNOWLEDGMENT

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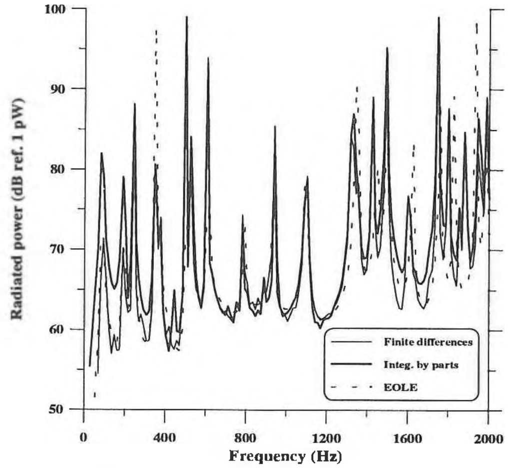

Figure 2 : Comparison of the two approaches

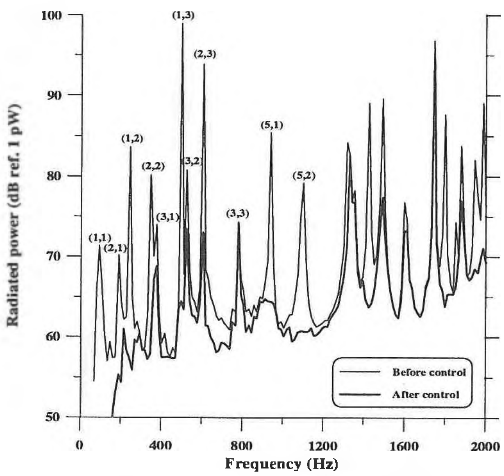

Figure 3 : Effect of active control, 16 × 16 strain sensors

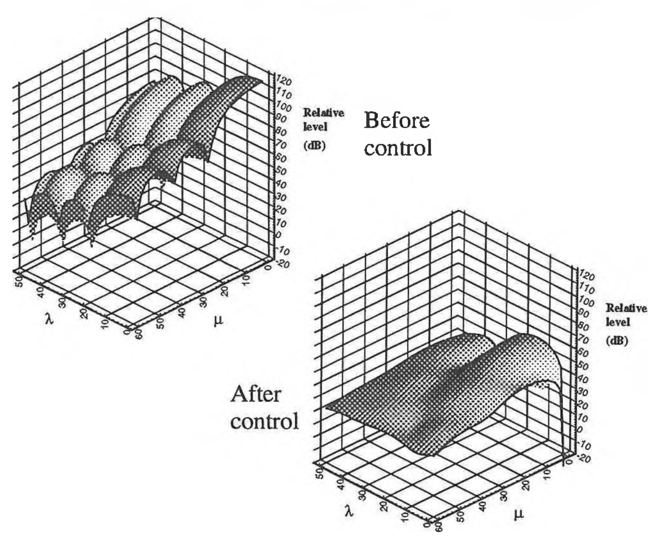

Figure 4 : Effect of control on the wavenumber spectrum, mode (1,1)